An Incoherence in the Tractatus

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1. After he had rejected much of the Tractatus, Wittgenstein, upon at least one occasion, still “thought that in the Tractatus he had provided a perfected account of a view that is the only alternative to the viewpoint of his later work”—a perfected account: that is to say (at least) a well-knit, coherent one. It seems to me that this merit must be denied the whole account presented in the Tractatus and I would like to explain why.

The Tractatus holds that every true or false proposition is analyzable as a truth-functional compound of elementary propositions. It further holds that elementary propositions are completely independent of one another. “The simplest kind of proposition, an elementary proposition, asserts the existence of a state of affairs” (4.21). “States of affairs are independent of one another. From the existence of one state of affairs it is impossible to infer the existence or non-existence of another” (2.061-2.062). Thus no elementary proposition implies the truth or falsity of any other elementary proposition. What is more, no consistent conjunction of elementary propositions and negations of elementary propositions can logically imply the truth or falsity of any other elementary proposition; for, as we are told in 4.27-4.28, given any \( n \) different elementary propositions any of the \( 2^n \) different assignments of truth-values to those \( n \) propositions is logically possible.

The requirement of the complete independence of the elementary propositions is troublesome. “It is extremely unlikely that it can be satisfied by the semantic structure of any natural language or, indeed, any possible language capable of describing reality as fully as a natural language. It is not surprising that this requirement seems to have been one of the first points in the Tractatus that Wittgenstein rejected."

But it is not only externally that the independence of the elementary propositions gives trouble. It is also incompatible with the

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2 Quotations from the Tractatus are from the Pears-McGuinness translation (London, 1961).
picture theory of propositions which is, perhaps, the most central and interesting feature of the Tractatus account of how language (thought) is able to represent reality. They are incompatible because the picture theory requires that whatever renders something a true or false picture of how things are related to one another must involve the possibility of co-ordinate alternative pictures of incompatible ways that things might be related. This point is not explicitly stated in the Tractatus but it is, as I shall try to bring out (sec. 3), implicit in what the Tractatus says about picturing. Before that (sec. 2), a little reflection on simple examples of truth-valued pictures will make the point clear; and there can be no doubt that Wittgenstein wished to rely on the term picture itself to carry considerable explanatory weight, to call upon our common understanding of picturing to show how a proposition's being a picture explains how it can be true or false of reality.

2. As good an example as any for seeing the point is the one that Wittgenstein uses to introduce the theory in the pre-Tractatus Notebooks. Consider the use, in a courtroom, say, of toy cars and lines on a surface to depict how certain actual cars were disposed on the street at the moment of their colliding. One who views a depicting model of this sort and understands how it shows how things actually were in the accident must also understand how a different arrangement of the toy cars and lines would show things wrong. To be competent to see that one arrangement is a true picture one must know what alterations in the arrangement would make it a false picture. Our arrangement cannot be a true picture unless some alternative arrangements are false ones. To make the right arrangement to correspond to the facts is to select one arrangement from various alternative, competing possibilities: it is to get it right instead of wrong. Likewise, one who knows how things actually were in the accident cannot grasp that a putative model of the facts is wrong unless he knows what changes in it would make it right. An eyewitness who claims to know how things actually were and claims that the plaintiff’s model is incorrect thereby claims to know how to put the model right. If he is quite baffled as to how to do so then it must be inferred that he either fails to know how things actually were or fails to understand how this method of picturing works and is in no position to judge any particular product of it as true or false of what he knows. He cannot know what a given arrangement of the depicting elements shows, truly or falsely, until he knows what are significant alternative arrangements and how to compare reality with them so as to see which one is correct. This point remains just as clear when it is generalized to all cases of putting together and arranging one set of things in order to show, truly or falsely, how another set of things is arranged, i.e., to all truth-valued picturing.

This understanding which makes a perceived combination of elements into a truth-valued picture and which must include understanding what kinds of variations in the arrangement or structuring of the elements can change the truth-value, is simply a necessary part of understanding what the significant or depicting features of the picture-fact are: seeing this is seeing what the Tractatus calls the “logico-pictorial form” that a picture-fact must have in common with what it truly or falsely depicts. A method for putting together certain sorts of elements in order to represent how things might be (in certain respects) must be a method for producing competing possible representations of how things might be. In order to produce any representation of how things are, either a correct or an incorrect one, the method must be able to produce both correct and incorrect representations. But if this is so, and if (as the Tractatus maintains) the elementary propositions (like all others) are pictures, then it follows that the method of representation that produces any elementary proposition must also be able to produce a proposition that is incompatible with it. That is, there must be a different combination of simple names (which are the elements of the picturing method at the elementary level), a different elementary proposition, that presents an incompatible possible structure of the objects (the simple elements of reality).

