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Procrastination on Long-Term Projects

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Abstract

Previous papers on time-inconsistent procrastination assume projects are completed once begun. We develop a model in which a person chooses whether and when to complete each stage of a long-term project. In addition to procrastination in starting a project, a naive person might undertake costly effort to begin a project but then never complete it. When the costs of completing different stages are more unequal, procrastination is more likely, and it is when later stages are more costly that people start but don't finish projects. Moreover, if the structure of costs over the course of a project is endogenous, people are prone to choose cost structures that lead them to start but not finish projects. We also consider several extensions of the model that further illustrate how people may incur costs on projects they never complete.

JEL Category: A12, B49, C70, D11, D91.

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1. Introduction

There is a growing literature in economics that explores the implications of self-control problems, conceived of as a time-inconsistent taste for immediate gratification. Such preferences can give rise to procrastination.[1] Existing research on procrastination assumes that a "project" requires only a single period of effort, and is completed once begun.[2] But most real-world projects, in contrast, take some duration to complete, involve effort costs that vary for different stages of the project, and can be abandoned after begun.

In this paper, we develop and analyze a simple model of long-term projects. In this environment, a person might not only procrastinate in starting a beneficial project, she might also start a project but then procrastinate in finishing it. We describe how the structure of costs over the course of a project plays an important role in whether and how a person procrastinates. Moreover, we show that if the cost structure is endogenous, naive procrastinators are prone to choose precisely the types of cost structure that make it most likely that they start but do not finish projects, and in some environments may incur large repeated costs on never-to-be-completed projects. Hence, in addition to all the rational reasons people may delay finishing long-term projects or jettison a project after putting in considerable effort, our analysis describes how and when *inefficient* delay and non-completion of projects can arise because of the human tendency to pursue immediate gratification.

In Section 2, we describe a formalization of time-inconsistent preferences originally developed by Phelps and Pollak (1968) in the context of intergenerational altruism, and later employed by Laibson (1994,1997) to capture self-control problems within an individual: In addition to time-consistent discounting, a person always gives extra weight to current well-being over future well-being. These "present-biased preferences" imply that each period a person tends to pursue immediate gratification more than she would have preferred if asked in any prior period. An important issue arises when a person has such self-control problems: How aware is the person of her future self-control problems? The results in this paper, as in previous papers, demonstrate the role that *naivete* — underestimation of future self-control problems — plays in procrastination. To emphasize this role, our analysis compares four types of people: *TCs* have standard time-consistent preferences; *sophisticates* have self-control problems and are fully aware of those problems; *naifs* have self-control problems and are fully *un*aware of those problems, and *partial naifs* have self-

[1] For recent papers discussing procrastination, see, for instance, Prelec (1989), Akerlof (1991), Fischer (1999), and O'Donoghue and Rabin (1999*a*, 1999*b*, 2001).

[2] A notable exception is Fischer (1999), who studies the behavior of sophisticates who must spend a fixed amount of time on a project before a deadline.

control problems and are aware of those self-control problems, but underestimate their magnitude.

In Section 3, we present our model of long-term projects. We assume for simplicity that a project has two stages, which we often refer to as "starting" and "finishing" the project. There is an infinite number of periods in which the person can work on the project. In each period the person can either complete the current stage, in which case she incurs an immediate cost associated with that stage, or she can do nothing. Completion of the first stage does not generate any benefits, but when (and if) the second stage is completed, an infinite stream of benefits begins the following period.

In Section 3, we take the structure of costs over the course of the project to be exogenous. In this environment, TCs immediately start and then finish the project if and only if the project is worth doing in terms of its net discounted present value. Otherwise, they never start the project. People with self-control problems likewise won't start projects that are not worth doing. But they might also delay on worthwhile projects.

People who are to some degree sophisticated about their self-control problems might not start a project because they expect that, once the second-stage cost becomes immediate, they will no longer deem it worthwhile to finish the project. Because they fully anticipate any such shift in desire, complete sophisticates, like TCs, never begin a project without completing it. Moreover, while they might delay a short while before completing projects, sophisticates complete the project in much the same circumstances as TCs: When their taste for immediate gratification is relatively small, the discounted benefits need only be a little larger than the discounted costs to guarantee that sophisticates complete the project.

Our main concern is a second source of delay for people with self-control problems: procrastination. People who are to any extent naive about their self-control problems may persistently *plan* to work on the project in the near future, but perpetually put off this work. As concluded in many previous analyses of procrastination, this means a person might never start a worthwhile project. But with long-term projects, a potentially more costly form of procrastination is possible: A person might start a project that she expects to finish, but then never finish. In this case, the cost of procrastination is not just the foregone benefits from a valuable project, but also the wasted effort incurred in working on a project that never produces any benefits. Indeed, we show that such wasted-effort costs can be substantial.

We also show in Section 3 that whether and how a person procrastinates depends crucially on the structure of costs over the course of the project. Procrastination is caused by a desire to put off incurring an immediate cost. The larger the cost, the stronger the urge to delay it. It is therefore the highest-cost stage on which people are most prone to procrastinate. Hence, for a fixed total cost, procrastination is least likely when costs are allocated evenly across stages, because this allocation minimizes the cost of the highest-cost stage. Moreover, when the allocation is uneven, the *order* of costs is important. When a project is difficult to start but easy to finish, a procrastinator is prone not to start. When a project is easy to start but hard to finish, a procrastinator is prone to start but not finish — and therefore incur costs without ever getting any benefits.

Our analysis in Section 3 shows how the structure of costs over the course of the project is an important determinant of procrastination. In Section 4, we endogenize this cost structure, allowing the person to choose an uneven allocation with a disproportionate share allocated to stage 1, or an uneven allocation with a disproportionate share allocated to stage 2, or anything in between including an even allocation. If, for instance, a person must put in a total of 12 hours of effort but cannot work for more than 8 hours on any given day, then she must (plan to) put in 4 to 8 hours of effort on each of two days in some combination that totals 12 hours. We show that, because the same preference for immediate gratification that leads a person to procrastinate also leads her to prefer deferring as much cost as possible to stage 2, the person is prone to choose a cost structure that maximizes the likelihood that she will start the project but not finish it.

In Section 5, we consider some extensions of our model to richer environments that further illustrate the possibility of naive people incurring costs without ever receiving benefits. Section 6 concludes.

2. Present-Biased Preferences

The standard economics model assumes that intertemporal preferences are *time-consistent*: A person's relative preference for well-being at an earlier date over a later date is the same no matter when she is asked. But there is a mass of evidence that intertemporal preferences take on a specific form of *time inconsistency*: A person's relative preference for well-being at an earlier date over a later date gets stronger as the earlier date gets closer. In other words, people have self-control problems caused by a tendency to pursue immediate gratification in a way that their "long-run selves" do not appreciate.[3]

[3] See, for instance, Ainslie (1975, 1991, 1992), Ainslie and Haslam (1992*a*, 1992*b*), Loewenstein and Prelec (1992), Thaler (1991), and Thaler and Loewenstein (1992). For a recent overview, see Frederick, Loewenstein, and O'Donoghue (2002). While the rubric of "hyperbolic discounting" is often used to describe such preferences, the qualitative feature of the time inconsistency is more general, and more generally supported by empirical evidence, than the specific hyperbolic functional form.

In this paper, we apply a simple form of such *present-biased preferences*, using a model originally developed by Phelps and Pollak (1968) in the context of intergenerational altruism and later used by Laibson (1994,1997) to model time inconsistency within an individual. Let u_t be the instantaneous utility a person gets in period t. Then her intertemporal preferences at time t, U^t, can be represented by the following utility function:

$$U^t(u_t, u_{t+1}, ..., u_T) \equiv \delta^t u_t + \beta \sum_{\tau=t+1}^{T} \delta^\tau u_\tau.$$

This two-parameter model is a simple modification of the standard one-parameter, exponential-discounting model. The parameter δ represents standard "time-consistent" impatience, whereas the parameter β represents a time-inconsistent preference for immediate gratification. For $\beta = 1$, these preferences are time-consistent. But for $\beta < 1$, at any given moment the person has an extra bias for now over the future.[4]

In this formulation, β represents a measure of the person's present bias or preference for immediate gratification. We often refer to β as representing a "self-control problem" because we interpret the preference for immediate gratification as being an "error" — it is a short-term feeling that the person disagrees with at every other moment in her life. While our interpretation motivates our language, it is not important that the reader agree with our interpretation.[5]

To examine intertemporal choice given time-inconsistent preferences, one must ask what a person believes about her own future behavior. Most of the literature has focused on two extreme assumptions: *Sophisticated* people are fully aware of their future self-control problems and therefore correctly predict how their future selves will behave, and *naive* people are fully *un*aware of their future self-control problems and therefore believe their future selves will behave exactly as they currently would like them to behave.[6] Recently (O'Donoghue and Rabin (2001)), we have

[4] This model has since been used by numerous authors, including Laibson (1998), Laibson, Repetto, and Tobacman (1998), Angeletos, Laibson, Repetto, Tobacman and Weinberg (2001), O'Donoghue and Rabin (1999*a*, 1999*b*, 2001), Fischer (1999), Carrillo and Mariotti (2000), and Benabou and Tirole (2000).

[5] While one's interpretation of β might influence one's belief about the proper welfare criterion, many welfare results would hold under essentially any reasonable welfare criterion. In this paper, we do not present any formal welfare results, although such results will be implicit. For formal welfare results in the realm of procrastination, see O'Donoghue and Rabin (1999*a*,2001).

[6] Strotz (1956) and Pollak (1968) carefully lay out these two assumptions (and develop the labels), but do not much consider the implications of assuming one versus the other. Most researchers assume sophisticated beliefs — e.g., Laibson (1994,1997,1998), Laibson, Repetto, and Tobacman (1998), Angeletos et al (2001), Fischer (1999), Carrillo and Mariotti (2000), and Benabou and Tirole (2000). O'Donoghue and Rabin (1999*a*) consider both, and explicitly contrast the two.

formulated an approach to the more realistic assumption of *partial naivete* wherein a person is aware that she will have future self-control problems but underestimates their magnitude. We suppose that a person has true self-control problem β, but perceives that in the future she will have self-control problem $\hat{\beta}$. In other words, in any given period the person's current preferences are characterized by β, but she perceives that in the future she will behave like a sophisticated person with preferences characterized by $\hat{\beta}$. With this formulation, people with standard time-consistent preferences — whom we refer to as TCs — have $\beta = \hat{\beta} = 1$, sophisticates have $\beta = \hat{\beta} < 1$, naifs have $\beta < \hat{\beta} = 1$, and partial naifs have $\beta < \hat{\beta} < 1$.

Our focus in this paper is on how naivete about future self-control problems can lead to procrastination on long-term projects. Much of our analysis will focus on people who are completely naive, for whom our results are strongest. But since the intuitions we identify apply even for people who are only partially naive, we also derive results for partial naifs. In the next section, we shall define a formal solution concept — that applies to sophisticates, naifs, partial naifs, and TCs — within our specific model.

3. Model with Exogenous Cost Structure

For most of our analysis, we focus for simplicity on two-stage projects; we discuss in Section 6 how our lessons extend to longer projects. A *long-term project* consists of two stages, and completing each stage is onerous in the sense that completing it requires that the person incur an immediate cost. In this section, we assume that the cost structure is exogenous, where the first stage requires cost $c > 0$ and the second stage requires cost $k > 0$. We endogenize this structure in Section 4.

A person carries out a long-term project because of the future benefits it creates. We assume that the person must complete both stages before she can reap any benefits; we discuss in Section 6 how our results extend to the case where some of the benefits start accruing upon partial completion. More precisely, we assume that completion of stage 2 in period τ initiates a stream of benefits $v \geq 0$ in each period from $\tau + 1$ onward.[7]

There is an infinite number of periods in which the person can work on the project, and in each period the person can take one of two actions: She can complete the current stage or do

[7] Hence, our formal assumption is that if the person completes stage 1 in period a and stage 2 in period $b > a$, then her instantaneous utilities are $u_a = -c$, $u_b = -k$, $u_\tau = v$ for all $\tau \in \{b+1, b+2, ...\}$, and $u_\tau = 0$ otherwise. The crucial feature of a procrastinatory environment is that costs are immediate whereas benefits are delayed.

nothing. Hence, in any period before which the person has not yet completed anything, she can choose either to do nothing or to complete the first stage; and in any period before which she has completed the first stage, she can choose either to do nothing or to complete the project.

Our solution concept, "perception-perfect strategies", requires that at all times a person have reasonable beliefs about how she would behave in the future following any possible current action, and that she choose her current action to maximize her current preferences given these beliefs. Perception-perfect strategies depend on the two attributes of a person discussed in Section 2 — her self-control problem β, and her perceptions of future self-control problems $\hat{\beta}$. We now define a formal solution concept within our specific model.

Let $A \equiv \{0,1\}$ be the set of *actions* available in each period, where $a = 0$ means "do nothing" and $a = 1$ means "complete the current stage". Let $h^t \in \{\emptyset, 1, 2, ..., t-1\}$ be a *history* in period t, where $h^t = \emptyset$ means the person has not completed stage 1 prior to period t, and $h^t = \tau \in \{1, 2, ..., t-1\}$ means the person completed stage 1 in period τ. A *strategy* is a function s such that if the history in period t is h^t, then strategy s specifies action $s(h^t, t) \in \{0,1\}$. In the usual game-theoretic sense, a strategy is a plan for what to do in all possible contingencies; but we shall use strategies to represent both a person's true behavior and her beliefs about future behavior, which may differ when $\hat{\beta} \neq \beta$.[8]

Let $V^t(a_t; h^t, s, \beta)$ represent the person's period-t preferences over current actions given history h^t and conditional on following strategy s beginning in period $t+1$. Then:

$$V^t(a_t; h^t, s, \beta) \equiv \begin{cases} -c + \beta\delta^d\left(-k + \frac{\delta v}{1-\delta}\right) & \text{if } h^t = \emptyset,\ a_t = 1, \text{ and} \\ & \quad d \equiv \min\{x > 0 | s(t, t+x) = 1\} \\[2ex] \beta\delta^d\left[-c + \delta^{d'}\left(-k + \frac{\delta v}{1-\delta}\right)\right] & \text{if } h^t = \emptyset,\ a_t = 0, \\ & \quad d \equiv \min\{x > 0 | s(\emptyset, t+x) = 1\}, \text{ and} \\ & \quad d' \equiv \min\{x > 0 | s(t+d, t+d+x) = 1\} \\[2ex] -k + \frac{\beta\delta v}{1-\delta} & \text{if } h^t = \tau \neq \emptyset, \text{ and } a_t = 1 \\[2ex] \beta\delta^d\left(-k + \frac{\delta v}{1-\delta}\right) & \text{if } h^t = \tau \neq \emptyset,\ a_t = 0, \text{ and} \\ & \quad d \equiv \min\{x > 0 | s(\tau, t+x) = 1\}. \end{cases}$$

The four cases in this equation correspond to four different possibilities of when, relative to period t, the person completes the two stages. In the first case, the person completes the first

[8] We define strategies to depend on t because our notation for histories does not identify the current period. Hence, $s(\tau, t)$ prescribes an action for period t conditional on having completed stage 1 in period τ. Also, our formulation rules out mixed strategies; it is perhaps best to interpret our analysis as applying to equilibrium strategies for an infinite horizon that correspond to some equilibrium strategy for a long, finite horizon, which (generically) does not involve mixed strategies.

stage now and the second stage in the future. In the second case, the person completes both the first stage and the second stage in the future. In the third case, the person has completed the first stage in the past (in period $\tau < t$) and completes the second stage now. In the fourth case, the person has completed the first stage in the past (in period $\tau < t$) and completes the second stage in the future.

