Endogenous Timing,Payoff Externalities and Informational Cascades

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September 19, 2007

Abstract

We study the phenomenon of herd behavior and the timing of actions of agents under the presence of pay-offs externalities. We characterize perfect bayesian equilibrium for a variety of pay-off externalities.

Under positive externalities, a unique symmetric pure strategy equilibrium exists where a herd always occurs and agents with a higher precision signal move first. On the other hand, under negative externalities, we show that there exists a unique equilibrium where a herd never arises and all agents act at the same time. An incentive structure with negative pay-off externalities leads to an un-mimicked and un-delayed disclosure of information.

*Thanks to my advisor Professor Kalyan Chatterjee for his support and encouragement. Thanks also to Professors Edward J. Green and Neil Wallace for valuable discussions.

1 Introduction

In a world of uncertainty, opinions and decisions are usually influenced by the actions and words of others. Uncertainty causes agents to "fall back on the judgment of the rest of the world. That is, people endeavor to conform to the behavior of the majority or the average, knowing that their own individual judgment is not sufficient" - Keynes [16]. Herding is the popular term associated with this aspect of human behavior. The importance of herd behavior stems from its applicability to domains such as *viral marketing*[1] and *forecasting*.

The most common explanation for the occurrence of herd behavior can be attributed to the presence of *informational externalities* [2] However, under most social and economic scenarios the actions of players also affect the pay-offs of other agents along with their own. This form of *pay-off externality* in the actions is found to co-occur with *Informational Externality* in the decision making of agents. To give a few examples :

- Consider the scenario of two firms planning to launch a new product into the market. Informational externalities in the interaction between the two firms comes in the form of valuable information derived about the market response of the product, from the observation of the actions of the forerunner. At the same time the decision of a firm to launch the product affects the profits accruing to the other firm due to pay-off externalities. The sole incumbent in the market reaps the benefit of greater profits and brand loyalty. However, simultaneous existence in the market is tantamount to sharing the pie between the two.

- Volatility in the housing market has been associated with the presence of convergence of behavior among buyers [5]. Uncertainty about the course of property prices induces buyers to wait and depend upon the information of neighbors before purchasing a housing asset. At the same time, as the occupancy in the locality increases, the quality of public services provided to each household may go down, exemplifying the presence of pay-off externalities.

- Studies like Hong [14] and Trueman[25] find that financial analysts tend to wait before they make predictions. Forecasts are found to be influenced by the first few predictions and the analysts are seen to herd towards a consensus. At the same time, empirical analysis [18, 23, 24] on the affect of reputation on the performance of analysts, find that one of the most important criteria while hiring financial analysts is their relative reputation or ranking on reports like *The Institutional Investor journal* highlighting the nature of pay-off externalities present in the business of forecasting.

[1] The influence of people on each other in their buying decisions has prompted businesses to use the customer in turn as the 'salesman' of their products.

[2] *Informational Externality is the phenomenon where the decision of an agent is influenced by the observation of the actions of other agents in the decision making process of agents.*

The co-occurrence of these externalities in real life motivates us to analyze them both together to produce a more realistic social learning model than past attempts[27, 20]. The choice of timing of actions taken by agents plays a critical role in the rise and delay of a herd. Literature so far [27, 20] analyzes herd behavior with endogenous timing of actions of agents under the presence of pure informational externalities. These analyses don't take into consideration the presence of pay-off externalities in their studies. In this paper, we extend the previous literature by studying the phenomenon of herd behavior and the timing of actions of agents under the presence of both informational and pay-off externalities.

We look at a broad class of pay-off externalities including negative, positive and mixed forms of externalities. Under negative externalities and some form of mixed externalities, we find that a herd would not occur and all agents would act independently at the earliest point in time. This is contrary to the result found by Zhang[27] and Rogers[20] where the highest precision agent would act first. Thus, we show that incorporating negative externalities or certain mixed externalities ensures that all information available among the agents is revealed. This helps prevent a 'bad' herd to rise and avoids inefficient delays in the revelation of information.

We first consider the scenario where there are two agents who can take one of the two irreversible actions. At any state of the world only one action is profitable. Each agent receives an independent signal about the unknown state of the world. Unlike Sgroi[21] where players are homogeneous in terms of their ability, agents in this model are allowed to vary according to the precision level of the signals that they receive. Each agent decides not only what action to take but also when to take that action. The choice of an action is made visible to both the agents once it is taken. The game ends when both the agents have taken an action. [3] The pay-offs to an agent depends upon the actions of both the players. An agent gets utility right from the period when he takes an action to the date when the state of the world is realized. Therefore, the incentive structure has two features-First mover advantage and pay-off externalities.

We analyze two types of pay-off externalities-positive and negative pay-off externalities. Under negative externalities the utility derived from an action decreases as more and more agents opt for the same action for example the choice of the restaurant to patronize. As the number of customers of a restaurant that you wish to patronize increases, the waiting time and the service quality of that restaurant decreases. Under such form of externality we find that the unique perfect bayesian equilibrium is characterized by no herd. And all agents would take an action independently and at the start of the game.

Under positive pay-off externalities, the pay-off from an action increases as more and more agents opt for the same action. For example, an agent purchasing a personal computer would be concerned about the number of other agents opting for the same hardware. This

[3] Agents are paid right from the period that they take an action till the end of the game. The time at which the state is realized does not effect the relative benefits of moving at different time periods. The results of the paper would not change even if the state is revealed some time after both the agents have moved or the game ends after a finite time T

is because as the number of units sold of a hardware increases, the quality and variety of software developed for use with that hardware would also increase [12]. Under such forms of pay-off externalities we find that the unique perfect bayesian equilibrium is characterized by the rise of a delayed herd. Also, agents with higher precision move first and lead the herd.

We also look at the nature of equilibrium when the number of agents increases in the environment. We find that under endogenous timing of actions the game ends in at most two periods. There would not be an instance where the herd would repeatedly break and rise again.

The rest of the article is organized as follows. Section 2 discusses some relevant literature. The section following that, sets up the simple two action,two state and two agent model. Section 4 first discusses some preliminary results for equilibrium analysis of all kinds of externalities and later characterizes the equilibrium for positive, negative and mixed pay-off externalities. Section 5 throws light on the extension of the model to more than two agents. Finally, Section 6 summarizes the findings of the paper and discusses possible future research directions.

2 Related Literature

The significance of the study of herd behavior is apparent from the extensive literature on the topic.[3, 4, 9, 15, 26].

One of the primary rationales provided for the existence of herd behavior is the presence of informational externalities in the interaction between agents. An *informational cascade* is said to have begun if the chosen action of an agent doesn't depend upon his own private signal and is solely based on the observation of the actions of others. Banerjee[3]and Bikhchandandi, Hirschleifer and Welch [4]in their pioneering work, model herding as a consequence of this informational externality. These papers treat the timing of actions of agents as exogenous. However, as agents vary in terms of their ability to judge the uncertainty in the environment, the number of agents that they wish to observe before taking an action should be a choice variable in the model. Chamley and Gale [6] look at this choice of timing of actions of agents in models of informational cascades and show that there is delay in the revelation of information and a bad herd prevails with a positive probability. Zhang [27] and Rogers [20] also analyze the timing of actions of agents under pure informational externalities. They observe that the most precise agent with the least informational incentive to wait, takes an action first. Given this knowledge a herd eventually arises.
Young-Ro Yoon[29] analyzes the timing of actions of agents under the presence of a very specific form of mixed externalities which in effect are positive. This paper on the other hand covers a much broader spectrum of pay-off externalities by analyzing positive, negative and all possible forms of mixed externalities. Yoon's analysis is deficient as it only analyzes the case where precision levels are common knowledge. We make a more realistic

assumption of precision levels being private information of each agent. Moreover, we also introduce first mover advantage in the model which is critical in many scenarios like a product launch.

The presence of pay-off and network externalities in the environment also explains the existence of herd behavior. Katz and Shapiro [17],Arthur[1] and Farrell and Saloner [12] champion this rationale. The argument also finds support from Diamond and Dyvbig [9] who observe that bank run models are better explained by the existence of direct pay-off externalities among agents.

Scharfstein and Stein[22] and Zweibel [28] are one of the first ones to integrate both informational externalities and pay-off externalities to analyze a more general model. They observe that absolute reputational concerns among managers drives them to mimic each other. Ottaviani and Sorensen [19] extend Scharfstein and Stein[22] to a general message space and find that in all equilibria experts do not fully reveal their signals and herd towards the prior mean. On the other hand, Effinger and Polborn [11] find evidence for anti-herding due to the presence of relative reputational concerns among managers. They show that if relative reputation concerns are high enough the successors always opposes their predecessors.

Drehmann etal[10] conduct experiments to incorporate different types of pay-off externalities into the standard herding models. They find little support for the lack of any pay-off externalities in most of the environments. However, in the literature dealing with relative pay-off externalities the timing of actions of agents is taken to be exogenous. The attempt of this paper is to bridge this gap in the literature by providing a theoretical analysis of timing of actions of agents under the presence of both informational and pay-off externalities.

3 Model

There is a countable set of dates, $T = \{1l, 2l, 3l, \dots\} \cup \{\infty\}$ for some $l > 0$. [4] To begin with there are two agents in the model. Each agent,$i \in \{1, 2\}$ decides a date $\tau_i \in T$ to take one of the two irreversible actions, $a_i \in A = \{a_G, a_B\}$. The return from each action depends upon the unknown state of the world ω which takes one of the two values from the set, $\Omega = \{G, B\}$. The prior probability of each state is common knowledge and is given by

$$Pr(\omega = G) = Pr(\omega = B) = \frac{1}{2}$$

Each agent,i receives a signal, $s_i \in \{s_G, s_B\}$, of certain precision level, p_i, about the uncertain event. The signals are distributed independently given the state of the world. The distribution of the signal given the state is given by the Table 1. Both the signal s_i and its quality p_i are private information of each agent i. However other agents are aware of

[4] The results of the model hold even if the game ends in a finite period of time.

the distribution of s_i conditional on p_i. Also, each agents signal quality, $p_i = Pr(s_G|G) = Pr(s_B|B)$ is drawn from a continuous distribution, $F(p)$ over [0.5,1]. At every date $t \in T$

Table 1: Signal Structure

$P(s_i\|\omega)$	s_G	s_B
G	p_i	$1-p_i$
B	$1-p_i$	p_i

each agent decides whether to take an action $a_i \in A$ or wait, w. Hence, the strategy of each agent i with precision level p_i and signal s_i is as follows

$$\sigma_i(s_i, p_i) : H \rightarrow A \cup w$$

where H is the set of all histories in the game. $\sigma_i(h^\tau; s_i, p_i) = \sigma_i(h^\tau)$ describes the action prescribed by strategy $\sigma_i(s_i, p_i)$ at the history h^τ. Once an action other than waiting is taken at any time t, it is irreversible i.e. if $\sigma_i(h^\tau; s_i, p_i) \neq w$ then $\sigma_i(h^\tau; s_i, p_i) = \sigma_i(h^{\tau+l}; s_i, p_i) \neq w$ for all $l \geq 0$. Agents observe the actions of their predecessors. The game ends once both the agents have taken an action.[5] Agents obtain utility according to Table 2 and 3 right from the period when they take the action till the state is realized. There is a common discount factor δ to account for the temporal preferences.

