Indexed Bonds And Default in Emerging Economies

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Abstract

In this paper I analyze a quantitative model of international lending with default and indexed debt. I show that there exists a large gain to using optimal financial contracts. However, the fully optimal financial contract is informationally demanding and the maximum gain may not be achievable; so I study a simple indexed bond. I show that a large fraction of the maximum gain can be utilized by augmenting the financial markets with an indexed bond. I also analyze alternative measures of financial markets inefficiency and find that possible gains to financial intermediaries are even larger than those of the borrower.

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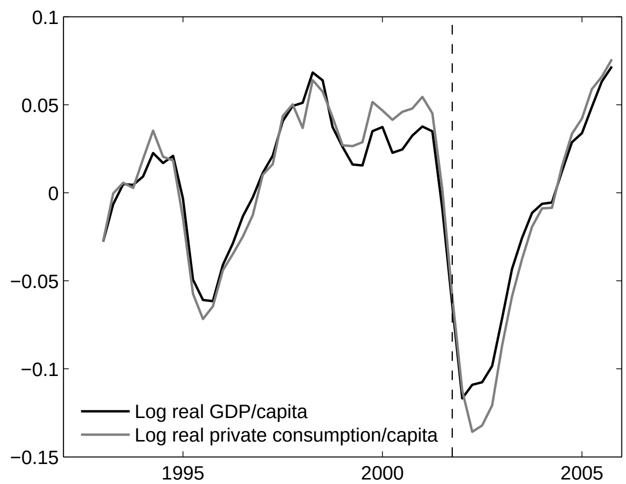
1 Introduction

Emerging economies are well known to be default prone. And all of the default periods in the emerging economies are associated with a large output and consumption drop. For example, in Argentina during a year prior to the default in December of 2001 real GDP per capita in Argentina declined by 11.8% and consumption declined by even more – 13.6% (see figure 1).

Figure 1: HP-filtered log real GDP per capita and real private consumption per capita in Argentina, 1993:1–2005:4



Economists offered many suggestions on how emerging economies could avoid the costly default periods. Among these are prudent fiscal policies, structural reforms and improvement of the financial markets. But, in one way or another, all of the default periods were related to financial market failures. Moreover, it is financial innovation whose effect can be readily quantified.

In this work I provide a quantitative assessment of the effect of the financial market improvement on default and welfare in the emerging economies.

To this end I analyze the model with incomplete financial markets and risk of repudiation that was introduced by Eaton & Gersowitz (1981) and later extended by Arellano (2007). This model features default in equilibrium and is known to be overall consistent with the emerging markets' business cycles. Using this model, calibrated to the Argentina's economy, I show that there exist large gains to the optimal financial contracting However, the optimal financial arrangement is a fully state contingent contract, *e.g.*Thomas & Worrall (1990); hence, it demands a lot of information to be specified. To address this issue I analyze what fraction of the total surplus can be extracted using a simple financial instrument. I find that in the case of a simple indexed bond approximately 2/3 of the total surplus can be recovered.

The idea of using GDP-indexed bonds has been considered at the IMF for many years now. Example studies are Borenstein & Mauro (2005) that discusses the issues with using indexed bonds in the sovereign debt market and Chamon & Mauro (2005) analyzing how should sovereign indexed debt be priced.[1] Chamon & Mauro (2005) simulate evolution of the sovereign's debt under the two borrowing regimes – with un-contingent debt and with the GDP-indexed debt. Debt-GDP ratio evolves according to the following equation

$$\frac{D_t}{Y_t} = \frac{D_{t-1}}{Y_{t-1}} \frac{\alpha(e_t/e_{t-1})(1+i_\$) + (1-\alpha)(1+i)}{(1+g_t)(1+\pi_t)} - pb_t \tag{1}$$

where α is the fraction of USD denominated debt, e_t is the exchange rate at date t, g_t is the growth rate and π_t is the inflation rate and pb_t is the ratio of the primary balance to GDP. Parameter α is chosen to match the fraction of debt to GDP ratio $\bar{d}$ is chosen so that the borrower would default a certain number of times. Using the threshold $\bar{d}$ the authors simulate the evolution of debt to GDP ratio in the economy with GDP-indexed bonds. While this is

[1]Pricing in this Chamon & Mauro (2005) is viewed as the most important obstacle to introduction of state contingent debt.