With this notation, we can provide a formal definition of perception-perfect strategy:

Definition 1. Given $\hat{\beta}$, strategy $\widehat{s}$ represents **$\hat{\boldsymbol{\beta}}$-consistent beliefs** if for all t and h^t,

$$\widehat{s}(h^t, t) = \arg\max_{a \in \{0,1\}} V^t(a_t; h^t, \widehat{s}, \hat{\beta}).$$

Given β and $\hat{\beta}$, strategy s is a **perception-perfect strategy** if there exists $\hat{\beta}$-consistent beliefs $\widehat{s}$ such that for all t and h^t,

$$s(h^t, t) = \arg\max_{a \in \{0,1\}} V^t(a_t; h^t, \widehat{s}, \beta).$$

A perception-perfect strategy represents how a person with self-control problem β and perceptions of future self-control problems $\hat{\beta}$ would actually behave in all contingencies. A perception-perfect strategy requires that in any situation the person must have some beliefs $\widehat{s}$ for how she would behave in the future, and that she choose an optimal action given these beliefs and her current preferences (which depend on β).[9] In addition, a perception-perfect strategy requires that the beliefs $\widehat{s}$ must be "consistent" with the person's perception of future self-control problems $\hat{\beta}$. Definition 1 imposes two aspects of consistency. Beliefs are internally consistent in that, for all possible contingencies, $\widehat{s}$ specifies an action that is optimal given her beliefs for subsequent periods. Internal consistency implies the person perceives that in the future she will behave like a sophisticated person with self-control problem $\hat{\beta}$. Beliefs are also externally consistent in that the person has the same beliefs across contingencies — that is, for all $t < \tau$ she has the same belief for what she would do in period τ following history h^τ. This restriction rules out procrastination arising from a form of irrational expectations that goes beyond merely mispredicting self-control, because without it the person could repeatedly reconstruct beliefs that permit her to delay.[10]

Before proceeding, we must address a technical issue. For $\hat{\beta} = 1$ — that is, for TCs and naifs — there is a unique set of $\hat{\beta}$-consistent beliefs and therefore a unique perception-perfect strategy

[9] Throughout we assume for simplicity that when a person is indifferent between $a = 0$ and $a = 1$, she chooses $a = 1$.

[10] The restriction of external consistency matters only if there are multiple $\hat{\beta}$-consistent beliefs. The restrictions imposed by external consistency essentially correspond to the additional restrictions which subgame-perfect equilibrium imposes beyond non-equilibrium backwards induction. By the same token, these restrictions would be unnecessary in generic, finite-period situations where "perceptual backwards induction" would yield a unique prediction.

(which we prove in Lemma 1 below). But for $\hat{\beta} < 1$ — that is, for sophisticates and partial naifs — there can exist multiple sets of $\hat{\beta}$-consistent beliefs and therefore multiple perception-perfect strategies. To avoid some difficulties, our main analysis focuses on perception-perfect strategies with "optimistic beliefs", which means that the person believes either that she will complete stage 2 immediately after completing stage 1 or that she will never complete stage 2. Formally:

Definition 2. $\hat{\beta}$-consistent beliefs $\widehat{s}$ are **optimistic** if for all $\tau \in \{1, 2, ...\}$ either $\widehat{s}(\tau, \tau + 1) = 1$ or $\widehat{s}(\tau, \tau') = 0$ for all $\tau' > \tau$.

What does our optimistic-beliefs restriction rule out? The problematic source of multiplicity is a cyclicality in beliefs about stage-2 behavior. As will become clear, for any $\hat{\beta}$, either the unique $\widehat{s}$ involves $\widehat{s}(\tau, \tau') = 0$ for all $\tau' > \tau$, or there exists $z \in \{1, 2, ...\}$ such that every $\widehat{s}$ involves $\widehat{s}(\tau, \tau + m) = 1$ if and only if $m \in \{m', m' + z, m' + 2z, ...\}$ for some $m' \in \{1, ..., z\}$. The problem arises in the latter case, because for $z > 1$ there is indeterminacy in the first date of completion, as determined by m'. Optimistic beliefs select $m' = 1$. Our main concern shall be how many stages a person completes, and not when she completes them. By restricting attention to optimistic beliefs (for sophisticates and partial naifs), we get uniqueness in how many stages are completed (which we also prove in Lemma 1 below). We note, however, that perception-perfect strategies with optimistic beliefs and perception-perfect strategies with non-optimistic beliefs need not involve the same number of stages completed. We discuss such issues more in Appendix A, including how some difficulties disappear when $\delta \to 1$.

In this environment, there are two main reasons why a person might not complete the project. The first revolves around whether the project is worth doing. The following definition will prove useful for describing such effects.

Definition 3. Given β and δ, stage 1 is **β-worthwhile** if $-c + \beta\delta\left(-k + \frac{\delta v}{1-\delta}\right) \geq 0$, and Stage 2 is **$\beta$-worthwhile** if $-k + \frac{\beta\delta v}{1-\delta} \geq 0$.

Stage $n \in \{1, 2\}$ is β-worthwhile if the person prefers completing the project starting from now as opposed to never completing the project. Given optimistic beliefs, stage 1 is β-worthwhile if the person prefers to complete stage 1 now and stage 2 next period as opposed to never starting the project. Stage 2 is β-worthwhile if a person who has completed stage 1 prefers to complete stage 2 now as opposed to never completing stage 2. Clearly a person will complete stage $n \in \{1, 2\}$ only if that stage is β-worthwhile. Moreover, because a person starts the project only if she believes that she will later finish it, and because given perceptions $\hat{\beta}$ she predicts she'll complete stage 2 only if stage 2 is $\hat{\beta}$-worthwhile, the person will complete stage 1 only if stage 2 is $\hat{\beta}$-worthwhile. Notice that since TCs have time-consistent preferences, stage 1 being ($\beta = 1$)-worthwhile necessarily implies that stage 2 is ($\beta = 1$)-worthwhile. For a person with present-biased preferences, in contrast, it could be that stage 1 is β-worthwhile and yet stage 2 is not β-worthwhile.

Whether the project is worth doing in any of these senses is primarily driven by the person's time-consistent impatience parameter δ. In other words, it depends on whether the person gives sufficient weight to the future benefits to justify incurring the immediate costs. Indeed, for any β, $\hat{\beta}$, c, k, and v, there exists $\bar{\delta} < 1$ such that for all $\delta \geq \bar{\delta}$, stage 1 is β-worthwhile and stage 2 is both β-worthwhile and $\hat{\beta}$-worthwhile. In order to abstract away from worthwhileness issues, we often examine the limit case as δ approaches one.

The second main reason why a person might not complete the project is "procrastination" — she views completion starting today as better than never completing the project, and she expects to complete the project, but she repeatedly plans to start completion in the near future rather than now. Throughout this paper, we use this very precise definition of procrastination.

Definition 4. A person **procrastinates stage 1** if she never completes stage 1 despite it being β-worthwhile and stage 2 being $\hat{\beta}$-worthwhile. When a person completes stage 1, she **procrastinates stage 2** if she never completes stage 2 despite it being β-worthwhile.

We use the term procrastination to mean *repeatedly* choosing to delay now based on a plan to work in the near future, but then changing one's mind when that near-future date arrives.[11] In richer, real-world environments, this cycle may eventually be broken, either because of eventual deadlines, or because of non-stationarities in the costs and benefits, or because the person eventually decides to jettison the project. In the stark, stationary environment we consider in this paper, procrastination takes the form of an infinite sequence of decisions to work on the project in the near future, and hence involves infinite delay, but it is not the infinity of delay that is the crux.

The logic of procrastination in this environment is very much like that in O'Donoghue and Rabin (2001). Given her self-control problem β, for each stage the person will have some maximum tolerable delay for completion of that stage. Formally, given β and given optimistic beliefs, the maximum tolerable delay on stage $n \in \{1, 2\}$, which we denote by $d(\beta, n)$, is given by

[11] Hence, our definition of procrastination makes it almost tautological that full sophisticates cannot procrastinate. But since our results suggest that any delay by sophisticates is limited in severity to the influence the taste for immediate gratification has on the overall cost-benefit evaluation, we feel this use is correct.

$$d(\beta,1) \equiv \max\left\{d\in\{0,1,...\} \mid -c+\beta\delta\left(-k+\frac{\delta v}{1-\delta}\right) < \beta\delta^{d+1}\left[-c+\delta\left(-k+\frac{\delta v}{1-\delta}\right)\right]\right\}$$
$$d(\beta,2) \equiv \max\left\{d\in\{0,1,...\} \mid -k+\frac{\beta\delta v}{1-\delta} < \beta\delta^{d+1}\left(-k+\frac{\delta v}{1-\delta}\right)\right\}.$$

In words, $d(\beta,n)$ is the maximum delay d such that the person prefers completing stage n in d periods rather than now. If no maximum delay exists — which holds if and only if stage n is not β-worthwhile — we define $d(\beta,n)=\infty$.

Whether the person completes stage n depends on whether she ever perceives that waiting now would lead to a delay of more than $d(\beta,n)$ periods. Her perceptions of future delay depend on her perceived *future* tolerance for delay, which is given by $d(\hat{\beta},n)$, where $\hat{\beta}\geq\beta$ implies $d(\hat{\beta},n)\leq d(\beta,n)$. If $d(\hat{\beta},n)+1\leq d(\beta,n)$, then the maximum delay she could ever predict beginning next period is $d(\hat{\beta},n)$, and therefore she never perceives that waiting now would lead to a delay of more than $d(\beta,n)$ periods. If, in contrast, $d(\hat{\beta},n)=d(\beta,n)$, then in some period she will predict delay $d(\hat{\beta},n)$ beginning next period, and therefore in that period she will perceive that waiting now would lead to an intolerable delay of $d(\beta,n)+1$ periods. We can conclude that the person procrastinates stage n if and only if $d(\hat{\beta},n)+1\leq d(\beta,n)$ — that is, if and only if her perceived future tolerance for delay of stage n is at least one period shorter than her current tolerance for delay of stage n.

Because $\hat{\beta}=\beta$ implies $d(\hat{\beta},n)=d(\beta,n)$, TCs and sophisticates never procrastinate. For naifs, the logic of procrastination is quite simple: Naifs procrastinate whenever they prefer completing stage n next period to completing stage n now (which follows formally from $d(\hat{\beta},n)=0$ for $\hat{\beta}=1$ whenever the project is worth doing). For partial naifs, the logic of procrastination is perhaps less transparent, although a simple example helps. Consider a partial naif with $d(\hat{\beta},1)=2$ and $d(\beta,1)=3$. Given future tolerance for delay $d(\hat{\beta},1)=2$, the person's perceptions for future stage-1 behavior must involve completing stage 1 every 3 periods — e.g., $\widehat{s}(\emptyset,t)=1$ if and only if $t\in\{1,4,7,...\}$. Now consider the mindset of a partial naif with these beliefs. In period 1, she waits planning/expecting to complete stage 1 in period 4. She continues with this plan until period 4 arrives, at which point she changes her mind and decides instead to wait planning/expecting to complete stage 1 in period 7. She continues with this new plan until period 7 arrives, at which point she decides to wait until period 10, and so on.

Having outlined the two sources of delay, we now characterize perception-perfect strategies. Lemma 1 describes some basic properties (all proofs are collected in Appendix B):

Lemma 1. For all δ, c, k, and v:

(1) For $\beta \leq \hat{\beta} = 1$ (for TCs and naifs), there exists a unique perception perfect strategy; and

(2) For $\beta \leq \hat{\beta} < 1$ (for sophisticates and partial naifs), if the person has optimistic beliefs, then either

(a) There exists a unique perception-perfect strategy s, and s satisfies $s(\emptyset, t) = 0$ for all t;

(b) Any perception-perfect strategy s satisfies $s(\emptyset, t) = 1$ for some $t \in \{1, 2, ..., d(\beta, 1) + 1\}$ but $s(t, \tau) = 0$ for all $\tau > t$; or

(c) Any perception-perfect strategy s satisfies $s(\emptyset, t) = 1$ for some $t \in \{1, 2, ..., d(\beta, 1) + 1\}$ and $s(t, t + 1) = 1$.

In our environment, there are three possible outcomes in terms of which stages the person completes: She can never start the project, she can complete stage 1 but not stage 2, and she can complete both stages 1 and 2. Lemma 1 establishes that we have uniqueness in which of these occurs. Part 1 establishes that there is a unique perception-perfect strategy for TCs and for naifs. For sophisticates and partial naifs, where there may be multiple perception-perfect strategies, Part 2 establishes that, under the restriction to optimistic beliefs, how many stages are completed is uniquely determined. More precisely, Lemma 1 establishes that multiple perception-perfect strategies arise only when these types complete stage 1, and the indeterminacy is solely about *when* they complete the first stage.

Our main results in this paper all focus solely on how many stages people complete, rather than precisely when they complete them. This approach permits a more concise statement of our key conclusions.[12] Proposition 1 describes when a person completes the two stages:

Proposition 1. For all β, $\hat{\beta}$, δ, c, k, and v, under any perception-perfect strategy with optimistic beliefs:

(1) The person completes stage 1 if and only if stage 1 is β-worthwhile, stage 2 is $\hat{\beta}$-worthwhile, and $d(\hat{\beta}, 1) = d(\beta, 1)$;

(2) Conditional on completing stage 1, the person completes stage 2 if and only if stage 2 is β-worthwhile and $d(\hat{\beta}, 2) = d(\beta, 2)$; and

(3) $d(\hat{\beta}, 1) = d(\beta, 1)$ if $c \leq \frac{\beta\delta(1-\delta)}{1-\beta\delta}\left(-k + \frac{\delta v}{1-\delta}\right)$ and only if $c < \frac{\beta\delta(1-\delta)}{1-\beta\delta/\hat{\beta}}\left(-k + \frac{\delta v}{1-\delta}\right)$;
$d(\hat{\beta}, 2) = d(\beta, 2)$ if $k \leq \frac{\beta\delta(1-\delta)}{1-\beta\delta}\left(\frac{v}{1-\delta}\right)$ and only if $k < \frac{\beta\delta(1-\delta)}{1-\beta\delta/\hat{\beta}}\left(\frac{v}{1-\delta}\right)$.

[12] As discussed in some detail in O'Donoghue and Rabin (2001), there are ways of intuiting and formalizing how finite delays in completing a task (or, as here, the first stage of a task) are in this context qualitatively different and less important than the infinite delays we focus on.

Proposition 1 formalizes the points from our earlier discussion. There are three reasons a person might not start the project: She might feel the project is not worth doing (stage 1 is not β-worthwhile); she might predict that in the future she won't find the project to be worth continuing (stage 2 is not $\hat{\beta}$-worthwhile); or she might procrastinate ($d(\hat{\beta}, 1) < d(\beta, 1)$). Similarly, conditional on starting the project, there are two reasons a person might not finish the project: She might feel the project is not worth continuing (stage 2 is not β-worthwhile); or she might procrastinate ($d(\hat{\beta}, 2) < d(\beta, 2)$). Part 3 describes conditions for when a person procrastinates.