Table 2: Realized utility of agent i at dates when $a_i \in A$ and $a_j \notin A$

θ	$a_i = a_\theta$	$a_i \neq a_\theta$
$a_j = w$	$0, z$	$0, -z$

Table 3: Realized utility of agent i at dates when $a_i, a_j \in A$

θ	$a_i = a_\theta$	$a_i \neq a_\theta$
$a_j = a_\theta$	$z-x, z-x$	$z, -z$
$a_j \neq a_\theta$	$-z, z$	$-z+y, -z+y$

The incentive structure has two features-First mover advantage and Pay-off externalities.

[5]If we assume that the game ends after a fixed period the results of the model would not change.

An agent obtains utility depending upon the correctness of his action. The incentive structure is such that for every state there is only one action which gives a positive reward. [6] For the state $\omega \in \{G, B\}$, a_ω is thought to be as the correct action. In this model an agent obtains utility right from the period when he takes an action till the period when the state is realized. Therefore, speaking early is beneficial if the agent is right but contrary if not. This feature imparts a first mover advantage to the incentive structure as the player to take an action early gets utility for a larger number of periods. The results of the model do not change if the bonus for speaking early is linear in time.

At dates when agent i is the only one to take an action in $A = \{a_G, a_B\}$ his realized utility depends upon the absolute accuracy of his action as in Table 2. However, for dates when agent i is not the only one to take an action in $A = \{a_G, a_B\}$ his utility depends upon the accuracy of his action and that of the other agent as in Table 3. This feature incorporates pay-off externalities into the utility function.
The following section analyzes the equilibrium for different forms of externalities. The first part of the section provides preliminary results required for the equilibrium analysis and the latter section characterizes the equilibrium for different types of externalities.

4 Equilibrium Analysis

4.1 Preliminary Analysis

We first begin with results on the actions taken by the first and second mover in the game along the equilibrium path. The first lemma describes the equilibrium behavior of whoever is the first mover of the game.

Lemma 1 *The first mover with precision level $p_i > 0.5$ and signal s_i takes an action according to the signal received*

$$\sigma_i(h^t; s_i = s_\omega, p_i) = a_\omega \ \ for \ \ \omega \in \{G, B\}$$

Proof. Given that the two states are a priory equally likely and the precision of the signal received by the agent is greater than half, the updated belief about the state for which the agent received a signal is more than half. By the pay-off structure in Table 1 the expected utility from taking an action according to the signal received is greater than that obtained by taking an action opposite to the signal. Hence, agents would take an action according to their own signal. For a detailed proof refer to Appendix A □

The result above is an artifact of the assumption of equally likely prior. If we assume an unequal prior there would exists a threshold level of precision such that all first movers

[6] If we assume that both the agents receive positive but unequal amount the mail results of the paper are unaffected.

with precision level below it would always take the more likely action irrespective of the signals that they receive. The actions of these agents would not provide any additional information to their successors. Only first movers with precision level above the threshold would take an action according to their signals. Hence, only the actions of these agents with precision level greater than this threshold would be informative to their followers. Since we are interested in the learning of successors from the actions of their predecessors, the latter would be the only relevant set of agents for the study. Hence, there is no loss of generality in assuming that the two states are equally likely.

Assumption: *Agent with precision level 0.5 takes an action according to his signal.*

Given the prior belief that the two states are equally likely the agent with precision level 0.5 is indifferent between taking any of the two actions irrespective of the signal received. We therefore assume that the agent with precision level 0.5 takes an action according to the signal they received. The following section characterizes equilibrium for different forms of externalities.

4.2 Characterization of Equilibrium

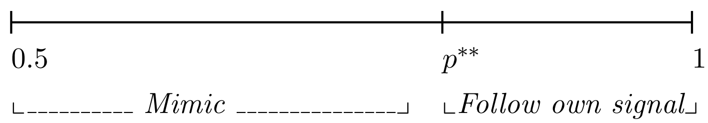
Under most scenarios informational externalities are accompanied with some form of pay-off externalities. Usually when agents interact with each other they not only learn through the actions of each other but the action of each agent affects the pay-offs of others too. The focus of this section is to characterize the nature of equilibrium when both informational and pay-off externalities are present in the environment.

4.2.1 Negative Pay-off Externalities

Negative pay-off externalities are said to be present if the pay-off from an action decreases as more and more individuals take that action. In this section we analyze the timing of actions of agents under negative pay-off externalities and first mover advantage. The following Lemma describes the action of the second mover under negative pay-off externalities.

Lemma 2 *Let $x \geq y$ and the action taken by the first mover be $a_1 \in A$. Let the belief of the second agent of the precision level of the first mover be $q(p_2)$ where p_2 is the precision level of the second agent.*
(i)If $p_2 \geq q(p_2)$ then there exists a precision level, p^* such that if $p_2 \leq p^*$ then the second agent would always anti herd. However, if $p_2 \geq p^*$ then the second mover would follow his signal.

(ii) If $p_2 \leq q(p_2)$ then there exists a precision level, p^{**} such that if $p_2 \leq p^{**}$ then the second agent would always mimic the first mover. However, if $p_2 \geq p^{**}$ then the second mover would follow his own signal.



Proof. *See appendix* □

Two forces effect the pay-offs- Accuracy and Negative Pay-off Externalities. Under the assumption $p_2 \geq q(p_2)$ and when agents get different signals, Accuracy and Negative Pay-off externalities indicates the agent to take an action according to his own signal. However, when the agents get the same signal, Accuracy and Negative Pay-off externalities move in the opposite direction. There exists a threshold level of precision, p^* such that Accuracy is dominated by Negative Pay-off Externalities for all precision levels less than p^* and agents move opposite to their signal. For all agents with precision level greater than p^* Accuracy dominates Negative Pay-off externalities and agents follow their signal. Therefore, all agents with precision less than the threshold would anti-herd and agents with precision level more than this threshold would always follow their own signal.

If on the other hand under the assumption $p_2 \leq q(p_2)$, negative pay-off externalities and Accuracy indicates the agent to take opposing actions. Following the Accuracy the agent should mimic its predecessor. However, following the negative pay-off externalities the agent should move according to its own signal. There exists a threshold level of precision level such that for all agents below it Accuracy dominates Negative Pay-off externalities and hence the second mover would mimic its predecessor. However, all agents above this threshold would follow their own signal as Negative Pay-off externalities dominates the Accuracy effect. The lemma below characterizes the stopping time of agents under pure negative pay-off externalities and mixed negative pay-off externalities.

Lemma 3 *If $z \geq x > 0, y < 0$ or $z \geq \bar{q}x \geq (1-\bar{q})y > 0$, then in any Perfect Bayesian Equilibrium $\tau(p_i)$ is a* **weakly increasing** *function of p_i. The agent with the lowest* *precision level would act first.*

where,
$\bar{q}$, $\int_{0.5}^{1} pdF(p)$
$\tau(p_i)$, *the time at which agent* p_i *would take an action given that no one else taken an action till then.*

Proof. Appendix □

$z \geq x > 0, y < 0$ is the regime of pure negative externalities where the pay-off from any action reduces as more and more agents take the same action. However, $z \geq \bar{q}x \geq (1-\bar{q})y > 0$ is a scenario of mixed externalities. There are negative externalities in taking the 'correct' action as the utility from being right reduces as more and more agents take that action. However, there are positive externalities from taking the 'incorrect' action as the benefit from being incorrect increases as more and more agents take that action. When $z \geq \bar{q}x \geq (1-\bar{q})y > 0$ these mixed externalities are effectively negative.

The intuition of the above lemma is as follows. Lets suppose that there is an equilibrium in which the agent with higher precision takes an action first. The follower with a lower precision level would would rightly mimic their successor [7] This would imply a mimicked continuation pay-off on an average to the first mover from the period when the second agent takes an action. The gain for the higher precision agent from taking an action early is to receive the un-mimicked pay-off one period early. However, the loss from taking the first action is to receive a mimicked pay-off from the second period. Due to negative pay-off externalities the loss from obtaining a continuation mimicked pay-off from the second period is greater than the gain from receiving the un-mimicked pay-off one period early. Therefore, there would be no equilibrium in which an agent with high precision would take an action before the one with low precision. Similarly, for the low precision agent the benefit of an un-mimicked two period continuation pay-off is higher than the informational gain from observing the action of the high precision agent. Hence, a low precision agent would always prefer to act before the high precision agent. Hence, $\tau(p_i)$ is a weakly increasing function. The following proposition characterizes the unique perfect bayesian equilibrium under negative pay-off externalities.

Proposition 1

If $z \geq x, y < 0$ *or if* $z \geq \bar{q}x \geq (1-\bar{q})y > 0$ *then there is a unique symmetric Perfect Bayesian Equilibrium in pure strategies where both the agents with any precision level take an action in the first period and take it according to their own signal.*

[7] All agents with precision level less than p^{**} would mimic the first mover and all agents with precision level more than p^{**} would would follow their own signal. Thus, the only time the two agents would not take the same action would be when the precision level of the second mover is more than p^{**} and it gets a signal other than that of the first mover.

Proof. See Appendix □

Proposition 1 describes that along the equilibrium path, all agents would take an action in the first period. By Lemma 3 it is common knowledge that an agent with the lower precision would take an action no later than the higher precision agent. The knowledge of this fact would induce the agent with the higher precision to also act in the first period and that to according to his signal. The higher precision agent has no incentive to wait for a period after the lower precision agent has already taken an action. Hence, both the agents would take an action in the same period.