3. Let us now see how this conclusion can be reached from what is said in the Tractatus (supplemented by some remarks in the pre-Tractatus Notebooks). 2.16 says: “If a fact is to be a picture, it must have something in common with what it depicts.” And 2.18: “What any picture, of whatever form, must have in common with reality in order to be able to depict it—correctly or incorrectly—in any way at all, is logical form, i.e., the form of reality.” Although Wittgenstein here ascribes logical form (also called pictorial form in 2.17) to pictures, it is clear from much else in the Tractatus that he thinks of logical form as primarily a property, not of facts or structures (whether depicting or depicted), but of the elements of the structures (see, for example, 2.032 where it is said that the simple objects constitute the unalterable form of the world). Further, it is clear that he thinks of the logical form of these elements as consisting, not in any actual combinations or structures they enter into (or in any fact that could have been otherwise), but in their possibilities of combination. (For example, in 2.15 the possibility of structure, explicitly distinguished from actual structure, is called the pictorial form of the picture.) Whatever the actual structures of its elements reality chooses to produce, the possible structures, from which the choice is made, are absolutely fixed and unalterable; these possibilities of structure are internal or necessary properties of the elements that are common to all the possible structures. Thus it is the elements of a picture that share logical form with the elements of the reality truly or falsely depicted.

A necessary condition of two sets of elements’ having the same logical form is that they be the same in number, be in a one-to-one correspondence (4.04). But, to have the same logical form, two sets of elements in one-to-one correspondence must also have the same possibilities of combination into structures. Furthermore, these two sets
of possible structures of the elements must also correspond one-to-one and must exhibit the same system; that is, corresponding structures must be derivable from one another by corresponding changes of the corresponding elements. It is this that makes it possible to compare the picture with reality (to "lay it against reality" (2.15.2)) in such a way as to see whether the picture is true or false. What one really compares are the whole systems of possible structures implicit in the picture and in the elements of reality that the picture picks out. One sees it, one sees that the two systems of possible structures and then sees whether the actualized structures on the two sides "touch" or coincide, i.e., whether the actuality on the one side occupies the same place in the common pattern of possibilities as the actuality on the other side. Thus the logical form that a picture must "share" with what it truly-or-falsely depicts is not a feature of just that particular picture but rather of the understood system or method for forming various depicting combinations of the picture-elements (of which the particular picture is just one possible product).³

The identity of logical form between picture and depicted requires that for every possibility on either side there is a corresponding possibility on the other side. And this means that whatever the actuality on the one side there is at least a corresponding possibility on the other side. Whatever actual structure of picture-elements is produced in accordance with the depicting system, there must be a corresponding possible structure of the reality-elements; and also whatever the actual structure of the reality elements the depicting method must yield a corresponding possible structure of the picture elements. The essential internal relation between picture and what it truly-or-falsely depicts, created by the method of representation, is a symmetrical relation. This relation is explicated in 2.151, by saying that, given any true or false picture, the elements in what is depicted "can be related to one another as are the [corresponding] elements of the picture." If this is so, and this relation is symmetrical, then it must also be that, given any reality, the elements that depict can, by the same method of representation, be related to one another as are the corresponding elements of reality. A picture's being false, says 2.21, is its failing to agree (or coincide) with reality. This failure to agree (or coincide) is surely a relation such that if its term on the picture side is a structure of elements then its term on the side of reality must be a structure of elements (one that occupies other than the coincident place in the common system or form composed of alternative possibilities of structure). If the actual elementary picture and the actual structure of objects in reality do not agree—the elementary proposition is false—it must be possible to bring them into agreement, to produce coinciding structures, by a change on either side alone. And this means that it must be possible, using the same elementary method of representation to produce another picture that

³ Cf. Notebooks, p. 19: "sign and method of symbolizing together must be logically identical with what is signified"; and p. 21: "what represents is not merely the sign or picture but also the method of representation."

will present the truth about the same reality. If an elementary proposition falsely depicts the arrangement of the objects named in it, then there must be another, incompatible elementary proposition that presents what instead actually is the arrangement of those objects.

4. Now someone might grant these implications of the Tractatus' picture theory and yet try to reconcile them with the independence of the elementary propositions. For the following he might say that for each possible combination of objects pictured by an elementary proposition there is just one alternative (incompatible) possibility, namely the non-obtaining of that combination of those objects, and this possibility is pictured by the negation of that elementary proposition: the negation represents the falling apart, as it were, of the structure pictured by the negated proposition. Thus logical form at the elementary level—the level of the system of possible relations among simple objects—divides throughout into independent pairs of incompatible possibilities, each pair consisting of the obtaining and the non-obtaining of a certain combination of objects. This form is mirrored on the picturing side by the pairs of the elementary propositions and their negations. Does this not meet the demand that any picturing method must provide for alternative incompatible pictures while retaining the independence of the elementary propositions?