As discussed above, TCs and sophisticates never procrastinate, and all that matters is the worthwhileness of the project. Corollary 1 compares TCs and sophisticates:

Corollary 1. (1) Under their unique perception-perfect strategy, TCs either complete both stages (immediately) or never start the project; and they complete both stages if and only if

$$-c - \delta k + \frac{\delta^2 v}{1-\delta} \geq 0.$$

(2) Under any perception-perfect strategy with optimistic beliefs, sophisticates either complete both stages (eventually) or never start the project; and they complete both stages if and only if

$$-c - \beta\delta k + \frac{\beta\delta^2 v}{1-\delta} \geq 0 \qquad \text{and} \qquad -k + \frac{\beta\delta v}{1-\delta} \geq 0.$$

TCs either complete the project or never start the project, and which they do merely depends on whether the project is worth doing. Hence, the condition for whether TCs complete the project is merely the standard net-present-value calculation (on utility) applied to our environment. An important implication is that, holding constant the stage-1 net present value — holding constant $-c - \delta k + \frac{\delta^2 v}{1-\delta}$ — changing the distribution of costs over the course of the project does not affect the behavior of TCs.[13]

Sophisticates behave much like TCs in that they either complete the project or never start the project. Like TCs, they might not start because the project is not worthwhile — as reflected by the first condition for sophisticates being identical to that for TCs except for incorporating the person's preference for immediate gratification. But unlike TCs, sophisticates also might not start because they expect not to want to finish, which occurs when stage 2 is not β-worthwhile — as reflected by the second condition for sophisticates. This latter intuition gives rise to one way in which the structure of costs matters for people with present-biased preferences in a way that would be irrelevant for people with time-consistent preferences. Specifically, the structure of costs affects whether the person will want to complete stage 2, and in particular, holding constant the NPV

[13] Indeed, changing the distribution of costs *and benefits* in a way that leaves the net present value unchanged cannot affect behavior for TCs, though it can for those with present-biased preferences.

$\left[-c - \beta\delta k + \frac{\beta\delta^2 v}{1-\delta}\right]$, decreasing c and increasing k makes it less likely that stage 2 is β-worthwhile, and therefore makes it less likely that sophisticates start.

While the behavior of sophisticates can differ from the behavior of TCs (in terms of whether they complete the project), Proposition 2 describes a sense in which sophisticates behave much like TCs unless their preference for immediate gratification is large (β far from 1).

Proposition 2. Define $C \equiv c + \delta k$ and $V \equiv \frac{\delta^2 v}{1-\delta}$, so that TCs never start if and only if $C/V > 1$. Then sophisticates never start only if $C/V > \beta$.

Proposition 2 establishes that if TCs complete the project while sophisticates don't (for whatever reason), it must be that the ratio of costs to benefits is "close" to one, in the sense of being between β and one. As our results below shall illustrate, procrastination can lead naifs and partial naifs to never start even as the ratio of costs to benefits approaches zero.

Worthwhileness matters for naifs and partial naifs much as it does for TCs and sophisticates. Both naifs and partial naifs might not start the project because it is not worth doing (when stage 1 is not β-worthwhile). And because partial naifs are partially sophisticated, they might not start because they expect not to want to finish (when stage 2 is not $\hat{\beta}$-worthwhile). This latter conclusion implies that the cost structure can matter for partial naifs in the same way that it does for sophisticates.

We doubt, however, the importance of misbehavior driven by worthwhileness concerns; for instance, one could formalize a sense in which the degree of harm from any such misbehavior can be large only if β is significantly less than 1. For the remainder of this section, we explore procrastination by people who are (at least partially) naive. In order to lay bare the forces that influence procrastination, we examine behavior when $\delta \to 1$. As discussed above, in this case everything is worthwhile, and therefore the only reason a person might not complete the project is procrastination.

Proposition 3 characterizes how the different types behave when $\delta \to 1$:

Proposition 3. When $\delta \to 1$, for any c, k, and v:

(1) If $\beta = \hat{\beta} \leq 1$, the person completes the project;

(2) If $\beta < \hat{\beta} = 1$, the person completes stage 1 if and only if $c < \frac{\beta v}{1-\beta}$, and if she completes stage 1, then she completes stage 2 if and only if $k < \frac{\beta v}{1-\beta}$; and

(3) If $\beta < \hat{\beta} < 1$, the person completes stage 1 if $c < \frac{\beta v}{1-\beta}$ and only if $c < \frac{\beta v}{1-\beta/\hat{\beta}}$, and if she completes stage 1, then she completes stage 2 if $k < \frac{\beta v}{1-\beta}$ and only if $k < \frac{\beta v}{1-\beta/\hat{\beta}}$.

Part 1 captures the intuition that everything is worth doing when $\delta \to 1$, so both TCs and sophisticates complete the project. Parts 2 and 3 characterize when naifs and partial naifs procrastinate. Proposition 3 yields several interesting conclusions about procrastination.[14]

First, notice that whether naifs procrastinate a stage is monotonic in the stage cost — that is, the larger is the stage cost, the more likely it is that naifs procrastinate that stage. Although this basic intuition appears in many previous papers on time-inconsistent procrastination, we emphasize it because our new results rely on it heavily. A similar intuition holds for partial naifs, except for a possible zone in which whether partial naifs procrastinate is non-monotonic in the stage cost. The source of this non-monotonicity is the discreteness of $d(\hat{\beta}, n)$ and $d(\beta, n)$. If the stage cost is small enough, then $d(\hat{\beta}, n) = d(\beta, n) = 0$ and therefore the person doesn't procrastinate. If the stage cost is large enough, then we can guarantee that $d(\hat{\beta}, n) < d(\beta, n)$, and therefore the person procrastinates. But for intermediate stage costs, we cannot tell whether the person procrastinates.[15]

Second, notice that two types of procrastination potentially arise when a person who is not completely sophisticated faces a long-term project. First, there is the "classical" form highlighted in previous procrastination papers, wherein a person plans to start a valuable project but never does so. But here a person might instead start a valuable project planning to finish it, but never finish it. This latter form of procrastination is clearly worse, because the person incurs the cost associated with starting the project without ever accruing any benefits. In fact, Proposition 4 establishes that there is in principle no bound on how much effort a person might exert in starting a project she does not finish.

Proposition 4. For all β, δ, and $\hat{\beta} > \beta$, for any c there exists k and v such that the person completes stage 1 but never completes stage 2.

The third — and most interesting — implication of Proposition 3 is that the structure of costs matters dramatically for whether a person procrastinates. Consider the behavior of naifs. Because Proposition 3 implies that naifs complete the project if and only if $\max\{c, k\} < \frac{\beta v}{1-\beta}$, the *allocation of costs* over the course of the project becomes crucial. In particular, for any fixed total cost, naifs are most likely to complete the project when these costs are allocated evenly over the course of

[14] All of our qualitative conclusions when $\delta \to 1$ also hold when $\delta < 1$, but the equations become more complicated.

[15] More precisely, $d(\beta, n)$ is the largest integer smaller than $\tilde{d}(\beta, n) \equiv (1-\beta)c/(\beta v)$, and $d(\hat{\beta}, n)$ is therefore the largest integer smaller than $\tilde{d}(\hat{\beta}, n)$. Clearly, $\tilde{d}(\beta, n) - \tilde{d}(\hat{\beta}, n)$ is strictly increasing in c. Indeed, it is the condition $\tilde{d}(\beta, n) - \tilde{d}(\hat{\beta}, n) > 1$ that guarantees $d(\hat{\beta}, n) < d(\beta, n)$. But for $\tilde{d}(\beta, n) - \tilde{d}(\hat{\beta}, n) < 1$, it is unclear whether $d(\beta, n) = d(\hat{\beta}, n)$ or $d(\beta, n) = d(\hat{\beta}, n) + 1$.

the project. If total costs are Γ, naifs are most likely to complete the project when $c = k = \Gamma/2$, and least likely to complete the project when $c = \Gamma$ or $k = \Gamma$. The intuition for the role of allocation is simple: Naifs complete the project if and only if they don't procrastinate the highest-cost stage. If a disproportionate share of the costs are allocated to stage 1, then the person is prone to procrastinate starting the project; and if a disproportionate share of the costs are allocated to stage 2, then the person is prone to procrastinate finishing the project.

When costs are allocated unevenly, the *order of costs* is important, because it determines whether naifs incur costs without accruing benefits. If both costs are sufficiently low that the person would not procrastinate either stage, or if both costs are sufficiently high that the person would procrastinate both stages, then the order of costs is irrelevant. But if costs are such that the person would procrastinate the high-cost stage but not the low-cost stage, then the person never starts when the high-cost stage comes first, whereas the person starts but doesn't finish when the high-cost stage comes second. Hence, naifs are better off when the high-cost stage comes first — that is, when $\min\{c, k\} = k$.

Proposition 3 suggests similar intuitions hold for partial naifs as well. Because partial naifs, like naifs, are more prone to procrastinate a stage the higher is the cost of that stage, the allocation of costs and order of costs can matter for partial naifs in the same way that it matters for naifs. But because of the non-monotonicities for partial naifs discussed above, it is possible to construct examples where these effects are reversed.[16]

4. Model with Endogenous Cost Structure

The previous section shows how the behavior of people with present-biased preferences depends critically on the structure of costs over the course of a project. In this section, we endogenize this structure. We show that in such situations people who are (at least partially) naive are in fact prone to choose cost structures for which they are likely to start but not finish the project.

To motivate our formal analysis, consider a person who must complete an unpleasant project at work that requires a total of 12 hours of effort. The person can put in these hours in any way

[16] Interestingly, there are situations where partial naifs can suffer worse outcomes than either extreme. Partial naifs are less likely to procrastinate any given stage than are naifs. But this means that in situations where naifs never start the project, if their partial sophistication overcomes procrastination at stage 1 but does not overcome procrastination at stage 2, then partial naifs start but do not finish the project. Hence, no matter whether sophisticates complete the project or never start in such situations, clearly both naifs and sophisticates experience better outcomes than do partial naifs.

that she wants, except that she cannot work for more than 8 hours on any given day, so that the project requires (at least) two days of work. But the person has discretion over how to allocate her time over those two days. What work schedule will the person choose?

Formally, we assume the person must choose a cost structure

$$(c, k) \in \{(x, y) \mid x + y = A, x \leq \bar{a}, y \leq \bar{a}\} \equiv \mathbf{P}^{A,\bar{a}}.$$

This formulation incorporates two assumptions. First, the total cost to be incurred, which we denote by A, is independent of the cost structure — that is, both the disutility and the benefits of an hour's worth of effort are assumed to be independent of when the effort is put in. We address at the end of this section how efficiency concerns might alter our conclusions. Second, there is a maximum cost $\bar{a}$ that can be incurred in any specific period. To make it meaningful that we are analyzing "long-term projects", it must be that the person cannot choose to complete the project all at once, because otherwise TCs would choose to do so and naifs would plan to do so. We assume $A \in (\bar{a}, 2\bar{a})$, which makes it a two-period project.[17]

Our formal analysis restricts the person to incur costs in *at most* two periods. This restriction is irrelevant for TCs and naifs: Because there is no value to dividing up the costs more than necessary, and because TCs and naifs are merely maximizing (actual or perceived) preferences, they always plan to complete the task in a total of two days. But this restriction can be substantive for sophisticates and partial naifs: Because they are worried about future misbehavior, there may be value to further dividing up the costs if doing so influences future behavior.

Importantly, the person makes her choice of cost structure at the moment of action. Hence, if in period t the person has waited in all prior periods, then in period t she can choose either to wait again or to incur any cost $x \in [A - \bar{a}, \bar{a}]$ while planning to incur cost $A - x$ next period (given optimistic beliefs). If, in contrast, the person has incurred cost $x \in [A - \bar{a}, \bar{a}]$ in the past, then in period t she can choose either to wait or to incur cost $A - x$ to finish the project. As in our basic model, the person receives an infinite stream of benefits with per-period benefit $v \geq 0$ upon completion of the project.

In this environment, the person is effectively making a choice between many possible projects — the person can choose any project in $\mathbf{P}^{A,\bar{a}}$. We define $\mathbf{p}^*(\mathbf{P})$ to be the person's *preferred project* within a set of projects $\mathbf{P}$. Formally,

[17] More generally, if A and $\bar{a}$ were such that $A \in ((N-1)\bar{a}, N\bar{a})$, then the project would be an N-period project.

$$\mathbf{p}^*(\mathbf{P}) \equiv \begin{cases} \arg\max_{(c,k)\in\mathbf{P}'(\mathbf{P})} \left[-c - \beta\delta k + \frac{\beta\delta^2 v}{1-\delta}\right] & \text{if } \mathbf{P}'(\mathbf{P}) \text{ is non-empty} \\ \emptyset & \text{if } \mathbf{P}'(\mathbf{P}) \text{ is empty} \end{cases}$$

where

$$\mathbf{P}'(\mathbf{P}) \equiv \{(c,k) \in \mathbf{P} \mid \text{stage 1 is } \beta\text{-worthwhile and stage 2 is } \hat{\beta}\text{-worthwhile}\}.$$

In words, the person's preferred project is the project that maximizes her preferences subject to the condition that she wants and expects to complete it. Note that the preferred project depends on β, and hence is the preferred project given the person's preference for immediate gratification. If there is no project that the person wants and expects to complete, then we say that the person does not have a preferred project. Clearly, if a person starts any project, it will be her preferred project; and if she has no preferred project, then she never does anything. The following lemma characterizes a person's preferred project.

Lemma 2. For any β, δ, v, A, and $\bar{a}$ such that $A \in (\bar{a}, 2\bar{a})$:

(1) If $\hat{\beta} = 1$ (for TCs and naifs), either $\mathbf{p}^*(\mathbf{P}^{A,\bar{a}}) = \emptyset$ or $\mathbf{p}^*(\mathbf{P}^{A,\bar{a}}) = (A - \bar{a}, \bar{a})$;

(2) If $\hat{\beta} < 1$ (for sophisticates and partial naifs), either $\mathbf{p}^*(\mathbf{P}^{A,\bar{a}}) = \emptyset$ or $\mathbf{p}^*(\mathbf{P}^{A,\bar{a}}) = (A - x_0, x_0)$ where $x_0 \equiv \min\left\{\bar{a}, \frac{\hat{\beta}\delta v}{1-\delta}\right\}$; and

(3) When $\delta \to 1$, $\mathbf{p}^*(\mathbf{P}^{A,\bar{a}}) = (A - \bar{a}, \bar{a})$ for all $\hat{\beta}$.

Lemma 2 captures the intuition that when the cost structure is endogenous, people will prefer to defer as much of the cost as possible to the second stage. Part 1 establishes that for TCs and naifs, the preferred project (if it exists) involves deferring the maximum possible amount $\bar{a}$ to the second stage. Part 2 establishes a similar result for sophisticates and partial naifs, but with a caveat: Because the person expects to have future self-control problems, she will not allocate so much cost to stage 2 so as to make stage 2 not $\hat{\beta}$-worthwhile, which requires that the stage-2 cost be no larger than $\frac{\hat{\beta}\delta v}{1-\delta}$. Finally, Part 3 establishes that when $\delta \to 1$, in which case everything is worthwhile, all types plan to incur the maximum cost $\bar{a}$ in stage 2. This propensity to defer costs is driven by both the person's time-consistent impatience, as captured by δ, and the person's preference for immediate gratification, as captured by β.

In order to explore the implications of "endogenizing the cost structure," we compare a person's behavior given exogenous cost structure (c, k) to her behavior given endogenous cost structure $\mathbf{P}^{\text{endog}}(c, k) \equiv \{(x, y) \mid x + y = c + k, x \leq \bar{a}, y \leq \bar{a}\}$. We assume $c \leq \bar{a}$, $k \leq \bar{a}$, and $c + k > \bar{a}$, so that we are comparing behavior under exogenous cost structure (c, k) to behavior under endogenous cost structure $\mathbf{P}^{\text{endog}}(c, k)$ when $(c, k) \in \mathbf{P}^{\text{endog}}(c, k)$. Proposition 5 describes how endogenizing the cost structure affects the worthwhileness of the project.