To motivate this result further let us consider the case of common knowledge of precision among agents. The above result also holds under complete information of the level of precision levels of agents and $x = y = \lambda$.

Lemma 4

If precision is common knowledge and $x = y = \lambda$ then there exists a pure strategy SPNE where the low and high precision agents take an action according to their own signal in the first period.

Proof. Appendix □

The intuition of the result above is as follows. Given the knowledge of the precision level of all agents, a lower precision agent has some informational gain by observing the actions of the high type agent. However, mimicking a higher type always implies a pay-off which is lower than the un-mimicked payoff obtained by taking an action independently. Also, a lower precision agent would receive an un-mimicked pay-off for an additional period by moving before the higher type. The gain of an un-mimicked pay-off for two periods outweighs the informational gain for the lower type. Hence, the lower type always prefers to move before the higher type. If the higher precision agent moves before the lower type he would always be mimicked. To avoid the mimicked pay-off, the higher precision agent always prefers to move with the lower type. This ensures that there is an equilibrium where all agents take an action in the first period according to their signals and no herd arises.

4.2.2 Positive Pay-off Externalities

Positive pay-off externalities are said to be present if the pay-off from an action increases as more and more individuals take that action. In this section we analyze the timing of actions of agents under positive pay-off externalities and first mover advantage. The following lemma characterizes the action of the second mover.

Lemma 5 *Let $x \leq y$ and the action taken by the first mover be $a_1 \in A$. Let the belief of the second agent about the precision level of the first mover be $q(p_2)$ where p_2 is the precision level of the second agent.*
(i) If $p_2 \geq q(p_2)$

then there exists a precision level, p^{***} such that if $p_2 \leq p^{***}$ the second agent always mimics the first mover. However, if $p_2 \geq p^*$ the second mover follows his signal.

(ii) If $p_2 \leq q(p_2)$ then the second agent would mimic the first mover.

Proof. *See appendix* □

Under the presence of positive pay-off externalities if the second mover believes that his precision level is more than that of the first mover it would always take an action according to its own signal. However, if the second mover believes that its precision is less than that of the first mover it would mimic the actions of the first mover irrespective of the signal that he received.
The lemma below characterizes the stopping time of agents under pure positive pay-off externalities and mixed positive pay-off externalities.

Lemma 6

If $z \geq y > 0, x < 0$ or if $z \geq y \geq x > 0$ then in any Perfect Bayesian Equilibrium $\tau(p_i)$ is a weakly decreasing function of p_i. The agent with the highest precision would move first.

where $\tau(p_i)$ is the time at which agent p_i would take an action given that no one else taken an action till then.

Proof. Appendix □

When $z \geq y > 0, x < 0$ we are under the regime of pure positive externalities where the pay-off from any action increases as more and more agents take the same action. However, $z \geq x, y > 0$ is a scenario of mixed externalities. There are negative pay-off externalities in taking the 'correct' action as the utility from being right reduces as more and more agents take that action. However, there are positive pay-off externalities from taking the 'incorrect' action as the benefit from being incorrect increases as more and more agents take that action. When $z \geq y \geq x > 0$ these mixed externalities take effectively the form of positive externalities.

The intuition of the above lemma is as follows. In equilibrium the higher precision agent would not prefer to take an action after the low precision agent. Delay is costly because of loss of per period pay-off and discounting. Higher precision agent also does not have any informational gain from taking an action after the lower precision agent. Moreover, due to the presence of positive payoff externalities, moving after the lower precision agent implies an un-mimicked pay-off which on an average is lower than the mimicked pay-off obtained by taking an action before the lower precision agent. Hence, the higher precision agent would prefer to take an action before the lower precision agent. The following proposition characterizes the unique perfect bayesian equilibrium under pure positive pay-off externalities.

Proposition 2

If $z \geq y > 0, x < 0$ or if $z \geq y \geq x > 0$ then there is a unique symmetric Perfect Bayesian Equilibrium in pure strategies characterized by a decreasing step function, $\tau(p_i)$ where the agent with higher precision level takes an action in the first period.

Proof. Appendix □

Proposition 2 describes that along the equilibrium path, the timing of the first mover is a decreasing function of his own precision level. By Lemma 4 it is common knowledge that the agent with the best knowledge would take an action first. Given this the lower precision level would rightly mimic their successors and take an action in the very next period. This would result in a rise of a cascade.

4.2.3 Mixed externalities with no first mover advantage

There is significant evidence suggesting that remuneration of forecasters depends upon the relative accuracy of their predictions. Stickel (1990,1992)[23] finds that annual rankings of forecasters as published by *Institutional Investor* effects the salaries of forecasters. Competition among the employers of the forecasters leads to relative performance remuneration among them. This section analyzes the timing of actions of agents when they are remunerated only according to their relative accuracy. Unlike the previous sections we assume that there is no first mover advantage in making predictions. This is a realistic formulation of rewards as it is generally observed that forecasters are remunerated only for their relative accuracy and not for giving predictions early. Following is the utility structure analyzed. The realized utility to an agent i from taking an action, $A = \{a_G, a_B\}$ is given by Table 3.

θ	$a_i = a_\theta$	$a_i \neq a_\theta$
$a_j = a_\theta$	$z - x, z - x$	$z, -z$
$a_j \neq a_\theta$	$-z, z$	$-z + y, -z + y$

The following Proposition characterizes the equilibrium in this scenario

Proposition 3

(a) Negative Payoff Externalities: If $x \geq y$ then all agents take an action together in the first period.There is no herding.

(b) Positive Payoff Externalities: If $y \geq x$ the highest precision agent takes an action first and the equilibrium is characterized by a decreasing step function.

Proof. Appendix □

Under the absence of first mover advantage we obtain multiplicity of equilibria. The nature of equilibrium depends upon the belief structure of the agents in the environment. Under the belief that low precision agents act first we find that all agents would move together in the first period.On the other hand, if it is believed that agents with higher precision level take an action first, there would be an equilibrium supporting this belief where high precision agents would move first.

5 Many players

A more realistic scenario for the social learning phenomenon is the case where more than two agents interact with each other. We analyze the timing of actions of agents when there are more than two agents in the economy under the presence of pay-off externalities.[8] The following are the claims for the nature of equilibrium in this setting.

Claim 1: *In equilibrium all agents with the same precision level take an action in the same period. i.e.* $\tau(p_i) = \tau(p_k) \forall i, k \in N(p)$

Proof. Let $N(p)$ be the set of agents in the economy with precision level p. Lets suppose that $\exists p$ and $i \in N(p) s.t. \tau(p_i) \neq \tau(p_k) = t_k \forall k \in N(p) - i$. Wlog lets assume that $\tau(p_i) > \tau(p_k) \forall k \in N(p) - i$. Therefore, for agent k $EU_k(\tau_k = t_k | h_t^0) > EU_k(w | h_t^0)$. Since,$p_i = p_k$ $EU_i(\tau_k = t_k | h_t^0) > EU_i(w | h_t^0)$.Hence, $\tau(p_i) = \tau(p_k)$ Which contradicts the assumption that $\tau(p_i) \neq \tau(p_k)$ □

As the number of players increases there is more information that can be learnt from the announcements of others. Thus, it would be expected that as the number of individuals increases, agents would be more patient. While this is in fact true, we can show that no equilibrium lasts for more than two periods from the period when the first action was taken. The intuition is as follows. Once an agent or a set of agents have taken an action their immediate successors would necessarily mimic any one of them. Given this no agent

[8]The details of this extended model are difficult to work out precisely, so we are not able to explicitly characterize the equilibria as in the case of two players. The main difficulty is the multiplicity of sub-games in the case with many players. This is not an issue in the case of n=2 as there are only two relevant histories.

would wait to take an action in the third period. The following claim points out that with more than two agents and pay-off externalities the game would end in at most two periods from the first prediction.

Claim 2: The agents who move in the second period mimic at least one agent in the first period. There is no equilibrium in which the game ends in more than two periods of the first action.

Proof. Lets assume there are N agents in the economy. Let P^k be the set of precision levels who take an action in the k-th period in equilibrium. We would first show that any group with precision level, $p_i \in P^2$ would mimic at least one agent who has moved in the first period.

Lets prove by assuming the contrary. Assume that an agent with precision level $p_i \in P^2$ always moves according to its own signal when it moves in the second period. Given this the expected utility of the agent, p_i from taking an action in the second period given the strategy of the others agents as σ^k is given by the following expression

$$\begin{aligned} EU(t=2;p_i) &= \sum_{n^1=0}^{N} Pr(n^1)\delta \sum_{n^2=n^1}^{N} Pr(n^2)\Big[\Gamma^2[\sigma^i,\{\sigma^k\}_{k=N-i};z,x,y,p_i,\vec{p_{n^2}} \\ &\quad ,n^2+1]+\delta \sum_{n^3=n^2}^{N} Pr(n^3)\Big[\Gamma^3[\sigma^i,\{\sigma^k\}_{k=N-i};z,x,y,p_i,\vec{p_{n^3}},n^3]\dots \end{aligned}$$

where, n^l is the number of agents who have taken an action till the l th period.
$\Gamma^s\Big[\sigma^i,\{\sigma^k\}_{k=N-i};z,x,y,p_i,\vec{p_{n^s}},n^s+1\Big]$ is the period s expected utility of agent, p_i^2 when he acts according to his own signal given the strategy others as σ^k. $\vec{p_{n^s}}$ is the belief of agent p_i about the precision level of agents who have taken an action till period s. Any agent who moves in the first period would move according to his or her signal. The expected utility of the agent with precision level $p_i \in P^2$ from taking an action in the first period is given by the following expression:

$$\begin{aligned} EU(t=1;p_i) &= \sum_{n=0}^{N} Pr(n^1=n)\Big[\Gamma^1\Big[\sigma^i,\{\sigma^k\}_{k=N-i};z,x,y,p_i^2,\vec{p_{n^1}},n^1+1\Big] \\ &+ \delta \sum_{n=n^1+1}^{N} Pr(n^2=n)\Big[\Gamma^2[\sigma^i,\{\sigma^k\}_{k=N-i};z,x,y,p_i^2,\vec{p_{n^2}},n^2+1] \\ &+ \delta \sum_{n=n^2}^{N} Pr(n^3=n)\Big[\Gamma^3[\sigma^i,\{\sigma^k\}_{k=N-i};z,x,y,p_i^2,\vec{p_{n^3}},n^3]\dots\Big] \end{aligned}$$

From the assumption that the second period movers move according to their own signal and do not mimic any of first period movers, the pay-off in the second period and the subsequent periods is the same for p_i irrespective of when it takes an action. It is easy to observe that the expected utility of agent $p_i \in P^2$ from moving in period 1 is greater than that from taking an action in period 2. Hence, under the assumption that the second period movers move only according to their own signals irrespective of when they take an action we reach a contradiction that these second movers prefer to take an action in the first period instead of the second period in equilibrium. Hence, all agents who move in the second period follow at least one agent in the first period.