the route often taken by practitioners, it ignores endogeneity of the interest rate paid on the debt, the default threshold, *etc.*

Another approach is taken by Durdu (2006) who studies the effect of introducing a GDP-indexed bond into a small open RBC economy. However, the model in Durdu (2006) is not consistent with the emerging markets' business cycles. In particular, it does not match the negative correlation between capital flows and output in emerging economies. But, first, the emerging economies are seen as the most important beneficiaries of this financial innovation. Second, since the indexed bond has to alleviate the balance of payments crises it seems crucial to have this correlation intact. In addition, this model does not contemplate a possibility of default. But a reduction in default rates is seen as the primary channel of welfare improvement in these economies.

Thus, this work brings together these two research lines by considering an open economy RBC model with risk of repudiation. In addition, I contemplate alternatives to the Lucas's (1987) measures of the cost of business cycle fluctuations. While the consumption based cost is in most cases small, the 'bank account' compensation is significant. In the case of Argentina, introducing a simple indexed bond is equivalent to the loan of 10% of Argentina's GDP. The latter constitutes approximately one third of the debt amount forgiven in Argentina after the default in 2001.

2 The Baseline Model

As the starting point I use the model pioneered by Eaton & Gersowitz (1981) and extended by Arellano (2007). This model is known to match the Argentina's business cycle properties well. As will be shown later, it matches the negative correlation between capital flows and output, interest rates and output, and positive correlation between capital flows and interest rates –the business cycle properties identifying emerging market economies.[2]

The economy, referred to as the borrower, is populated by an infinitely-lived representative borrower with time separable preferences given by

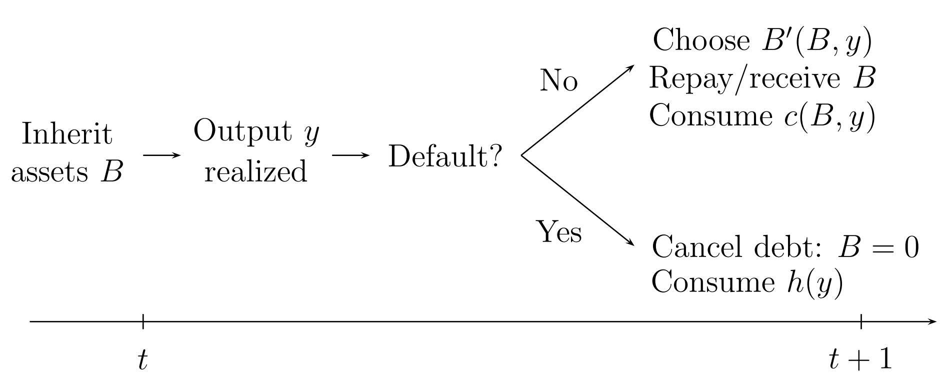
$$E\Big[\sum_{t=0}^{\infty}\beta^t u(c_t)\Big|I_0\Big], u' > 0, u'' < 0,$$

where I_0 is the information available to the borrower at date 0.

The borrower starts period 0 with assets B_0 and receives a stochastic endowment y_t. The endowment process is assumed to follow a first-order Markov process with a transition cdf $F(y'|y)$.

The decisions faced by the borrower that is not banned from the international credit market are represented by the time line in Figure 2. When the borrowing country has no access to the international credit market its decision is trivial. With no access to the financial market the borrower consumes a fraction of its output and with a known probability θ is given access to the market in the next period.

Figure 2: Decision time line



[2]See Aguiar & Gopinath (2006) for the statistics for a number of the emerging economies.

Let $V(B, y)$ be the value to the borrower with assets B and output y, $V^c(B, y)$ be the value function to the borrower with assets B and output y who chose not to default, and let also $V^d(y)$ be the value to the defaulting borrower with output y. Then the value functions V, V^c and V^d must satisfy the following Bellman equations

$$V(B, y) = \max_{C,D} \Big\{ V^c(B, y), V^d(y) \Big\} \tag{2}$$

$$V^c(B, y) = \max_{B'} \Big\{ u(c) + \beta \int V(B', y') dF(y'|y) \Big\} \tag{3}$$

$$s.t. \quad c + q(B', y)B' = y + B,$$

and

$$V^d(y) = u(h(y)) + \beta \int \Big[\theta V(B', y') + (1 - \theta) V^d(y') \Big] dF(y'|y).$$

The last equation states that in the financial autarky the borrower consumes $h(y) \leqslant y$ and continues in the autarky with probability θ and with the complementary probability is able to regain access to the credit market. Stochastic exclusion from the credit market models the fact that the borrowing countries are excluded from the market only temporarily.