Proposition 5. For any $\delta < 1$, v, $\bar{a}$, and (c, k) with $c, k \leq \bar{a}$ and $c + k > \bar{a}$:

(1) For any β and $\hat{\beta}$, if project (c, k) has stage 1 β-worthwhile and stage 2 $\hat{\beta}$-worthwhile, then $\mathbf{p}^*(\mathbf{P}^{\text{endog}}(c, k))$ exists and has stage 1 β-worthwhile and stage 2 $\hat{\beta}$-worthwhile; and

(2) For $\beta = \hat{\beta} \leq 1$ (for TCs and sophisticates), and given optimistic beliefs for sophisticates, if the person completes the project under exogenous cost structure (c, k), then she completes the project under endogenous cost structure $\mathbf{P}^{\text{endog}}(c, k)$.

Part 1 establishes that endogenizing the cost structure makes it more likely that the project is worth doing, and also makes it more likely that the person expects to want to finish. The intuition for the former result is that, with discounting, for any fixed total cost, allocating more of that cost to the second stage makes it more likely that the project is worth doing. The intuition for the latter result is that the person chooses the cost structure accounting for her perceived stage-2 incentives. Part 2 establishes that endogenizing the cost structure makes it more likely that TCs and sophisticates complete the project. This follows immediately from Part 1, because TCs complete the project if and only if stage 1 is $(\beta = 1)$-worthwhile, and sophisticates complete the project if and only if both stages are β-worthwhile.

Considerations of whether completing the project is worthwhile are relevant for naifs and partial naifs as well, and so for $\delta < 1$ endogenizing the cost structure may make it more likely that they complete the project. But endogenizing the cost structure might also influence whether naifs and partial naifs procrastinate. In order to lay bare the implications of endogenizing the cost structure for procrastination, we again examine behavior when $\delta \to 1$, in which case the above worthwhileness considerations disappear. Proposition 6 formalizes a stark contrast in the effects of endogenizing cost structure between sophisticates and naifs.

Proposition 6. When $\delta \to 1$, for any v, $\bar{a}$, and (c, k) with $c, k \leq \bar{a}$ and $c + k > \bar{a}$:

(1) If $\beta = \hat{\beta} \leq 1$, the person completes the project both under exogenous cost structure (c, k) and under endogenous cost structure $\mathbf{P}^{\text{endog}}(c, k)$; and

(2) If $\beta < \hat{\beta} = 1$, the person starts the project under endogenous cost structure $\mathbf{P}^{\text{endog}}(c, k)$ if she starts the project under exogenous cost structure (c, k), and she completes the project under endogenous cost structure $\mathbf{P}^{\text{endog}}(c, k)$ only if she completes the project under exogenous cost structure (c, k).

Part 1 merely restates our earlier conclusion that when $\delta \to 1$, everything is worth doing, and so TCs and sophisticates always complete the project. Part 2 establishes that for naifs, endogenizing the cost structure makes it more likely that they start the project, while at the same time makes it less likely that they complete the project. The intuition is simple. We saw in Section 3 that naifs are prone to start but not finish when a disproportionate share of the total cost is allocated to stage 2. When the cost structure is endogenized, the same preference for immediate gratification that leads the person to procrastinate also leads the person to defer as much of the total cost as is possible to stage 2. Hence, she is prone to choose a cost structure on which she is prone to start but not finish.[18]

Our results in Proposition 6 can be interpreted in terms of an intuition identified in O'Donoghue and Rabin (2001). One of the main results in that paper is that providing a person with additional options can make procrastination more likely. This outcome occurs when some new option is better than existing options from a long-run perspective but more onerous to carry out, because then the person will plan to do the new option, but may repeatedly put off incurring the high immediate cost. While that paper demonstrates that one can construct examples in which expanded choice exacerbates procrastination, one can also construct examples in which it mitigates procrastination. Here, by contrast, we show that a natural way to expand choice — giving people discretion over how to schedule their efforts — unambiguously makes (pure) naifs more prone to procrastinate.

Our results above are also suggestive of the implications of endogenizing the cost structure in a different way. Suppose that, rather than being able to allocate the total costs in a continuous fashion, a person must complete a project that consists of specific sub-component tasks, and can choose only the order of these tasks. In other words, given exogenous cost structure (c, k), endogenizing the cost structure in this way means the set of possible projects is $\mathbf{P} \equiv \{(c, k), (k, c)\}$. Applying the logic from Lemma 2, the preferred project $\mathbf{p}^*(\mathbf{P})$ involves doing the low-cost stage first and the high-cost stage second — that is, deferring as much cost as possible to stage 2. Applying the logic from Proposition 5, endogenizing the order can make it more likely that any type completes the project because she finds it worthwhile. In terms of procrastination, applying the logic from Proposition 6, endogenizing the order does not change whether the person completes the project. But if she doesn't complete the project, endogenizing the order makes it more likely that the person starts the project without finishing it.

[18] For partial naifs, a similar intuition holds. Just like naifs, endogenizing the cost structure leads partial naifs to allocate a disproportionate share of the costs to stage 2, and hence makes them prone to start but not finish the project. But because of the non-monotonicities discussed in Section 3, endogenizing the cost structure can have essentially any effect on partial naifs.

We conclude this section by extending our analysis to the case where the choice of cost structure has efficiency implications — that is, different cost structures imply different total costs. In order to incorporate this possibility, we modify the set of possible cost structures to be of the form

$$\{(x,y) \mid h(x)+h(y)=A, x\le \bar{a}, y\le \bar{a}\}$$

where $h'>0$ and $A\in(h(\bar{a}),2h(\bar{a}))$. We can interpret A to be the number of "effective hours" required, and $h(a)$ to be the number of effective hours accumulated when a hours are spent.[19] We distinguish between two cases, increasing returns to effort, $h''>0$, and decreasing returns to effort, $h''<0$.

Some additional notation will be useful. First, we define $\tilde{\mathbf{P}}^{\text{endog}}(c,k)$ analogously to $\mathbf{P}^{\text{endog}}(c,k)$: Given exogenous cost structure (c,k), we define

$$\tilde{\mathbf{P}}^{\text{endog}}(c,k)\equiv\{(x,y) \mid h(x)+h(y)=h(c)+h(k), x\le \bar{a}, y\le \bar{a}\},$$

where we assume $h(c)+h(k)\in(h(\bar{a}),2h(\bar{a}))$. Second, we define x^* to satisfy $2h(x^*)=h(c)+h(k)$. That is, x^* represents the per-stage cost for the project if it were to be completed with an even allocation of effort. Finally, we define $x(y)=h^{-1}\left([h(c)+h(k)]-h(y)\right)$. If y is the cost allocated to one stage, then $x(y)$ is the cost required in the other stage.

The following proposition characterizes behavior for naifs:

Proposition 7. When $\delta\to 1$ and $\beta<\hat{\beta}=1$, for any v, $\bar{a}$, and (c,k) with $c,k\le\bar{a}$ and $h(c)+h(k)\in(h(\bar{a}),2h(\bar{a}))$:

(1) If $h''>0$, then $\mathbf{p}^*\left(\tilde{\mathbf{P}}^{\text{endog}}(c,k)\right)=(x(\bar{a}),\bar{a})$, and the person completes the project under endogenous cost structure $\tilde{\mathbf{P}}^{\text{endog}}(c,k)$ only if she completes the project under exogenous cost structure (c,k); and

(2) If $h''<0$, then $\mathbf{p}^*\left(\tilde{\mathbf{P}}^{\text{endog}}(c,k)\right)=(x(y^*),y^*)$ for some $y^*\in(x^*,\bar{a}]$. If $y^*\ge\max\{c,k\}$, then the person completes the project under endogenous cost structure $\tilde{\mathbf{P}}^{\text{endog}}(c,k)$ <u>only if</u> she completes the project under exogenous cost structure (c,k). If $y^*<\max\{c,k\}$, then the person completes the project under endogenous cost structure $\tilde{\mathbf{P}}^{\text{endog}}(c,k)$ <u>if</u> she completes the project under exogenous cost structure (c,k).

Proposition 7 describes how the interaction of cost allocation and efficiency matters for procrastination. Efficiency considerations represent another force that influences the choice of cost structure. Part 1 establishes that if there are increasing returns to effort, then the conclusions in Part 2 of Proposition 6 still hold. Intuitively, if there are increasing returns to effort, then efficiency

[19] Alternatively, we could interpret A to be the number of actual hours required, and x to be the immediate disutility associated with working $h(x)$ hours.

considerations militate in favor of an uneven allocation (since it is most efficient to try to do as much as possible on the longer day), and hence reinforce the propensity of naive procrastinators to defer as much cost as possible to the future.[20] Part 2 establishes that if there are decreasing returns to effort, in contrast, it is ambiguous whether endogenizing the cost structure leads to more or less procrastination. With decreasing returns to effort, efficiency considerations militate in favor of an even allocation, and hence the person's preferred project need not involve deferring as much cost as possible to stage 2. As a result, the effects of endogenizing the cost structure depend on how the allocation associated with the preferred project compares to the initial exogenous cost structure.

5. Recurrent Procrastination Costs

In this section we consider some extensions of our model that further illustrate the possibility of naive people incurring costs without ever receiving benefits.

Our basic model explores when a person might procrastinate a single long-term project. Our first extension examines how the person's behavior might change when she can complete a series of independent long-term projects. Suppose there are N identical long-term projects that the person might complete, each with cost structure (c, k), and completion of a project in period τ initiates a stream of benefits $v > 0$ each period from $\tau + 1$ onward. In each period, the person can complete at most one stage of at most one project. Hence, she has three types of options each period: start a new project (if not all projects have been started), complete a started project (if one exists), or do nothing. For $N = 1$, this model is equivalent to our basic model in Section 3. To focus our discussion on procrastination, we examine behavior when $\delta \to 1$.

In this environment, TCs start and finish all N projects in succession — that is, in period 1 they start a project, in period 2 they finish that project; in period 3 they start a second project, in period 4 they finish the second project; and so forth until they have completed all N projects. Intuitively, when $\delta \to 1$, all projects are worth doing, and, to initiate the benefits of each project as soon as possible, it is always better to finish a started project before starting a new project.

Proposition 8 characterizes the behavior of naifs in this environment:

[20] Indeed, endogenizing the cost structure makes procrastination more likely even if the person has the option to complete the entire project on one day, which would hold if $h(\bar{a}) = h(c) + h(k)$. In that case, increasing returns to effort lead naifs to always plan to complete the project on one day, but that one day might always be tomorrow. In this case, the person would not incur costs without benefits.

Proposition 8. When $\delta \to 1$ and $\beta < \hat{\beta} = 1$, for any c, k, v, and N:

(1) Naifs start a new project only if they have (strictly) more than n_{PS} unstarted projects, and they finish a started project only if they have (strictly) more than n_{PF} unfinished projects, where

$$n_{PS} \equiv \max\left\{n \in \{0,1,...\} \mid c \geq \tfrac{\beta v}{1-\beta} n\right\}$$
$$n_{PF} \equiv \max\left\{n \in \{0,1,...\} \mid k \geq \tfrac{\beta v}{1-\beta} n\right\};$$

(2) Naifs start $n_S^* \equiv \max\{0, N - n_{PS}\}$ projects and finish $n_F^* \equiv \min\{\max\{0, N - n_{PF}\}, n_s^*\}$ projects; and

(3) If $c \geq k$ and $n_S^* > 0$, naifs start and finish n_S^* projects in succession; and if $c < k$ and $n_S^* > 0$, naifs first start n_I projects, and then alternate between finishing and starting until they have started n_S^* projects and finished n_F^* projects, where $n_I \equiv \min\left\{n \in \{1,...,N\} \mid k - c \leq \frac{\beta v}{1-\beta} n\right\}$.

In Part 1 of Proposition 8, the variables n_{PS} and n_{PF} represent the number of projects that naifs might procrastinate starting and finishing, respectively. Part 2 uses these conclusions to characterize how many projects naifs start, n_S^*, and how many projects they finish, n_F^*. Specifically, the person starts $N - n_{PS}$ projects, unless $N \leq n_{PS}$, in which case she never starts any project. Similarly, the person finishes $N - n_{PF}$ projects, unless either $N \leq n_{PF}$, in which case she never finishes any started projects, or $n_S^* < N - n_{PF}$, in which case she finishes every project that she starts. Parts 1 and 2 demonstrate how the possibility of procrastination extends to the availability of multiple projects. Both n_S^* and n_F^* are increasing in N — the more projects a person faces, the more projects she starts, and the more projects she finishes. Intuitively, when a person would procrastinate on a single project, facing multiple projects can help motivate her not to procrastinate because waiting imposes a delay on all projects that the person expects to complete in the future. But while facing multiple projects can help counteract procrastination, it doesn't eliminate it. In particular, both n_{PS} and n_{PF} are independent of N. Hence, if, for instance, $n_{PS} = 4$, then for $N \leq 4$ naifs never start any projects, and for $N > 4$ naifs never start exactly four projects.

Part 3 demonstrates how our results about the allocation of costs extends to facing multiple projects. When for each project the high-cost stage comes first, naifs finish immediately all projects that they start. Much as in our basic model, naifs are more prone to procrastinate the high-cost stage, and so if the high-cost stage comes first, naifs finish every project that they start. In contrast, when the low-cost stage comes first, naifs might start projects but then never finish them. Indeed, it is easy to construct examples in which $n_S^* = N$ and $n_F^* = 0$, so that naifs start all N projects and never finish *any* of them.[21] Also notice that, whereas the person should finish started projects before she starts new projects, when the low-cost stage comes first she might instead start new projects before she finishes started projects, and hence carry around an "inventory" of started but not completed projects. This distortion is driven by the same preference for immediate gratification that generates procrastination: When choosing which of two onerous activities to do first, people are biased towards choosing the less onerous activity.

Our second extension examines a natural class of long-term projects wherein stage 1 represents preliminary "search" while stage 2 represents "development". Consider, for example, a Ph.D. student writing her dissertation. Stage 1 can be interpreted as the period of time during which the graduate student pursues a new idea and develops some preliminary results. After completing stage 1, the student, with guidance from her advisor, can assess the likely quality of a dissertation based on these preliminary results. The student must then decide whether to pursue those results — i.e., to complete stage 2 of that project — or to begin working on some new idea — i.e., to complete stage 1 of a new project.

To capture such situations, we suppose that "completing stage 1" means the person incurs an immediate cost $c \geq 0$ and then learns the flow benefit v that she would receive upon completing that project. We assume v is distributed according to cumulative distribution function $F(\cdot)$ that is continuous, strictly increasing, and differentiable over support $[0, \infty)$. The student knows $F(\cdot)$ before completing the first stage but learns v only upon completing the first stage. Once she has drawn such a v, in any later period she can finish that project by incurring cost $k \geq 0$, after which she will receive the benefit v in all subsequent periods. But the person can also choose to complete stage 1 of a new project so as to find a higher benefit, where we assume the benefits are independent across draws, and the person can complete stage 2 of at most one project. Hence, in each period the person has three options: she can complete stage 1 of a new project, she can complete stage 2 of any previously begun project (clearly choosing the highest benefit v), or she can do nothing.

Lemma 3 characterizes the behavior of TCs and naifs in this environment.

[21] If we were to endogenize the cost structure as in Section 4, naifs would choose to decrease c and increase k, which would imply that n_S^* increases while n_F^* decreases. We are wary of such results, however, because it's unclear how valid it is to merge our new assumption of completing at most one stage of at most one project per day with our earlier interpretation of allocating hours over the day.

Lemma 3. For all c, k, and $F(\cdot)$:

(1) There is a unique perception-perfect strategy for TCs. This strategy generates one of the following two paths of behavior: (i) they never do anything; or (ii) there exists $\bar{v}^* > 0$ such that they complete stage 1 each period until they draw a benefit $v \geq \bar{v}^*$, and then finish that project the subsequent period.

(2) There is a unique perception-perfect strategy for naifs. This strategy generates one of the following two paths of behavior: (i) they never do anything; or (ii) there exists $\bar{v}^n > 0$ and $\bar{v}^{nn} \geq \bar{v}^n$ such that they complete stage 1 each period until they draw a benefit $v \geq \bar{v}^n$, and then they finish that project the subsequent period if $v \geq \bar{v}^{nn}$, and otherwise they never finish (i.e., they do nothing in all subsequent periods).