All agents who prefer to wait till period 2 to take an action would wish to copy at least one agent in the first period. Lets assume that there is a precision level, $p_j \notin P^1$ who prefers to wait till period 3 to move. Given that the second period movers also copy at least one agent in the first period the information gain from waiting till period 2 is the same as that till period 3. However, due to discounting waiting till period 3 is more costly than that till period 2. Hence, any agent who wishes to not take an action in period 1 would wish to do so in period 2. Thus the game would end in two periods of its start. $\square$

The example below throws light on the possible different types of equilibrium that may exist. This example highlights that there may be a case where the high type agent follows the low type agents.

Example 1 : *Low precision agents take action in the first period and the high precision agents follow in the next period.*

Consider the case where the total number of agents is N and $p_L = 0.6, n_L = N-1, p_H = 0.65, N_H = 1, x = y = \lambda$. All first movers would take an action according to their signal. The optimal strategy for the agents is to move according to their own signal when less than two actions are observed. However, when more than two actions are observed then agents would move according to the signal with a majority. Given this, the high type agent always prefers to move in the second period after the lower precision agent. Thus if the number of agents of the low precision group is high making its informational content high enough then the high precision agent would choose to move later.

6 Conclusion

In almost all instances, informational externalities coexist with pay-off externalities. In such scenarios the choice of timing of actions plays a critical role in the rise and delay of herds. In this paper we analyze a framework of social learning with endogenous timing under the presence of both of these externalities.

Unlike the previous literature we find that under negative pay-off externalities no herd arises and all agents take an action together without any delay. Using such an incentive mechanism helps prevent a 'bad' herd and leads to an un-delayed revelation of information. On the other hand, incentive structures with positive pay-off externalities gives rise to a herd led by the most precise agent. Positive externalities does not eliminate the possibility of a bad herd from rising. Such an incentive structure also delays the disclosure of information. From the perspective of firms or investors looking for financial advice deploying negative externalities or some form of mixed externalities with first mover advantage would be the most efficient form of incentive structure. This form of incentive structure not only ensures an un-mimicked financial advice but also leads to an un-delayed information disclosure.

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Appendix

A Proof of Lemma 1

Assume agent i is the first mover and moves at time period t. The expected utility of agent i at time period t from taking action:
$\sigma_i^*(h^t; s_i = s_\omega, p_i) = \sigma_i^*(h^t) = a_\omega$ where $\omega \in \{G, B\}$ is

$$\begin{aligned} EU_i(\sigma_i^*(h^t) = a_\omega, \sigma_k | p_i, s_i = s_\omega) &= Pr(\tau_k \neq t | h^t)\Big[(2p_i - 1)z + \delta Pr[\sigma_k(h^{t+1}) = \sigma_i^*(h^t)] \\ & \quad [p_i(z - x) + (1 - p_i)(-z + y)] \\ &+ \delta Pr[\sigma_k(h^{t+1}) \neq \sigma_i^*(h^t)](2p_i - 1)z\Big] \\ &+ Pr(\tau_k = t)\Big[(2p_i - 1)z + (1 - p_i)(1 - \bar{q})y - p_i\bar{q}x\Big] \end{aligned}$$

where p_i, s_i is the precision level and signal received by agent i. $\bar{q}$ is the mean precision level or expected precision if the first mover given the distribution of the precision level, $F(p_i)$. σ_k is the strategy followed by agent k.τ_k is the time at which agent k would take an action given that no one has taken an action till then. If agent i takes an action in period t then the best response of agent k at time period $t+1$ would be to take an action at period $t+1$. The expected utility obtained by the agent from taking an action, $\sigma_i^*(h^t; s_i = s_\omega, p_i) = \sigma_i^*(h^t) \neq a_\omega$ is

$$\begin{aligned} EU_i(\sigma_i^*(h^t) \neq a_\omega, \sigma_k | p_i, s_i = s_\omega) &= (-1)\Big[Pr(\tau_k \neq t)\Big[(2p_i - 1)z \\ &+ \delta Pr[\sigma_k(h^{t+1}) = \sigma_i^*(h^t)] \\ & \quad [p_i(z - x) + (1 - p_i)(-z + y)] \\ &+ \delta Pr[\sigma_k(h^{t+1}) \neq \sigma_i^*(h^t)](2p_i - 1)z\Big] \\ &+ Pr(\tau_k = t)\Big[(2p_i - 1)z + (1 - p_i)(1 - \bar{q})y - p_i\bar{q}x\Big]\Big] \\ &= (-1)EU_i(\sigma_i^*(h^t) = a_\omega, \sigma_k | p_i, s_i = s_\omega) \end{aligned}$$

The expected utility obtained from taking an action according to the signal received is higher than that from an action opposite to the signal. Hence, the first mover would always take an action according to his signal irrespective of his precision level.

B Proof of Lemma 2

Assume agent 1 is the first mover and takes an action a_1 at time period t. s_2 and p_2 are the signal and precision level of agent 2 which are private information of agent 2. However, from Lemma 1 and the knowledge of the action of agent 1, agent 2 can derive the true signal,s_1 received by agent 1. In the following two segments we describe the decision rule of the second mover.

(A) If $s_1 = s_2$

The expected utility to agent 2 from taking an action $\sigma_2(h^{t+1}; s_2 = s_\omega, p_2) = a_\omega$ is given by the following expression

$$\begin{aligned} EU\Big[\sigma_2(h^{t+1}; s_2, p_2) = a_\omega, \sigma_1^*|p_2, s_2 = s_\omega\Big] &= Pr[\omega|s_\omega = s_1 = s_2](z-x) \\ &+ [1 - Pr[\omega|s_\omega = s_1 = s_2]](-z+y) \end{aligned}$$

The expected utility to agent 2 from taking an action $\sigma_2(h^{t+1}; s_2 = s_\omega, p_2) \neq a_\omega$ is given by the following expression

$$\begin{aligned} EU\Big[\sigma_2(h^{t+1}; s_2, p_2) \neq a_\omega, \sigma_1^*|p_2, s_2 = s_\omega\Big] &= Pr[\omega|s_\omega = s_1 = s_2](-z) \\ &+ [1 - Pr[\omega|s_\omega = s_1 = s_2]](z) \end{aligned}$$

The second mover's decision rule is the following : If $Pr[\omega|s_\omega = s_1 = s_2] \geq \frac{2z-y}{4z-x-y} \geq \frac{1}{2}$ then it takes an action according to its own signal else goes against its signal.

(B)If $s_1 \neq s_2$

The expected utility to agent 2 from taking an action $\sigma_2(h^{t+1}; s_2 = s_\omega, p_2) = a_\omega$ is given by the following expression

$$\begin{aligned} EU\Big[\sigma_2(h^{t+1}; s_2, p_2) = a_\omega, \sigma_1^*|p_2, s_2 = s_\omega \neq s_1\Big] &= Pr[\omega|s_\omega = s_2 \neq s_1](z) \\ &+ [1 - Pr[\omega|s_\omega = s_2 \neq s_1]](-z) \end{aligned}$$

The expected utility to agent 2 from taking an action $\sigma_2(h^{t+1}; s_2 = s_\omega, p_2) \neq a_\omega$ is given by the following expression

$$\begin{aligned} EU\Big[\sigma_2(h^{t+1}; s_2, p_2) \neq a_\omega, \sigma_1^*|p_2, s_2 = s_\omega \neq s_1\Big] &= Pr[\omega|s_\omega = s_2 \neq s_1](-z+y) \\ &+ [1 - Pr[\omega|s_\omega = s_2 \neq s_1]](z-x) \end{aligned}$$

The second mover's decision rule is the following: If $Pr[\omega|s_\omega = s_2 \neq s_1] \geq \frac{2z-x}{4z-x-y}$ then it takes an action according to its own signal else goes against its signal.

We will now describe the equilibrium strategy for the second mover.

Case (i) $p_2 \geq q(p_2)$

If $s_1 = s_2$, then $Pr[\omega|s_\omega = s_1 = s_2, p_2] = \frac{p_2 q(p_2)}{p_2 q(p_2)+(1-p_2)(1-q(p_2))}$. We assume that $q'(p_2) > 0$ therefore $\frac{\partial Pr[\omega|s_\omega = s_1 = s_2]}{\partial p_2} > 0$. Since $Pr[\omega|s_\omega = s_1 = s_2, p_2 = 0.5] = 0.5$. Thus, there exists a precision level p^* such that $\forall p_2 \leq p^*$ $Pr[\omega|s_\omega = s_1 = s_2] \leq \frac{2z-y}{4z-x-y}$. However, $\forall p_2 \geq p^*$ $Pr[\omega|s_\omega = s_1 = s_2] \geq \frac{2z-y}{4z-x-y}$. Thus for all $p_2 \leq p^*$, agent 2 does not follow its own signal and for all $p_2 \geq p^*$ agent 2 follows its signal.

Now if $s_1 \neq s_2$, $Pr[\omega|s_\omega = s_2 \neq s_1, p_2] = \frac{p_2(1-q(p_2))}{p_2(1-q(p_2))+(1-p_2)(q(p_2))}$. Since, $p_2 \geq q(p_2)$, $Pr[\omega|s_\omega = s_2 \neq s_1, p_2] \geq \frac{1}{2}$. However, since $x \geq y$ $\frac{2z-x}{4z-x-y} \leq \frac{1}{2}$. Therefore, the second agent would always follow his signal if $s_1 \neq s_2$ and $p_2 \geq q(p_2)$

Thus, if $x \geq y$ and $p_2 \geq q(p_2)$ then there exists a precision level p^* such that $\forall p_2 \leq p^*$ agent two would anti herd and $\forall p_2 \geq p^*$ agent 2 would take an action according to its own signal.