2.1 Default penalty

The default entails direct penalty on the borrower. This is modeled by function $h(y) \leqslant y$, a portion of output that is available for consumption. With direct output penalty default is more costly and hence larger debt levels can be sustained in equilibrium. In reality such penalty corresponds to the sum of litigation and trade sanction costs imposed on the borrower.

In this work I consider two possibilities for the function h. First is

$$h(y) = \begin{cases} \lambda E(y), & \text{if } y > \lambda E(y) \\ y, & \text{if } y < \lambda E(y) \end{cases} . \tag{4}$$

This specification is reasonable if it is easier to punish the borrower when output is high rather than low. This is also the penalty function used by Arellano (2007). Under the specification (4) attractiveness of default is being shifted towards low output states. Under the alternative specification, a constant fraction of output is lost in each state.

$$h(y) = \lambda y. \tag{5}$$

This specification is used for the sensitivity analysis. Fraction λ can be estimated by the ratio of pre-default output level to its post-default level.

2.2 Financial Intermediary

A large number of risk-neutral financial intermediaries form a competitive industry and each faces the price of bonds $q(B, y)$. Zero profit condition requires that the bond pricing function satisfies the zero-profit condition

$$q(B, y) = \beta_c \int_{y' : V^c(B, y') > V^d(y')} dF(y'|y), \tag{6}$$

where $\beta_c = 1/(1 + r)$ is an intermediary's discount factor.

2.3 Competitive Equilibrium

Competitive equilibrium is the list (B', D, q), where

1. B' and D are respectively the borrower's optimal asset holding policy and default decision that solve (2) given the bond pricing function q;
2. the bond pricing function q satisfies (6).

3 Complete Financial Markets

Now consider the economy in which fully contingent contracting is allowed. That is the only friction in the economy is that the borrower cannot commit to repay a financial intermediary. If fully contingent contracting is allowed the intermediary will never lend beyond what would be fully repaid. Thus there is no default in the equilibrium – that is one of the benefits of the state contingent contracting. Hence, the price of the contingent bond paying one good unit in state y' is priced at $\beta_c f(y'|y)$.

Let $V_{cm}(B, y)$ be the value to the borrower with debt B and output y and $V^d_{cm}(y)$ be the value to the borrower with output y who decided to default. Then these value functions must satisfy the following Bellman equations:

$$V_{cm}(B, y) = \max_{B'} \left\{ u(c) + \beta \int \max[V_{cm}(B'(y'), y'), V^d_{cm}(y')] dF(y'|y) \right\}$$
$$s.t. \quad c + \beta_c \int B'(y') dF(y'|y) = y + B$$

and, similarly to above,

$$V^d_{cm}(y) = u(h(y)) + \beta \int \left[\theta V_{cm}(B'(y'), y') + (1 - \theta) V^d_{cm}(y') \right] dF(y'|y).$$

For the case of the complete financial markets, the optimal financial contract is described in Thomas & Worrall (1990). While the latter work assumes that both the creditor and the borrower discount the future at the same rate, the basic structure of the contract remains unchanged when the borrower is less patient than the creditor: $\beta < \beta_c$. The optimal financial contract is described in the following proposition.

Proposition 1 (Monotonicity of repayment schedule)**.**

$$B'(B, Y_i) \geqslant B'(B, Y_j), \quad \forall B, i \geqslant j.$$

Proof. See appendix. □

That is, like in the case with equally patient borrower and creditor, the better the realization of output the better is the financial position of the borrower next period. However, unlike in the other case, the borrower's financial position can deteriorate over time. This happens because the impatient borrower saves less than the 'benevolent' creditor would require.

4 A Simple Indexed Bond

In this section I consider the economy in which the asset structure is expanded to include the bond that pays only in the event of low output. Such the economy can be thought of as the one in which it is possible to re-negotiate the repayment when output is low. A possibility to contract upon the low output states also resembles the IMF's emergency financing policy.