To see how present-biased preferences can distort behavior, consider how naifs behave in comparison to TCs. If TCs never search, then clearly naifs also never search. But when TCs find it optimal to search, naifs might differ from TCs in three ways. First, naifs might never search, either because they view searching as not β-worthwhile or because they procrastinate. Second, when naifs search and eventually draw a benefit large enough that they plan to complete that project, they may never complete the project due to procrastination. Finally, naifs might search with a different cutoff than TCs — that is, $\bar{v}^n \neq \bar{v}^*$. Proposition 9 describes how the cost structure influences which of these distortions might occur:

Proposition 9. Suppose c, k, and $F(\cdot)$ are such that TCs search with cutoff $\bar{v}^* > 0$.

(1) If $c = k$, naifs either never search or behave exactly like TCs;

(2) If $c > k$, naifs either never search or search with cutoff $\bar{v}^n < \bar{v}^*$ and finish that project; and

(3) If $c < k$, naifs either never search or search with cutoff $\bar{v}^n \geq \bar{v}^*$ and then finish that project if and only if $v \geq \bar{v}^{nn}$, where $\bar{v}^{nn} \geq \bar{v}^n$.

Proposition 9 establishes that, once again, naifs incur costs without benefits only if finishing is more onerous than starting. More precisely, for $c \geq k$, if naifs search at all, then they will eventually complete some project, while for $c < k$, they might search but never complete any project. To illustrate this latter result, suppose $c = 20$, $k = 95$, and v is distributed uniformly on $[0, 1]$. TCs with $\delta = .9999$ search with cutoff $\bar{v}^* = .9353$ and then immediately finish; naifs with $\delta = .9999$ and $\beta = .99$ search with cutoff $\bar{v}^n = .9364$ but finish that project if and only if $v \geq \bar{v}^{nn} = .9692$. In this example, it is well worth finding a project with high benefits, and so TCs and naifs both search extensively — they both sample, on average, between 15 and 16 projects. But while TCs immediately finish once they find a satisfactory project, naifs might procrastinate. Indeed, in this example, despite their extensive sampling, the likelihood that naifs complete their chosen project is only 48%. But it is also the case that naifs who finish have projects that are on average 2% better than TCs.

Proposition 9 also establishes that, when deciding whether to finish a started project vs. searching further, naifs are biased towards choosing the activity that is less onerous. Specifically, conditional on searching, if $c > k$, naifs are too prone to finish an existing project and therefore $\bar{v}^n < \bar{v}^*$; and if $c < k$, naifs are too prone to start a new project and therefore $\bar{v}^n > \bar{v}^*$. When $c = k$, there is no distortion of this type, and so conditional on searching, naifs behave exactly like TCs.

Consider our example of a graduate student working on her dissertation. Proposition 9 implies that people who find preliminary work enjoyable but dislike the process of revising and polishing (i.e., have a low c and high k) tend to keep starting projects and not finish ones that they should. They will take too long to finish their dissertations, and end up with a dissertation whose quality is "too high" in the sense that their long-run well-being would have been higher had they had lower standards for their topic. In contrast, people who dislike preliminary work but enjoy the process of revising and polishing (i.e., have a high c and low k) tend not to search long enough. While they are prone to never get started on their dissertations, they finish whenever they start, but are likely to end up with a low-quality dissertation.[22]

Our final extension is perhaps our most striking demonstration of people incurring recurrent costs without ever receiving benefits. For many long-term projects, if a person starts a project but then delays before finishing, she must repeat some of her initial efforts. If, for instance, she works out preliminary results on a research project but then procrastinates in writing the paper, after a while she will not be able to write the paper without reviewing her earlier analysis. In such environments, naive people might repeatedly work out the same preliminary results without ever writing the paper.

To formalize this situation, we return to the case of a single two-stage project, where the cost structure (c, k) is exogenous and completion of the project in period τ initiates a stream of benefits $v > 0$ in each period from $\tau + 1$ onward. To introduce decay of earlier efforts in a particularly stark manner, we suppose that if the person does not finish the project immediately after starting, then she will have to completely re-do the first stage. To focus on the implications for procrastination, we once again examine behavior when $\delta \to 1$.

When there is decay, TCs clearly start and then finish the project immediately just as they would without decay. Proposition 10 describes the behavior of naifs:

[22] It is interesting to note that when $c > k$, sophistication can exacerbate the problem of settling for low benefits, because knowing she will settle for low benefits in the future in fact makes a person more prone to settle for a low benefits now.

Proposition 10. Suppose $\delta \to 1$ and $\beta < \hat{\beta} = 1$. If $c \geq \frac{\beta v}{1-\beta}$, then naifs never start the project; if $c < \frac{\beta v}{1-\beta}$ and $k < \frac{\beta(2v+c)}{1-\beta}$, then naifs start and then finish the project; and if $c < \frac{\beta v}{1-\beta}$ and $k \geq \frac{\beta(2v+c)}{1-\beta}$, then naifs repeatedly start the project but then delay finishing.

A comparison of Proposition 10 to Proposition 3 reveals three things of note. First, the possibility of decay does not affect whether naifs start the project. Intuitively, naifs never expect to delay, so the possibility of decay caused by delay seems irrelevant to them. Second, the possibility of decay makes it less likely that naifs procrastinate stage 2. Without decay, waiting merely means delaying finishing by one period. With decay, in contrast, waiting means delaying finishing by two periods and, importantly, having to incur the stage-1 cost for a second time. Hence, the costs of delay are larger with decay, making naifs more motivated to complete stage 2. But third, when this extra motivation is not enough to prevent procrastination at stage 2, naifs suffer a particularly unfortunate outcome: They repeatedly start the project but delay finishing, and hence incur the stage 1 cost over and over again.

6. Discussion and Conclusion

Our analysis in this paper identifies a number of results about time-inconsistent procrastination on long-term projects. While our formal analysis focuses on a highly stylized environment, we are confident that most of our conclusions apply qualitatively to more general settings. For instance, while our model focuses on two-stage projects, the lessons can be readily extended to longer-term projects. The key intuition that drives many of our results is that a person is most prone to procrastinate on the highest-cost stage, and this intuition clearly generalizes. Hence, for many-stage projects, if the highest-cost stage comes first, naive people will either complete the project or never start, whereas if the highest-cost stage occurs later, they might start the project but never finish. Indeed, if the highest-cost stage comes last, naive people might complete every stage of a many-stage project except the last stage, and as a result may expend nearly all of the total cost required to complete the project without receiving benefits.[23]

While our model assumes that no benefits accrue until after the entire project is completed, our results would also hold qualitatively if the person gets partial benefits from partial completion. Moreover, fixing the total benefit, if some of this benefit accrues upon partial completion, the person is, in fact, *less likely* to complete the project. Intuitively, allocating more benefit following

[23] As an extreme example, if a project involves 1000 days of effort $c = 8$ and just one day of effort $c = 16$, a naive person might put in 1000 days of "low" effort and yet never finish the project.

completion of stage 1 and less benefit following completion of stage 2 is much like allocating less cost to stage 1 and more cost to stage 2. Hence, if the person gets partial benefits from partial completion, then she is more prone to start but not finish the project, although the cost of doing so may be smaller.

The results in this paper highlight some more general themes. First, the microstructure — or fine details — of environments are important for people with time-inconsistent preferences in ways that don't matter for people with standard time-consistent preferences. Time-inconsistent people react to the same long-run incentives that time-consistent people react to — e.g., *ceteris paribus*, the higher the benefits and the lower the total effort costs, the more likely are naive procrastinators to complete a project quickly. But time-inconsistent people also react to other, short-run details. Our analysis in Section 4 on endogenizing the microstructure (the cost structure) extends this theme by illustrating that we should not necessarily expect people to choose the best microstructure.

Our analysis in Section 5 suggests that procrastination need not take the form of doing nothing at all, but rather might take the form of performing a low-cost activity rather than a high-cost activity. This distortion can be important whenever a person has a number of activities she might carry out and, because of time constraints, can only carry out some of them. For instance, new assistant professors have limited time to allocate between research and teaching. From numerous anecdotal conversations with colleagues, we suspect that some failures to do significant research can be attributed to procrastination in the form of allocating too much time to lower-cost teaching activities.

Finally, we note that the results in this paper could, if fleshed out, have implications in designing incentives to combat procrastination — both from a managerial perspective and from a government-policy perspective. In O'Donoghue and Rabin (1999*b*), we explore a simple model of designing a reward scheme to combat procrastination. Our analysis in the present paper suggests not just the importance of designing reward schemes, but also — to the extent possible — of designing projects themselves. For instance, because our analysis in Section 4 shows the potential drawbacks of giving people too much flexibility in how they pursue a project, it suggests that a firm might want to demand a particular schedule of work on a project. Taking our simple model literally, for instance, would suggest that even if there were variation among employees in their disutility of different parts of a project, it might help to impose virtually *any* schedule on employees rather than to leave it to each employee's own discretion.

Appendix A: Non-Optimistic Beliefs

In this appendix, we describe how our conclusions might change if we were to relax the optimistic-beliefs restriction. We remind the reader that there is a unique perception-perfect strategy for TCs and naifs, who have $\hat{\beta} = 1$; the statements in this appendix apply only for sophisticates and partial naifs, who have $\hat{\beta} < 1$.

Consider first what non-optimistic beliefs are possible. Suppose the person's perceived tolerance for delay of stage 2 is $d(\hat{\beta}, 2) = z > 0$. If $\widehat{s}$ represents $\hat{\beta}$-consistent beliefs, then for every $\tau \in \{1, 2, ...\}$ there exists $m' \in \{1, ..., z+1\}$ such that $\widehat{s}(\tau, \tau + m) = 1$ if and only if $m \in \{m', m' + (z+1), m' + 2(z+1), ...\}$. The optimistic-beliefs restriction requires $m' = 1$ for all τ. But clearly many other, non-optimistic beliefs are possible. We use the following terminology below: Beliefs are *pessimistic* if $m' = z + 1$ for all τ; and beliefs are *stationary* if m' is the same for all τ (so optimistic and pessimistic beliefs are both stationary). Also, we use the abbreviation PPS for perception-perfect strategy.

To illustrate one way in which non-optimistic beliefs might change our conclusions, consider how a PPS for sophisticates with optimistic beliefs might differ from a PPS for sophisticates with pessimistic beliefs. With optimistic beliefs, stage 1 is β-worthwhile if $-c + \beta\delta\left(-k + \frac{\delta v}{1-\delta}\right) \geq 0$; with pessimistic beliefs, stage 1 is β-worthwhile if $-c + \beta\delta^{z+1}\left(-k + \frac{\delta v}{1-\delta}\right) \geq 0$. If the former inequality holds while the latter does not, then any PPS with optimistic beliefs involves completing the project, whereas any PPS with pessimistic beliefs involves never starting the project.[24] This example reflects a more general result: For worthwhileness reasons, having non-optimistic beliefs can make it more likely that sophisticates never start the project (relative to having optimistic beliefs). However, because such effects disappear when $\delta \to 1$ — that is, when $\delta \to 1$ every PPS for sophisticates involves completing the project — we feel such effects are unimportant.

Having non-optimistic beliefs can also make it more likely that partial naifs never start the project for worthwhileness reasons. But for partial naifs, relaxing the optimistic-beliefs restriction can also change conclusions about procrastination on stage 1. Consider first the behavior of partial naifs given stationary beliefs that differ only in m'. With stationary beliefs, we can analyze

[24] Permitting non-optimistic beliefs requires a modified definition of stage 1 being β-worthwhile: Stage 1 is β-worthwhile in period t if $-c + \beta\delta^{m'+1}\left(-k + \frac{\delta v}{1-\delta}\right) \geq 0$ where $m' = \min\{x \in \{1, 2, ...\} | \widehat{s}(t, t+x) = 1\}$. Given this definition, we can show that sophisticates either complete the project or never start, and they never start if and only if either (i) stage 2 is not β-worthwhile or (ii) stage 1 is not β-worthwhile *in all periods*. Because different beliefs can alter the latter conclusion, for fixed parameters some PPS's might involve completing the project while other PPS's involve never starting the project.

procrastination merely by redefining the tolerance for delay of stage 1 to depend on m'. In other words, defining

$$d(\beta,1,m') \equiv \max\left\{d \in \{0,1,...\} \mid -c+\beta\delta^{m'}\left(-k+\frac{\delta v}{1-\delta}\right) < \beta\delta^{d+1}\left(-c+\delta^{m'}\left(-k+\frac{\delta v}{1-\delta}\right)\right)\right\},$$

the person procrastinates stage 1 if and only if $d(\beta,1,m') > d(\hat{\beta},1,m')$. The more pessimistic are beliefs, the more the person is willing to tolerate delay — that is, $d(\beta,1,m')$ and $d(\hat{\beta},1,m')$ are both (weakly) increasing in m'. And due to the non-monotonicities discussed in Section 3 (see in particular Footnote 15), having more pessimistic beliefs could make the person more or less likely to procrastinate on stage 1. But since these effects disappear when $\delta \to 1$ — that is, when $\delta \to 1$, $d(\beta,1,m')$ is independent of m' — we again believe such effects are unimportant.

Having non-optimistic beliefs can have more substantive effects on partial naifs when we allow non-stationary beliefs. To illustrate, suppose $d(\hat{\beta},2) = 1$ and $d(\beta,1) = 1 > 0 = d(\hat{\beta},1)$. If the person has optimistic beliefs, she procrastinates stage 1. Suppose instead that the person has non-stationary (and date-specific) beliefs $\widehat{s}$ that satisfy $\widehat{s}(\tau,t) = 1$ if and only if $t \in \{2,4,6,...\}$ for any $\tau \in \{1,2,...\}$ — that is, the person believes she would work on stage 2 only in periods 2, 4, 6,..., regardless of when she completes stage 1. One can show that, given $d(\beta,1) = 1$, such beliefs imply the person completes stage 1 in period 1, because she is unwilling to wait two periods until period 3 to start working. This example reflects a more general result: Non-stationary beliefs can create "deadlines" — dates on which there is a larger incentive to act driven by one's beliefs about future inaction — that help motivate partial naifs not to procrastinate stage 1. Such effects do not disappear as $\delta \to 1$. It is worth noting that relaxing the optimistic-beliefs restriction does not change our conclusions for behavior on stage 2. Hence, relaxing the optimistic-beliefs restriction can make procrastination on stage 1 less likely while not changing the likelihood of procrastination on stage 2, and hence might make the person more prone to incur costs without benefits.[25]

Finally, we note that our analysis in Section 4 of endogenous cost structure can also be sensitive to the optimistic-beliefs restriction (for sophisticates and partial naifs). In particular, with endogenous cost structure, in addition to the question of whether beliefs are stationary, there is also the question of whether beliefs depend on the cost structure chosen. If beliefs are both stationary and independent of the cost structure chosen, then our conclusions in Section 4 are mostly unchanged. But otherwise belief-driven deadlines can motivate action, as above, and moreover belief-driven incentives can influence the choice of cost structure.

[25] This conclusion further reinforces our point in Footnote 16 that partial naifs can suffer worse outcomes than naifs on long-term projects.

Appendix B: Proofs

Recall that we assume for simplicity that when a person is indifferent between $a = 0$ and $a = 1$, she chooses $a = 1$ (see Footnote 9). Also, we use the abbreviation PPS for perception-perfect strategy.