Case (ii) $p_2 \leq q(p_2)$

If $s_1 = s_2$, $Pr[\omega|s_\omega = s_1 = s_2, p_2] = \frac{p_2 q(p_2)}{p_2 q(p_2)+(1-p_2)(1-q(p_2))}$ We assume that $q'(p_2) > 0$ therefore $\frac{\partial Pr[\omega|s_\omega = s_1 = s_2]}{\partial p_2} > 0$. Since $Pr[\omega|s_\omega = s_1 = s_2, p_2 = 0.5] \geq \frac{2z-y}{4z-x-y}$ Therefore, if $s_1 = s_2$ and $p_2 \leq q(p_2)$ then the second agent always follows its own signal.

Now if $s_1 \neq s_2$ $Pr[\omega|s_\omega = s_2 \neq s_1, p_2] = \frac{p_2(1-q(p_2))}{p_2(1-q(p_2))+(1-p_2)(q(p_2))}$.

Since, $\frac{p_2(1-p_1))}{p_2(1-p_1)+(1-p_2)p_1}$ is concave in p_1 therefore

$$\frac{p_2(1-q(p_2))}{p_2(1-q(p_2))+(1-p_2)(q(p_2))} \geq E\Big[\frac{p_2(1-p_1)}{p_2(1-p_1)+(1-p_2)p_1}|p_2 \leq p_1\Big]$$

Since, $\frac{\partial \frac{p_2(1-p_1)}{p_2(1-p_1)+(1-p_2)p_1}}{\partial p_2} > 0$ and $Pr[\omega|s_\omega = s_2 \neq s_2, p_2 = 0.5] \leq \frac{2z-x}{4z-x-y}$. Then there exists a precision level p^{**} such that $\forall p_2 \leq p^*$ $Pr[\omega|s_\omega = s_2 \neq s_1] \leq \frac{2z-x}{4z-x-y}$. However, $\forall p_2 \geq p^{**}$ $Pr[\omega|s_\omega = s_2 \neq s_1] \geq \frac{2z-x}{4z-x-y}$. Thus for all $p_2 \leq p^{**}$, agent 2 does not follow its own signal and for all $p_2 \geq p^{**}$ agent 2 follows its signal.

Thus, if $x \geq y$ and $p_2 \leq q(p_2)$ then there exists a precision level p^{**} such that $\forall p_2 \leq p^*$ agent two would mimic the first mover and $\forall p_2 \geq p^{**}$ agent 2 would take an action according to its own signal.

C Proof of Lemma 3

Let the two agents be denoted by i and k. Let h_t^0 be the history such that no agent has taken an action till date t. We would first show that under the common belief that a lower precision agent moves first, if agent i with precision level p_i prefers to take an action at date t given history h_t^0 then all agents with precision level lower than p_i would also prefer to do so. The expected utility of agent i from taking an action at period t given history h_t^0 is

$$\begin{aligned} EU_i(t, p_i|h_t^0) &= Pr(\tau_k = t|h_t^0)\delta^{t-1}\Big[(2p_i - 1)z + (1-p_i)(1-\bar{q})y - p_i\bar{q}x\Big] \\ &+ Pr(\tau_k > t|h_t^0)\delta^{t-1}\Big[(2p_i-1)z(1+\delta) + \delta[1-F(p^*)][(1-p_i)(1-\bar{q})y - p_i\bar{q}x]\Big] \end{aligned}$$

δ is the per period discount factor. $\bar{q}$ is the mean precision level or the expected precision of the first mover given the distribution of precision, $F(p_i)$. p^* is the critical precision level from Lemma 2 such that all agents with precision level $p_i < p^*$ would always anti-herd and all agents with precision level $p_i > p^*$ would move according to their own signal. $Pr(\tau_k = u|h_t^0)$ is the belief that agent k would take an action at period u given history h_t^0. Its assumed that $Pr(\tau_k = t|h_t^0) = Pr(\tau_k = u|h_t^0)$ for any $t, u \in T$. Also, all agents have the same common belief about the timing of the other player. The latter follows from the assumption that the precision of each agent is private information.
The expected utility of agent i with precision level $p_i \geq p^*$ from waiting till period u to take an action given history h_t^0

$$\begin{aligned} EU_i(u, p_i|h_t^0) &= Pr(t \leq \tau_k < u|h_t^0)\delta^{\tau_k}\Big[(2p_i - 1)z + (1-p_i)(1-\bar{q})y - p_i\bar{q}x\Big] \\ &+ Pr(\tau_k = u|h_t^0)\delta^{u-1}\Big[(2p_i - 1)z + \ (1-p_i)(1-\bar{q})y - p_i\bar{q}x\Big] \\ &+ Pr(\tau_k > u|h_t^0)\delta^{u-1}\Big[(2p_i-1)z(1+\delta) + \delta[1-F(p^*)][(1-p_i)(1-\bar{q})y - p_i\bar{q}x]\Big] \end{aligned}$$

The following inequality would hold for agent i who prefers to take an action at period t.

$$EU_i(t, p_i|h_t^0) - EU_i(u, p_i|h_t^0) \quad = \quad A(2p_i - 1)z + B[(1-p_i)(1-\bar{q})y - p_i\bar{q}x] \geq 0 \tag{1}$$

where,

$$\begin{aligned} A &= \Big[Pr(\tau_k = t|h_t^0)\delta^{t-1} + Pr(\tau_k > t|h_t^0)\delta^{t-1}(1+\delta) \\ &- Pr(t \leq \tau_k < u|h_t^0)\delta^{\tau_k} - Pr(\tau_k = u|h_t^0)\delta^{u-1} - Pr(\tau_k > u|h_t^0)\delta^{u-1}(1+\delta)\Big] \\ B &= \Big[Pr(\tau_k = t|h_t^0)\delta^{t-1} + [1-F(p^*)]Pr(\tau_k > t|h_t^0)\delta^{t} - \delta^{\tau_k}Pr(t \leq \tau_k < u) \\ &- Pr(\tau_k = u)\delta^{u-1} - [1-F(p^*)]Pr(\tau_k > u)\delta^{u}\Big] \end{aligned}$$

Equation (1) can be re-written as

$$\frac{2p_i - 1}{(1-p_i)(1-\bar{q})y - p_i\bar{q}x} \geq \frac{-B}{A}$$

If $\bar{q}x > (1-\bar{q})y > 0$ or $x > 0, y < 0$ then

$$\frac{\partial \frac{2p_i - 1}{(1-p_i)(1-\bar{q})y - p_i\bar{q}x}}{\partial p_i} < 0$$

Therefore, if $\bar{q}x > (1-\bar{q})y$ or $x > 0, y < 0$

$$\frac{2p-1}{(1-p)(1-\bar{q})y - p\bar{q}x} \geq \frac{-B}{A} \quad \forall p \leq p_i \ \ s.t. p, p_i \geq p_i^*$$

Hence,

$$EU_k(t, p_k|h_t^0) - EU_k(u, p_k|h_t^0) \geq 0 \quad \forall p_k \leq p_i \quad s.t. \quad p_k, p_i \geq p^*$$

Now, lets analyze the nature of $\tau(p_i)$ for agents with precision level $p_i \leq p^*$. The expected utility of agent i with precision level $p_i \leq p^*$ from waiting till period u to take an action given history h_t^0 is given by

$$\begin{aligned} EU_i(u, p_i|h_t^0) &= Pr(t \leq \tau_k < u|h_t^0)\delta^{\tau_k}\Big[(2\bar{q}-1)z + (1-\bar{q})y - \bar{q}x\Big] \\ &+ Pr(\tau_k = u|h_t^0)\delta^{u-1}\Big[(2p_i-1)z + \ (1-p_i)(1-\bar{q})y - p_i\bar{q}x\Big] \\ &+ Pr(\tau_k > u|h_t^0)\delta^{u-1}\Big[(2p_i-1)z(1+\delta) + \delta[1-F(p^*)][(1-p_i)(1-\bar{q})y - p_i\bar{q}x]\Big] \end{aligned}$$

The following inequality holds for agent i who prefers to take an action at period t

$$EU_i(t, p_i|h_t^0) - EU_i(u, p_i|h_t^0) = A(2p_i-1)z + B[(1-p_i)(1-\bar{q})y - p_i\bar{q}x] - C \geq 0 \tag{2}$$

where,

$$\begin{aligned} A &= \Big[Pr(\tau_k = t|h_t^0)\delta^{t-1} + Pr(\tau_k > t|h_t^0)\delta^{t-1}(1+\delta) \\ &- Pr(\tau_k = u|h_t^0)\delta^{u-1} - Pr(\tau_k > u|h_t^0)\delta^{u-1}(1+\delta)] \\ B &= \Big[Pr(\tau_k = t|h_t^0)\delta^{t-1} + [1-F(p^*)]Pr(\tau_k > t|h_t^0)\delta^t \\ &- Pr(\tau_k = u)\delta^{u-1} - [1-F(p^*)]Pr(\tau_k > u)\delta^u] \\ C &= Pr(t \leq \tau_k < u|h_t^0)\delta^{\tau_k}\Big[(2\bar{q}-1)z + (1-\bar{q})y - \bar{q}x\Big] \end{aligned}$$

If $\bar{q}x > (1-\bar{q})y > 0$ or $x > 0, y < 0$ then

$$\frac{\partial \frac{(2p_i-1)z}{(1-p_i)(1-\bar{q})y - p_i\bar{q}x}}{\partial p_i} < 0$$

Therefore, if $\bar{q}x > (1-\bar{q})y$ or $x > 0, y < 0$

$$\frac{(2p-1)z}{(1-p)(1-\bar{q})y - p\bar{q}x} \geq \frac{-B}{A} \quad \forall p \leq p_i$$

Hence,

$$EU_k(t, p_k|h_t^0) - EU_k(u, p_k|h_t^0) > 0 \ \ \forall p_k \leq p_i \quad s.t. \quad p_k, p_i \leq p^*$$

Thus, if $\bar{q}x \geq (1-\bar{q})y > 0$ or $x > 0, y < 0$ then if p_i prefers to move at period t then all agents with precision level lower than p_i would also prefer to take an action at period t. Thus $\tau(p_i)$ is increasing in p_i.