The difficulty with this suggestion is that it puts the elementary proposition and its negation too much on a par. It makes them coordinate depictions, at the same logical level, and thus gives up all reason for saying that the positive one is the elementary one. We may be able to distinguish them as positive and negative, but there will be no reason to regard one as elementary and the other not.

The Tractatus clearly does not regard the negation of an elementary proposition as itself elementary, although it does think of the negative proposition as depicting the non-existence of the possible structure of objects depicted by the negated proposition. But it does not regard the negative proposition as depicting by presenting a coordinate alternative arrangement of elements of the same picturing method used in the negated proposition. The two propositions do not use the same method of representation at all. In the sense in which an elementary proposition presents or shows the possibility whose existence would make it true, the negation of an elementary proposition (or any other non-elementary proposition) does not present the possibility whose obtaining would make it true.⁴ 4.0312 says: "The possibility of propositions is based on the principle that objects have signs as their representatives . . . the 'logical constants' are not representatives" (my emphasis). This means that propositions formed by combining logical constants (truth-functional connectives) with elementary propositions, i.e., non-elementary or molecular propositions, do not represent or depict in the same fundamental, direct way in which elementary pro-

⁴ Cf. Max Black's comment in A Companion to Wittgenstein's Tractatus (Ithaca, New York, 1964) p. 111: "The sense on which a general proposition is a 'logical picture' can hardly be exactly the same as the sense in which an elementary proposition is one."
position, i.e., combinations of simple names, do. When one introduces negation and the other truth-functional constants one introduces a different logical level of representation, not coordinate with representation at the level of simple names and their combinations. There are two levels of pictorial or logical form. At the atomic level the elements are the names of simple objects; the possible structures of the elements are the elementary propositions, the system of which constitutes atomic logical form. At the molecular level the elements are the elementary propositions; and the possible structures of these elements are the truth-functionally compound propositions, the system of which constitutes molecular logical form. (Just as a name has no atomic structure, an elementary proposition has no molecular structure; the simplest proposition possessing molecular structure and depicting molecularly is the negation of an elementary proposition).

A proposition pictures atomically one or more possible structures of objects (states of affairs, Sachverhalten). A proposition pictures molecularly a certain possible pattern of existence and non-existence of the atomically pictured possibilities (termed a schlage in logical space in 2.11). Molecularly, a proposition draws boundaries in logical space—i.e., among the atomic possibilities which are the logical places—to show where "filling" (existence) will or will not be found. It presents a molecular shape into which reality's actual configuration of existence and non-existence of atomic states of affairs may or may not fit. A molecular picture can be such that reality may fit it, make it true, in any of several alternative ways; whereas reality can agree with an atomic picture in just one way. A molecular picture can be indeterminate but an atomic picture cannot.  

In an elementary proposition and in its negation there are pictured atomically exactly the same thing, a certain possible structure of objects; but there are pictured molecularly different and opposite things, the existence and the non-existence of that possible structure. The negation of an elementary proposition, in representing molecularly the non-existence of an atomic possibility, represents indeterminately. Its truth requires just that the objects named in the negated elementary proposition be arranged in some other way than the one pictured in the negated proposition, but it does not show how: it says that reality coincides with somewhere else in the associated atomic system of alternative possible structures, than the place which the negated proposition displays, but it does not say where. This view of the matter is strongly suggested by 4.0641: "One could say that negation must be related to the logical place determinated by the

negated proposition . . . The negating proposition determines a logical place with the help of the logical place of the negated proposition. For it describes it as lying outside the latter's logical place." And that is all that it describes it as. Each atomic possibility is a definite place in a logical space of competing possible structures of objects. Each place presupposes or carries within it the whole space; it is essentially a place in that space (a possibility in that system of possibilities). (Cf. 3.42(3): "The logical scaffolding surrounding a picture determines logical space. A proposition reaches through the whole of logical space.") Each place determines a twofold division within its space: it and the surrounding space. The elementary proposition points determinately to the place and its negation points indeterminately to the surrounding space. But within that surrounding space there are definite places and these may be pointed to determinately by other elementary propositions: The elementary proposition cannot be independent. As Wittgenstein put it in 1929 (Philosophische Bemerkungen, pp. 110-11), they must form "coordinates of description" for the elements of reality, i.e., systems of alternative possible structures.

Incidentally, there is a statement in the Tractatus which is, all by itself, incompatible with the doctrine of the complete independence of the elementary propositions, viz., 2.05: "The totality of existing states of affairs also determines which states of affairs do or do not occur.