Proof of Lemma 1. (1) The complete argument is long but straightforward, so we omit the details. In short, given $\hat{\beta} = 1$, one can show that if stage 1 is $\hat{\beta}$-worthwhile (which for $\hat{\beta} = 1$ implies stage 2 is $\hat{\beta}$-worthwhile), then any $\hat{\beta}$-consistent beliefs $\widehat{s}$ must have $\widehat{s}(h^t, t) = 1$ for all t and h^t. If stage 1 is not $\hat{\beta}$-worthwhile but stage 2 is $\hat{\beta}$-worthwhile, then any $\hat{\beta}$-consistent beliefs $\widehat{s}$ must have $\widehat{s}(\emptyset, t) = 0$ for all t and $\widehat{s}(h^t, t) = 1$ for all t and $h^t \neq \emptyset$. Finally, if stages 1 and 2 both are not $\hat{\beta}$-worthwhile, then any $\hat{\beta}$-consistent beliefs $\widehat{s}$ must have $\widehat{s}(h^t, t) = 0$ for all t and h^t. In each case, a unique $\widehat{s}$ satisfies these conditions. Given there is a unique set of $\hat{\beta}$-consistent beliefs, there exists a unique PPS.

(2) We fully characterize the set of PPS's with optimistic beliefs.

Consider behavior on stage 2. Suppose $d(\hat{\beta}, 2) = \infty$, or $-k + \frac{\hat{\beta}\delta v}{1-\delta} < 0$. Because $-k + \frac{\hat{\beta}\delta v}{1-\delta} < 0$ implies $-k + \frac{\hat{\beta}\delta v}{1-\delta} < \hat{\beta}\delta^x \left(-k + \frac{\delta v}{1-\delta}\right)$ for all $x \in \{1, 2, ...\}$, $\arg\max_{a\in\{0,1\}} V^t(a; h^t, s', \hat{\beta}) = 0$ for any s' and $h^t \neq \emptyset$. Hence, any $\hat{\beta}$-consistent beliefs $\widehat{s}$ must have $\widehat{s}(h^t, t) = 0$ for all t and $h^t \neq \emptyset$. Also, because $d(\hat{\beta}, 2) = \infty$ implies $d(\beta, 2) = \infty$, a similar logic yields that any PPS s must have $s(h^t, t) = 0$ for all t and $h^t \neq \emptyset$.

Suppose $d(\hat{\beta}, 2) = z_2 < \infty$. For any $h^t \neq \emptyset$, $\arg\max_{a\in\{0,1\}} V^t(a; h^t, s', \hat{\beta}) = 0$ if and only if $\min\{x \in \{1, 2, ...\} \mid s'(h^t, t + x) = 1\} \leq z_2$. It follows that any $\hat{\beta}$-consistent beliefs $\widehat{s}$ must satisfy the following condition: For every $h^t \neq \emptyset$ there exists $y \in \{1, ..., z_2 + 1\}$ such that $\widehat{s}(h^t, h^t + x) = 1$ if and only if $x \in \{y, y+(z_2+1), y+2(z_2+1), ...\}$. The restriction to optimistic beliefs then chooses $y = 1$. Hence, if $d(\hat{\beta}, 2) = z_2 < \infty$, then any optimistic $\hat{\beta}$-consistent beliefs $\widehat{s}$ must have, for every $h^t \neq \emptyset$, $\widehat{s}(h^t, h^t + x) = 1$ if and only if $x \in \{1, 1 + (z_2 + 1), 1 + 2(z_2 + 1), ...\}$.

Given any $\widehat{s}$ with this property, for any t and $h^t \neq \emptyset$, $V^t(0; h^t, \widehat{s}, \beta) \geq \beta\delta^{z_2+1}\left(-k + \frac{\delta v}{1-\delta}\right)$. Because $d(\beta, 2) > d(\hat{\beta}, 2)$ implies $V^t(1; h^t, \widehat{s}, \beta) = -k + \frac{\beta\delta v}{1-\delta} < \beta\delta^{z_2+1}\left(-k + \frac{\delta v}{1-\delta}\right)$, it follows that $d(\beta, 2) > d(\hat{\beta}, 2)$ implies $s(h^t, t) = 0$ for all t and $h^t \neq \emptyset$. In contrast, $d(\beta, 2) = d(\hat{\beta}, 2) = z_2$ implies $V^t(1; h^t, \widehat{s}, \beta) < \beta\delta^d\left(-k + \frac{\delta v}{1-\delta}\right)$ if $d < z_2 + 1$ but $V^t(1; h^t, \widehat{s}, \beta) \geq \beta\delta^d\left(-k + \frac{\delta v}{1-\delta}\right)$ if $d = z_2 + 1$. It follows that $d(\beta, 2) = d(\hat{\beta}, 2) = z_2$ implies any PPS s must have, for every $h^t \neq \emptyset$, $s(h^t, h^t + x) = 1$ if and only if $x \in \{1, 1 + (z_2 + 1), 1 + 2(z_2 + 1), ...\}$.

Now consider behavior on stage 1. If $d(\hat{\beta}, 2) = \infty$ and therefore any $\hat{\beta}$-consistent beliefs have $\widehat{s}(h^t, t) = 0$ for all t and $h^t \neq \emptyset$, then clearly any PPS s must have $s(\emptyset, t) = 0$ for all t. Suppose instead $d(\hat{\beta},2)=z_2<\infty$, so that any optimistic $\hat{\beta}$-consistent beliefs $\widehat{s}$ have $\widehat{s}(h^t,h^t+1)=1$ for every $h^t\neq\emptyset$. In this case, the logic for stage 1 is exactly analogous to that for stage 2 except that there is no analogue to the optimistic-beliefs restriction. The conclusions are (we omit the details): (i) If $d(\hat{\beta},1)=\infty$ then any PPS s must have $s(\emptyset,t)=0$ for all t; (ii) if $d(\beta,1)>d(\hat{\beta},1)$ then any PPS s must have $s(\emptyset,t)=0$ for all t; and (iii) if $d(\beta,1)=d(\hat{\beta},1)=z_1<\infty$ then for any PPS s there exists $y\in\{1,...,z_1+1\}$ such that $s(\emptyset,t)=1$ if and only if $t\in\{y,y+(z_1+1),y+2(z_1+1),...\}$.

Combining behavior on stages 1 and 2, there are four possible cases. (1) If $d(\hat{\beta},2)=\infty$, or if $d(\beta,2)>d(\hat{\beta},2)$ and either $d(\hat{\beta},1)=\infty$ or $d(\beta,1)>d(\hat{\beta},1)$, then there is a unique PPS s, which satisfies $s(h^t,t)=0$ for all t and h^t. (2) If $d(\beta,2)=d(\hat{\beta},2)=z_2$ and either $d(\hat{\beta},1)=\infty$ or $d(\beta,1)>d(\hat{\beta},1)$, then there is a unique PPS with optimistic beliefs s, which satisfies (i) $s(\emptyset,t)=0$ for all t and (ii) for every $h^t\neq\emptyset$, $s(h^t,h^t+x)=1$ if and only if $x\in\{1,1+(z_2+1),1+2(z_2+1),...\}$. (3) If $d(\beta,2)>d(\hat{\beta},2)$ and $d(\beta,1)=d(\hat{\beta},1)=z_1$, then there (may) exist multiple PPS's with optimistic beliefs, but any such strategy s satisfies (i) there exists $y\in\{1,...,z_1+1\}$ such that $s(\emptyset,t)=1$ if and only if $t\in\{y,y+(z_1+1),y+2(z_1+1),...\}$ and (ii) $s(h^t,t)=0$ for all t and $h^t\neq\emptyset$. (4) If $d(\beta,2)=d(\hat{\beta},2)=z_2$ and $d(\beta,1)=d(\hat{\beta},1)=z_1$, then there (may) exist multiple PPS's with optimistic beliefs, but any such strategy s satisfies (i) there exists $y\in\{1,...,z_1+1\}$ such that $s(\emptyset,t)=1$ if and only if $t\in\{y,y+(z_1+1),y+2(z_1+1),...\}$ and (ii) for every $h^t\neq\emptyset$, $s(h^t,h^t+x)=1$ if and only if $x\in\{1,1+(z_2+1),1+2(z_2+1),...\}$.

Cases 1 and 2 both satisfy statement 2a in Lemma 1, case 3 satisfies statement 2b, case 4 satisfies statement 2c, and so the result follows.

Proof of Proposition 1. Note that the four cases in the proof of Lemma 1 part 2 in fact hold for $\hat{\beta}=1$ as well, although $\hat{\beta}=1$ implies $d(\beta,n)\in\{0,\infty\}$ and $d(\hat{\beta},n)\in\{0,\infty\}$.

(1) The person completes stage 1 in cases 3 and 4, which hold if and only if $d(\hat{\beta},2)<\infty$ (stage 2 is $\hat{\beta}$-worthwhile) and $d(\beta,1)=d(\hat{\beta},1)<\infty$ (stage 1 is β-worthwhile and $d(\beta,1)=d(\hat{\beta},1)$).

(2) The person completes stage 2 (conditional on completing stage 1) in cases 2 and 4, which hold if and only if $d(\beta,2)=d(\hat{\beta},2)<\infty$ (stage 2 is β-worthwhile and $d(\beta,2)=d(\hat{\beta},2)$).

(3) Because $d(\beta,1)\geq d(\hat{\beta},1)$, $d(\beta,1)=d(\hat{\beta},1)$ if $d(\beta,1)=0$, which holds if $-c+\beta\delta\left(-k+\frac{\delta v}{1-\delta}\right)\geq -\beta\delta c+\beta\delta^2\left(-k+\frac{\delta v}{1-\delta}\right)$ or $c\leq\frac{\beta\delta(1-\delta)}{1-\beta\delta}\left(-k+\frac{\delta v}{1-\delta}\right)$.

$d(\beta,1)$ satisfies $\beta\delta^{d(\beta,1)}\left(-c-\delta k+\frac{\delta^2 v}{1-\delta}\right)>-c+\beta\delta\left(-k+\frac{\delta v}{1-\delta}\right)\geq\beta\delta^{d(\beta,1)+1}\left(-c-\delta k+\frac{\delta^2 v}{1-\delta}\right)$, or equivalently $\delta^{d(\beta,1)}\left(-c-\delta k+\frac{\delta^2 v}{1-\delta}\right)>-\frac{c}{\beta}+\delta\left(-k+\frac{\delta v}{1-\delta}\right)\geq\delta^{d(\beta,1)+1}\left(-c-\delta k+\frac{\delta^2 v}{1-\delta}\right)$. Similarly, $d(\hat{\beta},1)$ satisfies $\delta^{d(\hat{\beta},1)}\left(-c-\delta k+\frac{\delta^2 v}{1-\delta}\right)>-\frac{c}{\hat{\beta}}+\delta\left(-k+\frac{\delta v}{1-\delta}\right)\geq\delta^{d(\hat{\beta},1)+1}\left(-c-\delta k+\frac{\delta^2 v}{1-\delta}\right)$. It follows that $d(\beta,1)=d(\hat{\beta},1)$ only if $-\frac{c}{\beta}+\delta\left(-k+\frac{\delta v}{1-\delta}\right)>\delta\left(-\frac{c}{\hat{\beta}}+\delta\left(-k+\frac{\delta v}{1-\delta}\right)\right)$ or $c<$

$\frac{\beta\delta(1-\delta)}{1-\beta\delta/\hat{\beta}}\left(-k+\frac{\delta v}{1-\delta}\right)$. An analogous argument holds for stage 2.

Proof of Corollary 1. TCs and sophisticates both have $\beta=\hat{\beta}$, so $d(\beta,1)=d(\hat{\beta},1)$ and $d(\beta,2)=d(\hat{\beta},2)$, and moreover stage 2 being $\hat{\beta}$-worthwhile is equivalent to stage 2 being β-worthwhile. Hence, TCs and sophisticates complete the project if and only if both stages are β-worthwhile, and otherwise they never start. For TCs, given $\beta=\hat{\beta}=1$, stage 1 is β-worthwhile if and only if $-c-\delta k+\frac{\delta^2 v}{1-\delta}\geq 0$, and moreover this implies stage 2 is β-worthwhile. For sophisticates, given $\beta=\hat{\beta}<1$, stage 1 is β-worthwhile if and only if $-c-\beta\delta k+\frac{\beta\delta^2 v}{1-\delta}\geq 0$, and stage 2 is β-worthwhile if and only if $-k+\frac{\beta\delta v}{1-\delta}\geq 0$. The result follows.

Proof of Proposition 2. From Corollary 1, TCs never start if and only if $-c-\delta k+\frac{\delta^2 v}{1-\delta}=-C+V<0$, or $C/V>1$. Also from Corollary 1, sophisticates never start only if either $-c-\beta\delta k+\frac{\beta\delta^2 v}{1-\delta}<0$ or $-k+\frac{\beta\delta v}{1-\delta}<0$. In the former case, because $-c-\beta\delta k+\frac{\beta\delta^2 v}{1-\delta}=-C+\beta V+(1-\beta)\delta k>-C+\beta V$ (given $k>0$), $-c-\beta\delta k+\frac{\beta\delta^2 v}{1-\delta}<0$ implies $-C+\beta V<0$, or $C/V>\beta$. In the latter case, $-k+\frac{\beta\delta v}{1-\delta}<0$ implies $-\delta k+\frac{\beta\delta^2 v}{1-\delta}<0$; and because $-\delta k+\frac{\beta\delta^2 v}{1-\delta}>-c-\delta k+\frac{\beta\delta^2 v}{1-\delta}=-C+\beta V$, $-\delta k+\frac{\beta\delta^2 v}{1-\delta}<0$ implies $-C+\beta V<0$, or $C/V>\beta$. The result follows.

Proof of Proposition 3. For each statement, we prove there exists $\bar{\delta}<1$ such that the statement holds for all $\delta\in(\bar{\delta},1)$.

(1) Stage 1 is β-worthwhile if $-c-\beta\delta k+\frac{\beta\delta^2 v}{1-\delta}\geq 0$, and stage 2 is β-worthwhile (and, given $\hat{\beta}=\beta$, $\hat{\beta}$-worthwhile) if $-k+\frac{\beta\delta v}{1-\delta}\geq 0$. Because $\lim_{\delta\to 1}\left[-c-\beta\delta k+\frac{\beta\delta^2 v}{1-\delta}\right]=\lim_{\delta\to 1}\left[-k+\frac{\beta\delta v}{1-\delta}\right]=\infty$, there exists $\delta'<1$ such that for all $\delta\in(\delta',1)$ stages 1 and 2 are both β-worthwhile. Finally, because $\hat{\beta}=\beta$ implies $d(\hat{\beta},1)=d(\beta,1)$ and $d(\hat{\beta},2)=d(\beta,2)$, Proposition 1 implies that for all $\delta\in(\delta',1)$ the person completes both stages.

(2) Define δ' as in the proof of part 1, and note that because $-k+\frac{\beta\delta v}{1-\delta}$ is increasing in β, stage 2 being β-worthwhile implies stage 2 is also $\hat{\beta}$-worthwhile. Hence, for all $\delta\in(\delta',1)$, the person completes stage $n\in\{1,2\}$ if and only if $d(\hat{\beta},n)=d(\beta,n)$.

Given $\hat{\beta}=1$, Part 3 of Proposition 1 implies the person completes stage 1 if and only if $c\leq-\frac{\beta\delta(1-\delta)k}{1-\beta\delta}+\frac{\beta\delta^2 v}{1-\beta\delta}$. Suppose $c\geq\frac{\beta v}{1-\beta}$. Because $\frac{\beta v}{1-\beta}>\frac{\beta\delta^2 v}{1-\beta\delta}$ for all $\delta<1$, it follows that $c>-\frac{\beta\delta(1-\delta)k}{1-\beta\delta}+\frac{\beta\delta^2 v}{1-\beta\delta}$ for all $\delta<1$, and therefore for all $\delta<1$ the person does not complete stage 1. Suppose $c<\frac{\beta v}{1-\beta}$. Because $\lim_{\delta\to 1}\left[-\frac{\beta\delta(1-\delta)k}{1-\beta\delta}+\frac{\beta\delta^2 v}{1-\beta\delta}\right]=\frac{\beta v}{1-\beta}$, there exists $\delta''<1$ such that $c<-\frac{\beta\delta(1-\delta)k}{1-\beta\delta}+\frac{\beta\delta^2 v}{1-\beta\delta}$ for all $\delta\in(\delta'',1)$. Hence, defining $\bar{\delta}\equiv\max\{\delta',\delta''\}<1$, naifs complete stage 1 for all $\delta\in(\bar{\delta},1)$. The argument for stage 2 is analogous.