D Proof of Proposition 1

To prove that if $\bar{q}x > (1-\bar{q})y > 0$ or $x > 0, y < 0$, all agents will take an action together in the first period. I would first prove that given that $\tau'(p_i) \geq 0$ agents with precision level $p_i = 1$ would strictly prefer to act in period t given history h_t^0. The expected utility of agent i with precision level $p_i = 1$ at period t given h_t^0 is

$$\begin{aligned}
EU_i(t, p_i|h_t^0) &= Pr(\tau_k = t|h_t^0)\delta^{t-1}\Big[(2p_i-1)z + (1-p)(1-\bar{q})y - p_i\bar{q}x\Big] \\
&+ Pr(\tau_k > t|h_t^0)\delta^{t-1}\Big[(2p_i-1)z(1+\delta) + \delta[1-F(p^*)][(1-p_i)(1-\bar{q})y - p_i\bar{q}x]\Big] \\
&= Pr(\tau_k = t)\delta^{t-1}\Big[z - \bar{q}x\Big] + \delta^{t-1}Pr(\tau_k > t)\Big[z(1+\delta) - \delta[1-F(p^*)]\bar{q}x\Big]
\end{aligned}$$

The expected utility of agent i from waiting till period $u > t$ to take an action given that the history h_t^0 under the assumption that $p^* < 1$

$$\begin{aligned}
EU_i(u, p_i|h_t^0) &= Pr(t \leq \tau_k < u|h_t^0)\delta^{\tau_k}\Big[(2p_i-1)z + (1-p_i)(1-\bar{q})y - p_i\bar{q}x\Big] \\
&+ Pr(\tau_k = u|h_t^0)\delta^{u-1}\Big[(2p_i-1)z + \ (1-p_i)(1-\bar{q})y - p_i\bar{q}x\Big] \\
&+ Pr(\tau_k > u|h_t^0)\delta^{u-1}\Big[(2p_i-1)z(1+\delta) + \delta[1-F(p^*)][(1-p_i)(1-\bar{q})y - p_i\bar{q}x]\Big] \\
&= Pr(t \leq \tau_k < u|h_t^0)\delta^{\tau_k}\Big[z - \bar{q}x\Big] + Pr(\tau_k = u|h_t^0)\delta^{u-1}\Big[z - \bar{q}x\Big] \\
&+ Pr(\tau_k > u|h_t^0)\delta^{u-1}\Big[z(1+\delta) - \delta[1-F(p^*)]\bar{q}x\Big]
\end{aligned}$$

Therefore, for agents with precision level $p_i = 1$

E Proof of Lemma 4

Let the two agents be referred to as High, p_H and Low, p_L agents. There are two possible histories: $H_i(w)$ is the set of all histories, $h^t \in H$ such that no one has taken an action till period t. $H_i(a)$ is the set of all histories, h^t such that the i th agent hasn't taken an action and the k th agent has, till time period t. Consider the following candidate strategy profile for SPNE

$$\sigma_L^* = \begin{cases} \sigma_L(h^t) = a_j, s.t. s_L = s_j \forall j \in \{G, B\} & \forall h^t \in H_L(w) \\ \sigma_L(h^t) = \sigma_H(h^{t-1}), \forall h^t \in H_L(a) \end{cases}$$

$$\sigma_H^* = \begin{cases} \sigma_H(h^t) = a_j . s.t. s_H = s_j \forall j \in \{G, B\} & \forall h^t \in H_H(w) \\ \sigma_H(h^t) = a_j \forall h^t \in H_H(a) \end{cases}$$

The above strategy profile states that the high type agent would never wait and always move according to his signal at all histories. The low type would also not wait and would follow his signal at histories where no one else has yet taken an action. However, it would take an action immediately but mimic the high type at histories where the high type has already moved. We would show that the above strategy profile is a SPNE.

Low Type

First, we will show that the strategy of the Low type agent is a best response to the strategy of the High type, σ_H^* at all histories faced by the Low type.
Case(i) $h^t \in H_L(a)$

$$EU_L\Big(\sigma_L(h^t) = \sigma_H(h^{t-1}), \sigma_H^*|h^t\Big) = \delta^{t-1}[(2p_H - 1)(z - \lambda)]$$

$$EU_L\Big(\sigma_L(h^t) \neq \sigma_H(h^{t-1}), \sigma_H^*|h^t\Big) = \delta^{t-1}[(2p_H - 1)(-z)]$$

$$EU_L\Big(\sigma_L(h^t) = w, \sigma_H^*(h^t)|h^t\Big) = 0$$

Since, $z > \lambda$, $\sigma_L^*(h^t) = \sigma_H(h^{t-1})$ is the best response of the low type for the strategy at history $h^t \in H_L(a)$

Case (ii) $h^t \in H_L(w)$

The utility obtained by the Low type from following the strategy, $\sigma_L(h^t) = a_j$ where $s_L = s_j \forall j \in \{G, B\}$ at history $h^t \in H_L(w)$ given the strategy of the High type, σ_H^* is given by the following expression.

$$EU_L\Big(\sigma_L(h^t) = a_j, \sigma_H^*|h^t\Big) = \delta^{t-1}[(2p_L - 1)z + \lambda[(1 - p_H)(1 - p_L) - p_H p_L]]$$

Similarly, the utility obtained by the Low type agent when he takes the action ,$\sigma_L(h^t) = w$ is given by the following expression.

$$EU_L\Big(\sigma_L(h^t) = w, \sigma_H^*|h^t\Big) = \delta^{t-1}[(2p_L - 1)(z - \lambda)]$$

Similarly, the utility obtained by the Low type agent when he takes the action $\sigma_L(h^t) \neq a_j$ for $s_L = s_j$ and $j \in \{G, B\}$ is given by the following expression.

$$EU_L\Big(\sigma_L(h^t) \neq a_j, \sigma_H^*(h^t)|h^t\Big) = \delta^{t-1}[(1 - 2p_L)z + \lambda[p_H(1 - p_L) - p_L(1 - p_H)]]$$

Given that $z \geq \lambda$, $\sigma_L^*(h^t) = a_j$ is the best response of the low type for the strategy at history $h^t \in H_L(w)$

High Type

Case(i) $h^t \in H_H(a)$
The utility obtained by the High type agent from following the strategy, $\sigma_H(h^t) = a_j$ at history $h^t \in H_H(a)$ given the strategy of the Low type, σ_L^* is given by the following expression.

$$EU_H\Big(\sigma_H(h^t) = a_j, \sigma_L^*|h^t\Big) = \delta^t[(2p_H - 1)z + [(1 - p_H)(1 - p_L) - p_H p_L]\lambda]$$

The expected utility to the high type agent from taking an action opposite to his signal is given by the following expression

$$EU_H\Big(\sigma_H(h^t) \neq a_j, \sigma_L^*|h^t\Big) = \delta^t[(1 - 2p_H)z + \lambda[p_H(1 - p_L) - p_L(1 - p_H)]]$$

$$EU_H(\sigma_L(h^t), \sigma_H^*(h^t) = w|h^t) = 0$$

Since, $z > \lambda$, $\sigma_H^*(h^t) = a_j$ where $s_L = s_j \forall j \in \{G, B\}$ is the best response of the high type for the strategy at history $h^t \in H_H(a)$

Case (ii) $h^t \in H_H(w)$

The utility obtained by the High type from following the strategy, $\sigma_H(h^t) = a_j$ where $s_H = s_j \forall j \in \{G, B\}$ at history $h^t \in H_H(w)$ given the strategy of the Low type, σ_L^* is given by the following expression.

$$EU_H\Big(\sigma_H(h^t) = a_j, \sigma_L^*|h^t\Big) = \delta^{t-1}[(2p_H - 1)z + \lambda[(1 - p_H)(1 - p_L) - p_H p_L]]$$

Similarly, the utility obtained by the Low type agent when he takes the action
, $\sigma_H(h^t) = w$ or $\sigma_H(h^t) \neq a_j$ is given by the following expression.

$$EU_H\Big(\sigma_H(h^t) = w, \sigma_L^*|h^t\Big) = \delta^{t-1}[(1 - 2p_H)z + \lambda[p_H(1 - p_L) - p_L(1 - p_H)]]$$

Given that $z \geq \lambda$, $\sigma_H^*(h^t) = a_j$ is the best response of the low type for the strategy at history $h^t \in H_H(w)$

Hence, the strategy profile mentioned above is an SPNE. Thus when agents have common knowledge about their precision level then the unique sub game perfect nash equilibrium is for both the agents to take an action in the first period according to their own signal.

F Proof of Lemma 5

Assume agent 1 is the first mover and takes an action a_1 at time period t. s_2 and p_2 are the signal and precision level of agent 2 which are private information of agent 2. However, from Lemma 1 and the knowledge of the action of agent 1 agent 2 can derive the true signal, s_1 received by agent 1. In the following two segments we describe the decision rule of the second mover.

(A) If $s_1 = s_2$

The expected utility to agent 2 from taking an action $\sigma_2(h^{t+1}; s_2 = s_\omega, p_2) = a_\omega$ is given by the following expression

$$\begin{aligned} EU\Big[\sigma_2(h^{t+1}; s_2, p_2) = a_\omega, \sigma_1^*|p_2, s_2 = s_\omega\Big] &= Pr[\omega|s_\omega = s_1 = s_2](z - x) \\ &+ [1 - Pr[\omega|s_\omega = s_1 = s_2]](-z + y) \end{aligned}$$

The expected utility to agent 2 from taking an action $\sigma_2(h^{t+1}; s_2 = s_\omega, p_2) \neq a_\omega$ is given by the following expression

$$\begin{aligned} EU\Big[\sigma_2(h^{t+1}; s_2, p_2) \neq a_\omega, \sigma_1^*|p_2, s_2 = s_\omega\Big] &= Pr[\omega|s_\omega = s_1 = s_2](-z) \\ &+ [1 - Pr[\omega|s_\omega = s_1 = s_2]](z) \end{aligned}$$

The second mover's decision rule is the following:

If $Pr[\omega|s_\omega = s_1 = s_2] \geq \frac{2z-y}{4z-x-y}$ then take an action according to your own signal else go against your signal.