"This incompatibility, while striking enough by itself, becomes considerably more interesting when it is realized that 2.05 is not an isolated dictum but is implied by the centrally important picture theory. Max Black has pointed out (Companion, p. 72) that in virtue of 2.05 the Tractatus is committed to the view that one will, necessarily, tell the whole truth about reality if one merely asserts all the true elementary propositions. This would not be so if the elementary propositions were completely independent of one another; for then the true negation of an elementary proposition would give independent information not implied by any conjunction of true elementary propositions. This interesting tenet, to affirm all the true elementary propositions is to tell the whole truth about reality, is a consequence not merely of 2.05 but of the picture theory, as I have made it out; for according to it reality can fail to coincide with, and thus render false, a particular elementary picture only by exhibiting an alternative structure of objects that does coincide with and render true an alternative incompatible elementary picture; if one sees that the one picture is true, one must thereby see that the other is false through one's grasp of the whole elementary picturing method, i.e., of logical form at the atomic level.

5. The Tractatus could be coherent and contain the central idea of propositions as pictures only if the doctrine of the independence of the elementary propositions were purged, and, as far as I can see, the purge of that doctrine would do away with nothing of real interest and importance. An interesting and important principle may, in a sense, lie behind the assertion of the independence of the elementary pro-
positions, namely, the principle stated in 6.113 that a proposition is a logical truth if and only if it can be seen to be true from the symbol alone. It may be that this principle together with a desire for economy in his explanations led Wittgenstein to fasten on tautological truth-functional structure as the way in which a propositional symbol could show that it could not possibly disagree with reality. But this over-simplicity could be given up without abandoning the basic principle. It would only be necessary to recognize that aspects of a propositional symbol other than its molecular structure can contribute to its being unable to disagree with reality, namely, features of its atomic structures. For example, it could be part of grasping what certain elementary propositions “aRb” and “bRa” represent that one understand that the alternative incompatible arrangements of “a” and “b” in the two propositional signs signify alternative incompatible possible arrangements of a and b (i.e., R is an anti-symmetrical relation). Syntactical convention could, of course, dictate that the incompatibility in the two propositional signs does not symbolize incompatibility but only difference in the possible arrangements of objects that the two signs symbolize (i.e., R is asymmetrical), or even that it signifies nothing at all (R is symmetrical). But in any case, knowing what this difference in elementary propositional signs signifies is part of knowing the atomic logical form of the propositional elements as employed in the given system of representation. It is, after all, only a convention that determines that the perceptible signs “p” and “¬p” present competing molecular structures (and determines which is positive and which negative (see 4.0520)). If, in understanding what and how the elementary “aRb” and “bRa” symbolize, in grasping their logical form, we see that the one structure excludes the other—that the two propositions cannot both be true—then we see from the symbol “¬(aRb & bRa)” alone (if we understand negation and conjunction as well) that it cannot but be true.

The Tractatus allows that some interesting formal properties of reality show themselves in elementary propositions: for instance, that they present different sets of objects having the same logical form and the same structure (2.0233), or that the possible structures they present have objects in common (4.1211). It is not a huge step from this sort of thing to allowing that one might see that two elementary propositions present incompatible possible structures of objects. Indeed, the more one thinks about it the more astonishing it becomes that the Tractatus should have held (without any explanation that I can see) that logical truth is a matter entirely of molecular (truth-functional) form, that logical form at the elementary level can have nothing to do with it.

If the Tractatus were to abandon the independence of the elementary propositions it would not thereby abandon its atomicity, its conception that reality is a mosaic of independent atoms of fact. This is certainly a characteristic and important feature of the Tractatus' metaphysical view, without which it would not merit the label "logical atomism" that is usually given it. But the independence of atomic

facts is not in any way denied by the denial that atomic propositions are completely independent. That every elementary proposition should be incompatible with other elementary propositions is perfectly compatible with its being the case that every true elementary proposition is independent of, neither implying nor implied by, every other true elementary proposition. Given the principle that every proposition is a truth-function of elementary propositions, the principle that all compatible elementary propositions are also independent is enough to ensure the atomicity of the Tractatus ontology; and this principle is quite consistent with what we have found to be implicit in the picture theory, namely, that the elementary propositions must fall into sets the members of each of which are mutually incompatible.

6. Norman Malcolm reports part of Wittgenstein's Philosophische Bemerkungen (1929-30) as follows:

No longer is it thought that a single sentence is compared with reality, but rather a whole system. Now one might believe that the system is a picture. Wittgenstein does indeed say, in one place, that the sentences which compose a coordinate of description "form one picture."

Malcolm goes on to comment: "perhaps this is a tenable position. But surely the picture conception is losing its charm." Not at all. I would rather say that in the passages on which Malcolm here comments Wittgenstein more fully elucidates the picture theory in all its subtle charm. The subsequent course of Wittgenstein's thought makes it natural to look upon his rejection in 1929 of the independence of the elementary propositions as the beginning of his destruction of the Tractatus. I hope that I have shown that this was not because his rejecting that doctrine pulled out an essential prop in the Tractatus system. Rather it could equally well be viewed as a last effort to perfect the system, to make it cohere around its most important ideas.9

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