(3) Suppose $c<\frac{\beta v}{1-\beta}$. Part 3 of Proposition 1 implies that, just as for naifs, the person completes stage 1 if $c\leq-\frac{\beta\delta(1-\delta)k}{1-\beta\delta}+\frac{\beta\delta^2 v}{1-\beta\delta}$, and hence our argument in the proof of part 2 implies that there

exists $\bar{\delta} < 1$ such that the person completes stage 1 for all $\delta \in (\bar{\delta}, 1)$.

Suppose $c \geq \frac{\beta v}{1-\beta/\hat{\beta}}$. Because $\frac{\beta v}{1-\beta/\hat{\beta}} > \frac{\beta\delta^2 v}{1-\beta\delta/\hat{\beta}}$ for all $\delta < 1$, it follows that $c > -\frac{\beta\delta(1-\delta)k}{1-\beta\delta/\hat{\beta}} + \frac{\beta\delta^2 v}{1-\beta\delta/\hat{\beta}}$ for all $\delta < 1$. Hence, by Part 3 of Proposition 1, for all $\delta < 1$ the person does not complete stage 1.

Again, the argument for stage 2 is analogous.

Proof of Proposition 4. By Proposition 1, the person completes stage 1 if $-c-\beta\delta k+\frac{\beta\delta^2 v}{1-\delta} \geq 0$ (stage 1 is β-worthwhile), $-k+\frac{\hat{\beta}\delta v}{1-\delta} \geq 0$ (stage 2 is $\hat{\beta}$-worthwhile), and $c \leq -\frac{\beta\delta(1-\delta)k}{1-\beta\delta} + \frac{\beta\delta^2 v}{1-\beta\delta}$ (which guarantees $d(\hat{\beta}, 1) = d(\beta, 1)$). The person does not complete stage 2 if $-k+\frac{\beta\delta v}{1-\delta} < 0$ (stage 2 is not β-worthwhile) or $k > \frac{\beta\delta v}{1-\beta\delta/\hat{\beta}}$ (which guarantees $d(\hat{\beta}, 2) < d(\beta, 2)$). Note that $c \leq -\frac{\beta\delta(1-\delta)k}{1-\beta\delta} + \frac{\beta\delta^2 v}{1-\beta\delta}$ implies $-c - \beta\delta k + \frac{\beta\delta^2 v}{1-\delta} \geq 0$, and $-k + \frac{\beta\delta v}{1-\delta} < 0$ only if $k > \frac{\beta\delta v}{1-\beta\delta/\hat{\beta}}$. It follows that the person completes stage 1 but never completes stage 2 if three conditions hold: (i) $-k + \frac{\hat{\beta}\delta v}{1-\delta} \geq 0$, (ii) $c \leq -\frac{\beta\delta(1-\delta)k}{1-\beta\delta} + \frac{\beta\delta^2 v}{1-\beta\delta}$, and (iii) $k > \frac{\beta\delta v}{1-\beta\delta/\hat{\beta}}$. Hence, we need to prove that for all β, δ, $\hat{\beta} > \beta$, and c, there exists k and v such that all three conditions are satisfied.

Fix β, δ, $\hat{\beta} > \beta$, and c, and define $f(v) \equiv \frac{\beta\delta v}{1-\beta\delta/\hat{\beta}}$. Because $\hat{\beta} > \beta$ implies $\frac{\beta\delta v}{1-\beta\delta/\hat{\beta}} < \frac{\hat{\beta}\delta v}{1-\delta}$, for all $v > 0$ there exists $x(v) > 0$ such that for all $k \in (f(v), f(v) + x(v))$ conditions (i) and (iii) are both satisfied. Note that condition (ii) can be rewritten as $k \leq -\frac{(1-\beta\delta)c}{\beta\delta(1-\delta)} + \frac{\delta v}{1-\delta}$. Because $\frac{\beta\delta v}{1-\beta\delta/\hat{\beta}} < \frac{\delta v}{1-\delta}$, there exists $v' > 0$ such that for all $v > v'$ there exists $y(v) > 0$ such that for all $k \in (f(v), f(v) + y(v))$ conditions (ii) and (iii) are both satisfied. Hence, for any $v > v'$ and $k \in (f(v), f(v) + \min\{x(v), y(v)\})$, conditions (i), (ii), and (iii) are all satisfied.

Proof of Lemma 2. Because every $(c, k) \in \mathbf{P}^{A,\bar{a}}$ has $c+k = A$, we can transform the problem into choosing $k \in [A - \bar{a}, \bar{a}]$ to maximize $g(k) \equiv -(A - k) - \beta\delta k + \frac{\beta\delta^2 v}{1-\delta}$ such that stage 1 is β-worthwhile and stage 2 is $\hat{\beta}$-worthwhile. And since $g(k)$ is increasing in k, we merely choose the largest $k \in [A - \bar{a}, \bar{a}]$ such that stage 1 is β-worthwhile and stage 2 is $\hat{\beta}$-worthwhile.

(1) When $\hat{\beta} = 1$, stage 1 being β-worthwhile implies stage 2 is $\hat{\beta}$-worthwhile, and therefore the only constraint is stage 1 being β-worthwhile. Stage 1 is β-worthwhile when $g(k) \geq 0$, and since $g(k)$ is increasing in k, there exists k' such that $g(k) \geq 0$ for all $k \geq k'$. If $k' > \bar{a}$, $\mathbf{P}'(\mathbf{P}^{A,\bar{a}})$ is empty and therefore $\mathbf{p}^*(\mathbf{P}^{A,\bar{a}}) = \emptyset$. If $k' \leq \bar{a}$, the largest $k \in [A - \bar{a}, \bar{a}]$ such that stage 1 is β-worthwhile is $\bar{a}$, and therefore $\mathbf{p}^*(\mathbf{P}^{A,\bar{a}}) = (A - \bar{a}, \bar{a})$.

(2) Define k' as in part 1. Stage 2 is $\hat{\beta}$-worthwhile for all $k \leq \frac{\hat{\beta}\delta v}{1-\delta}$. If either $k' > \bar{a}$ or $k' > \frac{\hat{\beta}\delta v}{1-\delta}$, then $\mathbf{P}'(\mathbf{P}^{A,\bar{a}})$ is empty and therefore $\mathbf{p}^*(\mathbf{P}^{A,\bar{a}}) = \emptyset$. Otherwise, the largest $k \in [A - \bar{a}, \bar{a}]$ such that stage 1 is β-worthwhile and stage 2 is $\hat{\beta}$-worthwhile is $\min\left\{\bar{a}, \frac{\hat{\beta}\delta v}{1-\delta}\right\} \equiv x_o$, and therefore $\mathbf{p}^*(\mathbf{P}^{A,\bar{a}}) = (A - x_o, x_o)$.

(3) Since $\lim_{\delta\to 1} k' = -\infty$ and $\lim_{\delta\to 1} \frac{\hat{\beta}\delta v}{1-\delta} = \infty$, there exists $\bar{\delta} < 1$ such that $k' \leq \bar{a} \leq \frac{\hat{\beta}\delta v}{1-\delta}$ for

all $\delta \in (\bar{\delta}, 1)$. It follows that $\mathbf{p}^*(\mathbf{P}^{A,\bar{a}}) = (A - \bar{a}, \bar{a})$ for all $\delta \in (\bar{\delta}, 1)$.

Proof of Proposition 5. (1) Because $(c, k) \in \mathbf{P}^{\text{endog}}(c, k)$, if (c, k) has stage 1 β-worthwhile and stage 2 $\hat{\beta}$-worthwhile, then $\mathbf{P}'(\mathbf{P}^{\text{endog}}(c, k))$ is non-empty and therefore $\mathbf{p}^*(\mathbf{P}^{\text{endog}}(c, k))$ exists. Moreover, by definition, if $\mathbf{p}^*(\mathbf{P}^{\text{endog}}(c, k))$ exists, then it has stage 1 β-worthwhile and stage 2 $\hat{\beta}$-worthwhile.

(2) If the person completes the project under exogenous cost structure (c, k), then (c, k) has stage 1 β-worthwhile and stage 2 β-worthwhile (by Proposition 1), and so $\mathbf{p}^*(\mathbf{P}^{\text{endog}}(c, k))$ exists and has stage 1 β-worthwhile and stage 2 β-worthwhile. Suppose $\mathbf{p}^*(\mathbf{P}^{\text{endog}}(c, k)) = (c', k')$. Given stage 2 β-worthwhile, and given optimistic beliefs, the person always believes that if she completes stage 1 (incurs c') now then she will complete stage 2 (incur k') next period. That is, one option is to complete (c', k') beginning now. Because $\hat{\beta} = \beta$, the person has correct beliefs, and given stage 1 is β-worthwhile, the person waits only if she will complete the project in the future. It follows that she must (eventually) complete the project.

Proof of Proposition 6. (1) Follows from Proposition 3 part 1 and Proposition 5 part 2.

(2) Under exogenous cost structure (c, k), by Proposition 3 part 2, the person completes stage 1 if and only if $c < \frac{\beta v}{1-\beta}$, and the person completes the project if and only if $\max\{c, k\} < \frac{\beta v}{1-\beta}$. Under endogenous cost structure, Lemma 2 implies $\mathbf{p}^*(\mathbf{P}^{\text{endog}}(c, k)) = ((c+k) - \bar{a}, \bar{a})$. Applying the logic from Proposition 3 part 2, the person completes stage 1 if and only if $(c + k) - \bar{a} < \frac{\beta v}{1-\beta}$, and the person completes the project if and only if $\max\{(c+k) - \bar{a}, \bar{a}\} = \bar{a} < \frac{\beta v}{1-\beta}$. Because $(c+k) - \bar{a} \leq c$ (given $k \leq \bar{a}$), the person is more likely to complete stage 1 under $\mathbf{P}^{\text{endog}}(c, k)$ than under (c, k). Because $\bar{a} \geq \max\{c, k\}$, the person is less likely to complete the project under $\mathbf{P}^{\text{endog}}(c, k)$ than under (c, k).

Proof of Proposition 7. Note $x'(y) = \frac{-h'(y)}{h'(x(y))}$, which implies $x'(x^*) = -1$ (since $x(x^*) = x^*$) and *sign* $x'' =$ *sign* $-h''$. Also, for δ close enough to 1, all relevant possibilities have stages 1 and 2 both β-worthwhile, and so $\mathbf{p}^*\left(\tilde{\mathbf{P}}^{\text{endog}}(c, k)\right) = (x(y^*), y^*)$ where y^* is the $y \in [x(\bar{a}), \bar{a}]$ that maximizes $\tilde{g}(y) \equiv -x(y) - \beta\delta y + \frac{\beta\delta^2 v}{1-\delta}$. Finally, note $\tilde{g}'(y) = -x'(y) - \beta\delta$ and $\tilde{g}''(y) = -x''(y)$.

(1) $h'' > 0$ implies $x'' < 0$, which implies $\tilde{g}'' > 0$. Hence, we have a corner solution, and given $\beta\delta < 1$ the optimum involves $y^* = \bar{a}$, or $\mathbf{p}^*\left(\tilde{\mathbf{P}}^{\text{endog}}(c, k)\right) = (x(\bar{a}), \bar{a})$. Because this holds for any β, including $\beta = 1$, the person never even considers any project besides $(x(\bar{a}), \bar{a})$, and so the argument in the proof of Proposition 6 holds here.

(2) $h'' < 0$ implies $x'' > 0$, which implies $\tilde{g}'' < 0$ and so we might have an interior solution. Because $\tilde{g}'(x^*) = -(-1) - \beta\delta > 0$ implies $y^* > x^*$, it follows that $\mathbf{p}^*\left(\tilde{\mathbf{P}}^{\text{endog}}(c, k)\right) = (x(y^*), y^*)$ for

some $y^* \in (x^*, \bar{a}]$. The optimal y^* may depend on β, and so a person currently working on stage 1 may think she would complete a different project if she waits. Specifically, letting $(x(y^{**}), y^{**})$ be the project done by TCs (given $\beta = 1$), a person with $\hat{\beta} = 1$ who is working on stage 1 compares completing $(x(y^*), y^*)$ beginning now to completing $(x(y^{**}), y^{**})$ beginning next period. If she does complete stage 1, then her decision for stage 2 is identical to that in the basic model.

Posit $y^* \geq \max\{c, k\}$. When $\delta \to 1$, the person completes stage 2 under $\tilde{\mathbf{P}}^{\text{endog}}(c, k)$ if and only if $y^* < \frac{\beta v}{1-\beta}$. Under exogenous cost structure (c, k), she completes the project if and only if $\max\{c, k\} < \frac{\beta v}{1-\beta}$ (applying Proposition 3). It follows that she completes the project under $\tilde{\mathbf{P}}^{\text{endog}}(c, k)$ only if she completes it under (c, k). Note that if $y^* \geq \frac{\beta v}{1-\beta} > \max\{c, k\}$, the person does not complete the project under $\tilde{\mathbf{P}}^{\text{endog}}(c, k)$ but she does under (c, k), so the *if* direction does not hold.

Posit $y^* < \max\{c, k\}$, and note that in this case, $\min\{c, k\} < x(y^*) < x^* < y^* < \max\{c, k\}$. Under $\tilde{\mathbf{P}}^{\text{endog}}(c, k)$, the person completes stage 1 if and only if $-x(y^*) - \beta\delta y^* + \frac{\beta\delta^2 v}{1-\delta} \geq -\beta\delta x(y^{**}) - \beta\delta^2 y^{**} + \frac{\beta\delta^3 v}{1-\delta}$. By revealed preference, $-x(y^*) - \beta\delta y^* + \frac{\beta\delta^2 v}{1-\delta} \geq -x(y^{**}) - \beta\delta y^{**} + \frac{\beta\delta^2 v}{1-\delta}$, and so the person completes stage 1 if $-x(y^{**}) - \beta\delta y^{**} + \frac{\beta\delta^2 v}{1-\delta} \geq -\beta\delta x(y^{**}) - \beta\delta^2 y^{**} + \frac{\beta\delta^3 v}{1-\delta}$ or $\frac{\beta\delta^2 v}{1-\beta\delta} \geq x(y^{**}) + \frac{\beta\delta(1-\delta)}{1-\beta\delta} y^{**}$. Because $\lim_{\delta\to1} y^{**} = \lim_{\delta\to1} x(y^{**}) = x^*$, if $x^* < \frac{\beta v}{1-\beta}$ then there exists $\bar{\delta} < 1$ such that this last inequality holds for all $\delta \in (\bar{\delta}, 1)$. If the person completes the project under (c, k), then $\max\{c, k\} < \frac{\beta v}{1-\beta}$. Because this implies $x^* < \frac{\beta v}{1-\beta}$, the person completes stage 1 under $\tilde{\mathbf{P}}^{\text{endog}}(c, k)$. Because this also implies $y^* < \frac{\beta v}{1-\beta}$, the person also completes stage 2 under $\tilde{\mathbf{P}}^{\text{endog}}(c, k)$. It follows that she completes the project under $\tilde{\mathbf{P}}^{\text{endog}}(c, k)$ if she completes it under (c, k). Note that if $y^* < \frac{\beta v}{1-\beta} \leq \max\{c, k\}$, the person completes the project under $\tilde{\mathbf{P}}^{\text{endog}}(c, k)$ but does not under (c, k), and so the *only if* direction does not hold.