(B) If $s_1 \neq s_2$

The expected utility to agent 2 from taking an action $\sigma_2(h^{t+1}; s_2 = s_\omega, p_2) = a_\omega$ is given by the following expression

$$\begin{aligned} EU\Big[\sigma_2(h^{t+1}; s_2, p_2) = a_\omega, \sigma_1^*|p_2, s_2 = s_\omega\Big] &= Pr[\omega|s_\omega = s_2 \neq s_1](z) \\ &+ [1 - Pr[\omega|s_\omega = s_2 \neq s_1]](-z) \end{aligned}$$

The expected utility to agent 2 from taking an action $\sigma_2(h^{t+1}; s_2 = s_\omega, p_2) \neq a_\omega$ is given by the following expression

$$\begin{aligned} EU\Big[\sigma_2(h^{t+1}; s_2, p_2) \neq a_\omega, \sigma_1^*|p_2, s_2 = s_\omega\Big] &= Pr[\omega|s_\omega = s_2 \neq s_1](-z + y) \\ &+ [1 - Pr[\omega|s_\omega = s_2 \neq s_1]](z - x) \end{aligned}$$

The second mover's decision rule is the following:

If $Pr[\omega|s_\omega = s_2 \neq s_1] \geq \frac{2z-x}{4z-x-y}$ then take an action according to your own signal else go against your signal.

Now we would describe the equilibrium strategy of the second mover under two possible scenarios : (i) $p_2 \geq q(p_2)$ and (ii) $p_2 \leq q(p_2)$

Case (i) $p_2 \geq q(p_2)$

If $s_1 = s_2$, $Pr[\omega|s_\omega = s_1 = s_2, p_2] = \frac{p_2 q(p_2)}{p_2 q(p_2) + (1-p_2)(1-q(p_2))} \geq \frac{1}{2}$ Since, $y \geq x$,$\frac{2z-y}{4z-x-y} \leq \frac{1}{2}$. Therefore, if $s_1 = s_2, y \geq x$ and $p_2 \geq q(p_2)$ then all agents would follow their signal.

Now if $s_1 \neq s_2$, $Pr[\omega|s_\omega = s_2 \neq s_1, p_2] = \frac{p_2(1-q(p_2))}{p_2(1-q(p_2)) + (1-p_2)(q(p_2))}$. Since, $p_2 \geq q(p_2)$, $Pr[\omega|s_\omega = s_2 \neq s_1, p_2 = 0.5] = \frac{1}{2}$. However, since $y \geq x$ $\frac{2z-x}{4z-x-y} \geq \frac{1}{2}$. Therefore, the second agent would always follow his signal if $s_1 \neq s_2$ and $p_2 \geq q(p_2)$

Thus, if $y \geq x$ and $p_2 \geq q(p_2)$ then all agents would follow their own signal.

Case (ii) $p_2 \leq q(p_2)$

Then if $s_1 = s_2$ $Pr[\omega|s_\omega = s_1 = s_2, p_2] = \frac{p_2 q(p_2)}{p_2 q(p_2) + (1-p_2)(1-q(p_2))} \geq \frac{1}{2}$ Since, $y \geq x$ $\frac{2z-y}{4z-x-y} \leq \frac{1}{2}$. Therefore, if $s_1 = s_2, y \geq x$ and $p_2 \leq q(p_2)$ then all agents would follow their signal.

Now if $s_1 \neq s_2$ $Pr[\omega|s_\omega = s_2 \neq s_1, p_2] = \frac{p_2(1-q(p_2))}{p_2(1-q(p_2)) + (1-p_2)(q(p_2))} \leq \frac{1}{2}$. Since, $y \geq x$ $\frac{2z-x}{4z-x-y} \geq \frac{1}{2}$. Thus all second agents would mimic the first mover. Hence, if $y \geq x$ and $p_2 \leq q(p_2)$ then all second movers would mimic the first movers.

G Proof of Lemma 6

Let the two agents be denoted by i and k. Let h_t^0 be the history such that no agent has taken an action till date t. To prove that if $z \geq y \geq x > 0$ or $z \geq y > 0, x < 0$ $\tau(p_i)$ is decreasing in p_i we would first show that under the common belief that a higher precision agent moves first, if agent i with precision level p_i prefers to take an action at date t given history h_t^0 then all agents with precision level higher than p_i would also prefer to do so. The expected utility of agent i from taking an action at period t given history h_t^0 is

$$\begin{aligned} EU_i(t, p_i|h_t^0) &= Pr(\tau_k = t|h_t^0)\delta^{t-1}\Big[(2p_i - 1)z + -p_i\bar{q}x + (1-p_i)(1-\bar{q})y\Big] \\ &+ Pr(\tau_k > t|h_t^0)\delta^{t-1}\Big[(2p_i - 1)z(1+\delta) + \delta(-p_i x + (1-p_i)y)\Big] \end{aligned}$$

δ is the per period discount factor. $\bar{q}$ is the mean precision level or the expected precision of the first mover given the distribution of precision, $F(p_i)$. $Pr(\tau_k = u|h_t^0)$ is the belief that agent k would take an action at period u given history h_t^0. Its assumed that $Pr(\tau_k = t|h_t^0) = Pr(\tau_k = u|h_t^0)$ for any $t, u \in T$. Also, all agents have the same common belief about the timing of the other player. The latter follows from the assumption that precision of each individual is private information. The expected utility of agent i from waiting till period u to take an action given that no one has yet taken an action at date t is

$$\begin{aligned} EU_i(u, p_i|h_t^0) &= Pr(t \leq \tau_k < u|h_t^0)\delta^{\tau_k}\Big[(2\bar{q}-1)z + (1-\bar{q})y - \bar{q}x\Big] \\ &+ Pr(\tau_k = u|h_t^0)\delta^{u-1}\Big[(2p_i - 1)z + (1-p_i)(1-\bar{q})y - p_i\bar{q}x\Big] \\ &+ Pr(\tau_k > u|h_t^0)\delta^{u-1}\Big[(2p_i-1)z(1+\delta) + \delta(-p_i x + (1-p_i)y)\Big] \end{aligned}$$

For agent i who prefers to take an action at period t would have

$$EU_i(t, p_i|h_t^0) - EU_i(u, p_i|h_t^0) = Ap_i - B(1-p_i) \geq 0$$

where,

$$\begin{aligned} A &= \Big[Pr(\tau_k = t|h_t^0)\delta^{t-1}(z - \bar{q}x) + Pr(\tau_k > t|h_t^0)\delta^{t-1}z + Pr(\tau_k > t)\delta^t(z-x) \\ &- Pr(\tau_k = u|h_t^0)\delta^{u-1}(z-\bar{q}x) - Pr(\tau_k > u|h_t^0)\delta^{u-1}z - Pr(\tau_k > u)\delta^u(z-x)] \\ B &= \Big[Pr(\tau_k = t|h_t^0)\delta^{t-1}(z-(1-\bar{q})y) + Pr(\tau_k > t|h_t^0)\delta^{t-1}z + Pr(\tau_k > t|h_t^0)\delta^t \\ &\quad (z-y) - Pr(\tau_k = u|h_t^0)\delta^{u-1}(z-(1-\bar{q})y) + Pr(\tau_k > u|h_t^0)\delta^{u-1}z \\ &+ Pr(\tau_k > u|h_t^0)\delta^t(z-y)] \end{aligned}$$

Since, $z \geq x, y$, $A, B > 0$ The derivative of $EU_i(t|h_t^0) - EU_i(u|h_t^0)$ with respect to p_i is positive. Hence, if an agent with precision level p_i prefers to move at period t then all agents with precision level greater than that would also prefer to do that.

$$EU_k(t, p_k|h_t^0) - EU_k(u, p_k|h_t^0) > 0 \ \ \forall p_k \geq p_i$$

Therefore, $\tau(p_i)$ is decreasing in p_i and all agents with precision level higher than p_i would also prefer to take an action at period t

H Proposition 2

$\tau(p_i)$ is characterized by a step function, if there exists a number $k < \infty$ and numbers $0.5 = p_0^* < p_1^* < p_2^* \cdots < p_k^* = 1$ such that if $p_{r-1}^* < p_i < p_r^*$ then $\tau(p_i) = (r-1)l, \forall r = 1, \ldots, k$. We will characterize the equilibrium by uniquely identifying the set of the cut-off points $0.5 = p_0^* < p_1^* < p_2^* \ldots p_k^* = 1$ such that if $p_{r-1}^* < p_i < p_r^*$ then $\tau(p_i) = (r-1)l, \forall r = 1, \ldots, k$ is a perfect bayesian equilibrium. Agent with precision level $p_m^* \forall m \in \{0, 1, \ldots k\}$ is such that

$$EU[\tau(p_m^*) = (m-1)l|h^{(m-1)l}, p_m^*] = EU[\tau(p_m^*) = ml|h^{(m-1)l}, p_m^*]$$

If it is believed that the equilibrium is characterized by a step function of the form described above then the expected utility from taking an action at period $(m-1)l$ at history $h^{(m-1)l}$ for an agent with precision level p_i is given by the following expression

$$\begin{aligned}
EU[\tau(p_i) = (m-1)l|h^{(m-1)l}, p_i] &= \delta^{(m-1)l-1}\Big[F(p_m) - F(p_{m-1})\Big] \\
&\Big[(2p_i - 1)z + (1-p_i)(1-q(p_m, p_{m-1}))y \\
&- p_i q(p_m, p_{m-1})x\Big] + \delta^{(m-1)l-1} \\
&F(p_{m-1})\Big[(2p_i - 1)z(1+\delta) + \delta[(1-p_i)y - p_i x]\Big]
\end{aligned}$$

Where $q(p_m, p_{m-1})$ is the average precision level of agents who move at period $(m-1)l$. $q(p_m, p_{m-1})$ is constant with respect to p_i. The expected utility from taking an action at period ml at history $h^{(m-1)l}$ for an agent with precision level p_i is given by the following expression

$$\begin{aligned}
EU[\tau(p_i) = ml|h^{(m-1)l}, p_i] &= \delta^{ml-1}\Big[F(p_m) - F(p_{m-1})\Big]\Big[(2q(p_m, p_{m-1}) - 1)z \\
&+ (1-q(p_m, p_{m-1}))y - q(p_m, p_{m-1})x] + \delta^{ml-1} \\
&\Big[F(p_{m-1}) - F(p_{m-2})\Big]\Big[(2p_i - 1)z + (1-q(p_{m-1}, p_{m-2})) \\
&(1-p_i)y - q(p_{m-1}, p_{m-2})p_i x\Big] + \delta^{ml-1} \\
&F(p_{m-1})\Big[(2p_i - 1)z(1+\delta) + \delta[(1-p_i)y - p_i x]\Big]
\end{aligned}$$