Proof of Proposition 8. (1) Consider the person's decision when she has n unstarted projects and m started but unfinished projects. Given $\hat{\beta} = 1$ (and given everything is worthwhile when $\delta \to 1$), the person believes that (i) if she starts a project now, she will next finish the $m+1$ started projects, and then start and finish the remaining $n-1$ unstarted projects in succession; (ii) if she finishes a project now, she will next finish the remaining $m-1$ started projects, and then start and finish the n unstarted projects in succession; (iii) if she waits now, she will next finish the m started projects, and then start and finish the n unstarted projects in succession. Hence, given n and m, her payoffs from her three possible actions are:

Payoff if start: $-c + \beta \sum_{i=1}^{m+1} \delta^i \left(-k + \frac{\delta v}{1-\delta}\right) + \beta\delta^m \sum_{i=1}^{n-1} \delta^{2i} \left(-c - \delta k + \frac{\delta^2 v}{1-\delta}\right)$.

Payoff if finish: $\left(-k + \frac{\beta\delta v}{1-\delta}\right) + \beta \sum_{i=1}^{m-1} \delta^i \left(-k + \frac{\delta v}{1-\delta}\right) + \beta\delta^{m-2} \sum_{i=1}^{n} \delta^{2i} \left(-c - \delta k + \frac{\delta^2 v}{1-\delta}\right)$.

Payoff if wait: $0 + \beta \sum_{i=1}^{m} \delta^i \left(-k + \frac{\delta v}{1-\delta}\right) + \beta\delta^{m-1} \sum_{i=1}^{n} \delta^{2i} \left(-c - \delta k + \frac{\delta^2 v}{1-\delta}\right)$.

She prefers starting to waiting if and only if $\beta\delta^m \sum_{i=1}^{n} \delta^{2i} v \geq (1-\beta\delta^{m+1})c + \beta\delta^{m+1}(1-\delta)k + \beta\delta^m \sum_{i=1}^{n-1} \delta^{2i}(1-\delta)(-c-\delta k)$. Because as $\delta \to 1$ the left-hand side goes to $\beta n v$ and the right-hand side goes to $(1-\beta)c$, the logic from Proposition 3 implies that when $\delta \to 1$, the person prefers starting to waiting if and only if $c < \frac{\beta n v}{1-\beta}$. Defining $n_{PS} \equiv \max\{n \in \{0,1,...\} | c \geq \frac{\beta n v}{1-\beta}\}$, it follows that the person will start a new project only if $n > n_{PS}$.

Similarly, she prefers finishing to waiting if and only if $\beta \sum_{i=1}^{m} \delta^i v + \beta\delta^{m-2} \sum_{i=1}^{n} \delta^{2i} v \geq (1-\beta\delta^m)k + \beta\delta^{m-2} \sum_{i=1}^{n-1} \delta^{2i}(1-\delta)(-c-\delta k)$. Because as $\delta \to 1$ the left-hand side goes to $\beta(m+n)v$ and the right-hand side goes to $(1-\beta)k$, when $\delta \to 1$, the person prefers finishing to waiting if and only if $k < \frac{\beta(m+n)v}{1-\beta}$. Defining $n_{PF} \equiv \max\{n \in \{0,1,...\} | k \geq \frac{\beta n v}{1-\beta}\}$, it follows that the person will finish a started project only if $m+n > n_{PS}$.

(2) The logic above implies that the person will (eventually) start another project if and only if $n > n_{PS}$, and therefore she starts $n_S^* \equiv \max\{0, N - n_{PS}\}$ projects. Similarly, the person will (eventually) finish another project if and only if $m > 0$ and $m+n > n_{PS}$, and therefore she finishes $n_F^* \equiv \min\{\max\{0, N - n_{PS}\}, n_S^*\}$ projects.

(3) The person prefers finishing to starting if and only if $\beta \sum_{i=1}^{m} \delta^i v \geq (1-\beta\delta^m)(k-c)$. Because as $\delta \to 1$ the left-hand side goes to $\beta m v$ and the right-hand side goes to $(1-\beta)(k-c)$, when $\delta \to 1$, the person prefers finishing to starting if and only if $k - c < \frac{\beta m v}{1-\beta}$. Defining $n_I \equiv \min\{n \in \{1,...,N\} | k - c < \frac{\beta n v}{1-\beta}\}$, the person will finish rather than start if and only if $m > n_I$.

If $c \geq k$, then $n_I = 1$ and $n_{PS} \geq n_{PF}$. $n_I = 1$ implies the person always prefers finishing to starting. Because $n_{PS} \geq n_{PF}$ implies $n_F^* = n_S^*$, it follows that she starts and finishes n_S^* projects in succession. If $c < k$, the person must start n_I projects (or n_S^* if $n_S^* < n_I$), and then alternate between starting and finishing (because m alternates between n_I and $n_I - 1$) until she has started n_S^* projects and finished n_F^* projects (given part 1).

Proof of Lemma 3. (1) This is a standard, stationary search model, and so for TCs the optimal strategy involves the usual cutoff rule. We note for use below that given cutoff $\bar{v}^*$, the ex ante payoff for TCs is $U^* \equiv -c + \delta \left[F(\bar{v}^*)U^* + (1-F(\bar{v}^*)) \left(-k + \frac{\delta E(v | v \geq \bar{v}^*)}{1-\delta} \right) \right]$, and a necessary condition for the optimality is $U^* = -k + \frac{\delta \bar{v}^*}{1-\delta}$.

(2) Consider a naif whose highest draw from a prior period is v_o, where we say $v_o = 0$ if the person has not completed stage 1 in any prior period. This person believes that she will behave like a TC beginning next period. Hence, the person's perceived payoffs from completing stage 1 now, finishing now, and waiting now, which we denote by P^1, P^2, and P^W, respectively, are:

$$P^1(v_o) = \begin{cases} -(1-\beta)c + \beta U^* & \text{if } v_o \le \bar{v}^* \\ -c + \beta\delta\left[-k + F(v_o)\frac{\delta v_o}{1-\delta} + (1-F(v_o))\frac{\delta E(v|v\ge v_o)}{1-\delta}\right] & \text{if } v_o \ge \bar{v}^* \end{cases}$$

$$P^2(v_o) = -k + \frac{\beta\delta v_o}{1-\delta}$$

$$P^W(v_o) = \begin{cases} \beta\delta U^* & \text{if } v_o \le \bar{v}^* \\ \beta\delta\left(-k + \frac{\delta v_o}{1-\delta}\right) & \text{if } v_o \ge \bar{v}^*. \end{cases}$$

[Note that $-(1-\beta)c + \beta U^* = -c + \beta\delta\left[F(\bar{v}^*)U^* + (1-F(\bar{v}^*))\left(-k + \frac{\delta E(v|v\ge\bar{v}^*)}{1-\delta}\right)\right]$.]

If $-(1-\beta)c + \beta U^* < \beta\delta U^*$ or $U^* < \frac{(1-\beta)c}{\beta(1-\delta)}$, then $P^1(0) < P^W(0)$ and therefore naifs never do anything. Suppose otherwise. Since $\frac{dP^W}{dv_o} = \frac{dP^1}{dv_o} = 0$ for all $v_o < \bar{v}^*$ and $\frac{dP^W}{dv_o} > \frac{dP^1}{dv_o}$ for all $v_o > \bar{v}^*$, there exists $v_A \ge \bar{v}^*$ such that $P^1(v_o) \ge P^W(v_o)$ if and only if $v_o \le v_A$. Since $\frac{dP^2}{dv_o} > \frac{dP^W}{dv_o}$ for all v_o and $P^2(0) < 0 < P^W(0)$, there exists $v_B > 0$ such that $P^2(v_o) \ge P^W(v_o)$ if and only if $v_o \ge v_B$. Since $\frac{dP^2}{dv_o} > \frac{dP^1}{dv_o}$ for all v_o and $P^2(0) < 0 < P^1(0)$, there exists $v_C > 0$ such that $P^2(v_o) \ge P^1(v_o)$ if and only if $v_o \ge v_C$.

Because $\frac{dP^2}{dv_o} > \frac{dP^W}{dv_o} \ge \frac{dP^1}{dv_o}$ for all v_o, and because P^1, P^2, and P^W are all continuous, we must have either (1) $v_B < v_C < v_A$, (2) $v_B = v_C = v_A$, or (3) $v_B > v_C > v_A$. For cases (1) and (2), behavior path (ii) holds for $\bar{v}^n = \bar{v}^{nn} = v_C$. For case (3), behavior path (ii) holds for $\bar{v}^n = v_A$ and $\bar{v}^{nn} = v_B$.

Proof of Proposition 9. Using the notation from the proof of Lemma 3, $P^2(\bar{v}^*) = -k + \frac{\beta\delta\bar{v}^*}{1-\delta}$, and since $U^* = -k + \frac{\delta\bar{v}^*}{1-\delta}$, $P^1(\bar{v}^*) = -(1-\beta)c + \beta(-k + \frac{\delta\bar{v}^*}{1-\delta})$. It follows that $P^1(\bar{v}^*) \ge P^2(\bar{v}^*)$ if and only if $k - c \ge 0$.

(1) If $c = k$, then $v_C = \bar{v}^*$. Naifs might never search, but if they do, then $v_A \ge \bar{v}^*$, and therefore we must be in case (1) or case (2). Hence, if naifs search, they behave exactly like TCs.

(2) If $c > k$, then $v_C < \bar{v}^*$. Naifs might never search, but if they do, then $v_A \ge \bar{v}^*$, and therefore we must be in case (1). Hence, if naifs search, they search with cutoff $\bar{v}^n = v_C < \bar{v}^*$ and finish that project.

(3) If $c < k$, then $v_C > \bar{v}^*$. Naifs might never search, but if they do, any of the three cases can hold. We have $\bar{v}^n = \min\{v_A, v_C\} \ge \bar{v}^*$ and $\bar{v}^{nn} = \max\{v_B, v_C\} \ge \bar{v}^n$.

Proof of Proposition 10. Consider a naif who is considering whether to complete stage 1 (i.e., in period 1 or when she waited the previous period). Given $\hat{\beta} = 1$, she compares completion beginning now to completion beginning next period. Hence, she completes stage 1 if and only if $-c - \beta\delta k + \frac{\beta\delta^2 v}{1-\delta} \ge -\beta\delta c - \beta\delta^2 k + \frac{\beta\delta^3 v}{1-\delta}$, or $c \le -\frac{\beta\delta(1-\delta)k}{1-\beta\delta} + \frac{\beta\delta^2 v}{1-\beta\delta}$. Because this condition is identical to that in the basic model, the logic from Proposition 3 implies that when $\delta \to 1$ the person completes stage 1 if and only if $c < \frac{\beta v}{1-\beta}$.

Consider a naif who is deciding whether to complete stage 2 (i.e., when she completed stage 1 the previous period). Given $\hat{\beta} = 1$, she compares finishing now to completion of both stages beginning next period. Hence, she completes stage 2 if and only if $-k + \frac{\beta\delta v}{1-\delta} \geq -\beta\delta c - \beta\delta^2 k + \frac{\beta\delta^3 v}{1-\delta}$, or $k \leq \frac{\beta\delta(1+\delta)v}{1-\beta\delta^2} + \frac{\beta\delta c}{1-\beta\delta^2}$. A logic analogous to that in Proposition 3 implies that when $\delta \to 1$ the person completes stage 2 if and only if $k < \frac{\beta 2v+\beta c}{1-\beta}$.

Combining these two conclusions, the result follows.

References

Ainslie, George (1975). "Specious Reward: A Behavioral Theory of Impulsiveness and Impulse Control." *Psychological Bulletin*, **82**(4), 463-496.

Ainslie, George (1991). "Derivation of 'Rational' Economic Behavior from Hyperbolic Discount Curves." *American Economic Review*, **81**(2), 334-340.

Ainslie, George (1992). *Picoeconomics: The strategic interaction of successive motivational states within the person*. New York: Cambridge University Press.

Ainslie, George and Nick Haslam (1992*a*). "Self-Control," in George Loewenstein and Jon Elster, eds., *Choice Over Time*. New York: Russell Sage Foundation, 177-209.

Ainslie, George and Nick Haslam (1992*b*). "Hyperbolic Discounting," in George Loewenstein and Jon Elster, eds., *Choice Over Time*. New York: Russell Sage Foundation, 57-92.

Akerlof, George (1991). "Procrastination and Obedience." *American Economic Review*, **81**(2), 1-19.

Angeletos, George-Marios, David Laibson, Andrea Repetto, Jeremy Tobacman, and Stephen Weinberg (2001). "The Hyperbolic Buffer Stock Model: Calibration, Simulation, and Empirical Evaluation." *Journal of Economic Perspectives*, **15**(3), 47-68.

Benabou, Roland and Jean Tirole (2000). "Self-Confidence: Intrapersonal Strategies." Princeton University Discussion Paper #209.

Carrillo, Juan and Thomas Mariotti (2000). "Strategic Ignorance as a Self-Disciplining Device." *Review of Economic Studies*, **67**, 529-544.

Fischer, Carolyn (1999). "Read This Paper Even Later: Procrastination with Time-Inconsistent Preferences." Resources for the Future Discussion Paper 99-20.

Frederick, Shane, George Loewenstein, and Ted O'Donoghue (2002). "Time Discounting and Time Preference: A Critical Review." Forthcoming in *Journal of Economic Literature*.

Laibson, David (1994). "Essays in Hyperbolic Discounting." Dissertation, MIT.

Laibson, David (1997). "Hyperbolic Discounting and Golden Eggs." *Quarterly Journal of Economics*, **112**(2), 443-477.

Laibson, David (1998). "Life-Cycle Consumption and Hyperbolic Discount Functions." *European Economic Review*, **42**, 861-871.

Laibson, David, Andrea Repetto, and Jeremy Tobacman (1998). "Self-Control and Saving for Retirement." *Brookings Papers on Economic Activity*, **1**, 91-196.

Loewenstein, George and Drazen Prelec (1992). "Anomalies in Intertemporal Choice: Evidence and an Interpretation." *Quarterly Journal of Economics*, **107**(2), 573-597.

O'Donoghue, Ted and Matthew Rabin (1999a). "Doing It Now or Later." *American Economic Review*, **89**(1), 103-124.

O'Donoghue, Ted and Matthew Rabin (1999b). "Incentives for Procrastinators." *Quarterly Journal of Economics*, **114**(3), 769-816.

O'Donoghue, Ted and Matthew Rabin (2001). "Choice and Procrastination." *Quarterly Journal of Economics*, **116**(1), 121-160.

Phelps, E. S. and Robert A. Pollak (1968). "On Second-Best National Saving and Game-Equilibrium Growth." *Review of Economic Studies*, **35**, 185-199.

Pollak, Robert A. (1968). "Consistent Planning." *Review of Economic Studies*, **35**, 201-208.

Prelec, Drazen (1989). "Decreasing Impatience: Definition and Consequences." Mimeo, Harvard Business School.

Strotz, Robert H. (1956). "Myopia and Inconsistency in Dynamic Utility Maximization." *Review of Economic Studies*, **23**(3), 165-180.

Thaler, Richard H. (1991). "Some Empirical Evidence on Dynamic Inconsistency," in *Quasi Rational Economics*. New York: Russell Sage Foundation, 127-133.

Thaler, Richard H. and George Loewenstein (1992). "Intertemporal Choice," in Richard H. Thaler, ed., *The Winner's Curse: Paradoxes and Anomalies of Economic Life*. New York: Free Press, 92-106.