The difference between the expected utility from taking an action at period $(m-1)l$ and that from taking an action at period ml given history $h^{(m-1)l}$ is given by the following expression

$$\begin{aligned}
\mathbf{L(p_i)} &= EU[\tau(p_i) = (m-1)l|h^{(m-1)l}, p_i] - EU[\tau(p_i) = ml|h^{(m-1)l}, p_i] \\
&= \Big[F(p_m) - F(p_{m-1})\Big]\delta^{(m-1)l-1}\Big[(2p_i - 1)z - \delta(2q(p_m, p_{m-1}) - 1)z \\
&+ (1 - p_i - \delta)y(1 - q(p_m, p_{m-1})) + (\delta - p_i)q(p_m, p_{m-1})x\Big] \\
&+ \Big[F(p_{m-1}) - F(p_{m-2})\Big]\delta^{(m-1)l-1}\Big[(2p_i - 1)z + \delta[(1-p_i)q(p_{m-1}, p_{m-2})y \\
&- p_i(1 - q(p_{m-1}, p_{m-2}))x]\Big]
\end{aligned}$$

The derivative of $\mathbf{L}$ with respect to p_i is

$$\begin{aligned}
\frac{\partial \mathbf{L(p_i)}}{\partial p_i} &= \Big[F(p_m) - F(p_{m-1})\Big]\delta^{(m-1)l-1}\Big(2z - y(1-q(p_m, p_{m-1})) - q(p_m, p_{m-1})x\Big) + \\
&\Big[[F(p_{m-1}) + F(p_{m-2})]\delta^{(m-1)l-1}[2z - \delta q(p_{m-1}, p_{m-2})y - \delta(1 - q(p_{m-1}, p_{m-2}))x]\Big]
\end{aligned}$$

Since $z \geq x, y$
$\frac{\partial \mathbf{L(p_i)}}{\partial p_i} > 0$. Also, $L(p_i = 0.5) < 0$. Therefore, we define p^*_m such that

$$\mathbf{L}(p^*_{m-1}) = 0 \tag{3}$$

The agent with precision level p^*_{m-1} would be indifferent between taking an action at period $(m-1)l$ and ml. However,$\forall p_i > p^*_{m-1}, \mathbf{L}(p_i) > 0$. Hence all agents with precision level greater than p^*_{m-1}

would prefer to take an action at $(m-1)l$ given history $h^{(m-1)l}$. However, $L(p_i) < 0$ for all agents precision level $p_i > p^*_{m-1}$. All agents with precision with precision level less than p^*_{m-1} would prefer to delay at $h^{(m-1)l}$ given that $\mathbf{L}(p_i) < 0 \forall p_i < p^*_{m-1}$. Also, all agents above p_i prefer to move $(m-1)l$ given the history $h_0^{(m-1)l}$. Therefore, there exist a cut-off $p^*_{m-1}(p^*_m, p^*_{m-2})$ such that all agents with precision level lower than p_i prefer to not move at $(m-1)l$ period. Given that $p^*_0 = 0.5$ and $p^*_k = 1$ we can exactly identify the cut-offs.

Therefore, there exist cut-offs $\{p^*_j\}_{j=0}^k$, $0.5 = p^*_0 < p^*_1 < p^*_2 \cdots < p^*_k = 1$ such that if $p^*_{r-1} < p_i < p^*_r$ then $\tau(p_i) = (r-1)l, \forall r = 1, \ldots, k$ is a Perfect Bayesian equilibrium.

I Proof of Proposition 3

I.1 (a)

Lets assume that all agents believe that a lower precision agent moves first. The expected utility of agent i with precision level p_i from taking an action at time period t given the history h^0_t is

$$\begin{aligned} EU_i(t|h^0_t) &= Pr(\tau_k = t)[(2p_i - 1)z + [(1-p_i)(1-\bar{q})y - p_i\bar{q}x]] \\ &+ Pr(\tau_k > t)\Big[(2p_i - 1)z + \Big[(1-\bar{q})(1-p_i)y - \bar{q}p_i x\Big][1 - F(p^*)]\Big] \end{aligned}$$

p^* is the critical precision level from Lemma 2 such that all agents with precision level $p_i \leq p^*$ would always anti-herd and all agents with precision level $p_i \geq p^*$ would move according to their own signal.

Since, the pay-offs are not time dependent we do not consider any discounting in this case. The expected utility of agent i with precision level $p_i \geq p^*$ from taking the action at time period $u > t$ given the history h^0_t is given by the following expression.

$$\begin{aligned} EU_i(u|h^0_t) &= Pr(t \leq \tau_k \leq u)\Big[(2p_i - 1)z + (1-p_i)(1-\bar{q})y - p_i\bar{q}x\Big] \\ &+ Pr(\tau_k > u)\Big[(2p_i - 1)z + [(1-p_i)(1-\bar{q})y - p_i\bar{q}x][1 - F(p^*)]\Big] \end{aligned}$$

The difference between the two expected utilities for an agent i who prefers to wait till period u than to take an action at time period t is

$$EU_i(t|h^0_t) - EU_i(u|h^0_t) = -Pr(t < \tau \leq u)\Big[(1-p_i)(1-\bar{q})y - p_i\bar{q}xF(p^*)\Big] \geq 0$$

If $x \geq y$ then $(1-p_i)(1-\bar{q})y - p_i\bar{q}x < 0, \forall p_i$ as $p_i \in [0.5, 1]$. Therefore, all agents with precision level lower than p_i would also prefer to take an action at period t.

Similarly, the expected utility of agent i with precision level $p_i \leq p^*$ from taking the action at time period $u > t$ given the history h^0_t is given by the following expression.

$$\begin{aligned} EU_i(u|h^0_t) &= Pr(t \leq \tau_k \leq u)\Big[(2\bar{q} - 1)z + (1-\bar{q})y - \bar{q}x\Big] \\ &+ Pr(\tau_k > u)\Big[(2p_i - 1)z + [(1-p_i)(1-\bar{q})y - p_i\bar{q}x][1 - F(p^*)]\Big] \end{aligned}$$

The difference between the two expected utilities for an agent i who prefers to wait till period u than to take an action at time period t is

$$EU_i(t|h^0_t) - EU_i(u|h^0_t) = Pr(t \leq \tau \leq u)\Big[2(p_i - \bar{q})z - p_i(1-\bar{q})y + \bar{q}(1-p_i)x\Big] \geq 0$$

The derivative of this difference with respect to p_i is

$$\frac{\partial EU_i(t|h_t^0) - EU_i(u|h_t^0)}{\partial p_i} = Pr(t \leq \tau \leq u)\Big[2z - (1-\bar{q})y - \bar{q}x\Big] \tag{4}$$

Since, $z > x, y$ therefore, if an agent i with precision level $p_i \leq p^*$ prefers to move at period t given history h_t^0 then all agents with precision level $p \leq p_i \leq p^*$ would also prefer to do so. Thus $\tau(p_i)$ is increasing in p_i.

If $p^* > 1$ then the difference between the expected utility of agent with precision level $p_i = 1$ from taking an action at time period t and time period u given the history h_t^0 is given by the following expression

$$EU_i(t|h_t^0, p_i = 1) - EU_i(u|h_t^0, p_i = 1) \quad = \quad Pr(t \leq \tau \leq u)\Big[(1-\bar{q})(2z - y)\Big]$$

Since, $2z > y$, $EU_i(t|h_t^0, p_i = 1) - EU_i(u|h_t^0, p_i = 1) \geq 0$

If $p^* < 1$ then the difference between the expected utility of agent with precision level $p_i = 1$ from taking an action at time period t and time period u is given by the following expression

$$EU_i(t|h_t^0, p_i = 1) - EU_i(u|h_t^0, p_i = 1) \quad = \quad Pr(t < \tau \leq u)\Big[\bar{q}xF(p^*)\Big] \geq 0$$

Therefore, agent , $p_i = 1$ always prefers to take an action at time period t given the history h_t^0. Hence, under the belief that lower precision agents take an action first there is a unique equilibrium where all agents would take an action in the first period.

I.2 (b)

Assume that every agent believes that high precision agents move first. The expected utility of agent with precision level p_i is

$$\begin{aligned} EU_i(t|h_t^0) \quad &= \quad Pr(\tau_k = t|h_t^0)\Big[(2p_i - 1)z + (1-\bar{q})(1-p_i)y - p_i\bar{q}x\Big] \\ &+ \quad Pr(\tau_k > t)\Big[(2p_i - 1)z + (1-p_i)y - p_i x\Big] \end{aligned}$$

The expected utility of agent i from waiting till period u to take an action under the common belief that higher precision agents move first is

$$\begin{aligned} EU_i(u|h_t^0) \quad &= \quad Pr(t \leq \tau_k < u)\Big[(2\bar{q} - 1)z + (1-\bar{q})y - \bar{q}x\Big] \\ &+ \quad Pr(\tau_k = u|h_t^0)\Big[(2p_i - 1)z + [(1-\bar{q})(1-p_i)y - \bar{q}p_i x]\Big] \\ &+ \quad Pr(\tau_k > u)\Big[(2p_i - 1)z + (1-p_i)y - p_i x\Big] \end{aligned}$$

The difference between the two expected utilities for an agent i who prefers to wait till period u than to take an action at time period t is

$$EU_i(t|h_t^0) - EU_i(u|h_t^0) \quad = \quad p_i A - (1-p_i)B \geq 0$$

where,

$$
\begin{aligned}
A &= Pr(\tau_k = t)(z - \bar{q}x) + Pr(\tau_k > t)[z - x] - Pr(\tau_k = u)[z - \bar{q}x] - Pr(\tau_k > u)[z - x] \\
B &= Pr(\tau_k = t)(z - (1 - \bar{q})y) + Pr(\tau_k > t)[z - y] - Pr(\tau_k = u)[z - (1 - \bar{q})y] \\
&- Pr(\tau_k > u)[z - y]
\end{aligned}
$$

The derivative of $EU_i(t|h_t^0) - EU_i(u|h_t^0)$ with respect to p_i is positive. Hence, if an agent with precision level p_i prefers to move at period t then all agents with precision level greater than would also prefer to do that. Hence, $\tau(p_i)$ is decreasing in p_i. The cut-offs and the exact characterization is similar to that in Proposition 2.