Corsetti-Roubini (2004) study a global game between the IMF providing the emergency lending to the borrower. While their model cannot be used for quantitative analysis, they do show that provision of emergency lending can improve of the borrower's welfare as measured by output.

The economy described above is equivalent to the one in which a simple contingent bond is traded. Such an asset is represented by a vector $(B_1, B_2, \bar{y})$ where

$$\text{repayment} = \begin{cases} B_2, & \text{if } y \geqslant \bar{y} \\ B_1, & \text{if } y < \bar{y} \end{cases}. \tag{7}$$

First, note that such an asset requires substantially less information to price than the fully optimal contract. Second, note that magnitude $\bar{y}$ also could be indexed to the current economic conditions. It will be shown that even with so coarsely designed asset substantial gains are possible.

For the economy with the contingent bond, let $V_{cb}(B, y)$ be the value to the borrower with assets B and output y, $V_{cb}^c(B, y)$ be the value function to the borrower with assets B and output y who chose not to default, and let also $V_{cb}^d(y)$ be the value to the defaulting borrower with output y. Then the value functions V_{cb}, V_{cb}^c and V_{cb}^d must satisfy the following Bellman equations

$$V_{cb}(B, y) = \max\left\{V_{cb}^c(B, y), V^d(y)\right\} \tag{8}$$

$$V_{cb}^c(B, y) = \max_{B'}\left\{u(c) + \beta \int V(B', y')dF(y'|y)\right\} \tag{9}$$

$$s.t. \quad c + q(B', y) \cdot B' = y + B,$$

where

$$B' = (B'_1, B'_2), \quad q(B', y) \cdot B' = q_1(B_1, y)B_1 + q_2(B_2, y)B_2$$

and

$$V^d_{cb}(y) = u(h(y)) + \beta \int \Big[\theta V_{cb}(B', y') + (1-\theta)V^d_{cb}(y')\Big] dF(y'|y).$$

4.1 Intermediary Trading Contingent Bonds

As before, with competitive financial intermediaries profit in the industry is driven down to zero. The latter implies the following bond pricing functions

$$q_1(B, y) = \beta_c \int_{y' < E(y): V^c_{cb}(B, y') > V^d_{cb}(y')} dF(y'|y),$$
$$q_2(B, y) = \beta_c \int_{y' \geqslant E(y): V^c_{cb}(B, y') > V^d_{cb}(y')} dF(y'|y).$$

The only difference with the un-contingent bond pricing function is the domain of integration – low output states for q_1 and high output states for q_2.

5 Solution Method

I solve for the equilibrium numerically using the following algorithm.[3] Given the pricing function $q(B, y)$ the optimal value functions V, V^c, V^d are found by iteration on the Bellman equation. Consequently the pricing function is updated according to (6) to obtain $q'(B, y)$ using the value functions obtained on the previous step. The procedure is continued until convergence of the bond pricing function as measured by the L_1 norm, $\int\int |q(B, y) - q'(B, y)| dBdy$.

[3] This algorithm is similar to the one used by Arellano (2007); the difference is in the way the pricing function is updated. For the parameterizations used, both algorithms give the results identical up to the numerical error; however, updating used here is faster.

The space of asset holdings is discretized into 201-point equidistant grid. The space of output is discretized into 21-point grid using Tauchen & Hussey (1991) procedure based on Gauss-Hermite quadrature.[4]

6 Calibration

The model parameters are chosen to match the key facts of the Argentina's business cycles. The target moments are chosen to match the set of moments reported in table 2. The parameter β_c is fixed to match the 5.0% world real interest rate. The parameters ρ and σ are chosen to match the estimated output process for Argentina during the period 1980:Q1-2001:Q4.

Table 1: Parameters (time unit equals a quarter)

Parameter		Value	Type
borrower's discount factor	β	0.928	Free
lender's discount factor	β_c	0.988	Fixed
coef. of relative risk aversion	γ	2.0	Fixed
loss of output in default	λ	0.031	Fixed
probability of re-entering credit mkt	γ	0.282	Free
output process, autocorrelation	ρ	0.945	Fixed
output process, std. deviation	σ	0.025	Fixed

Table 6 reports parameters chosen for the model simulation. The default probability reported in the table is an annualized probability. It was chosen, similarly to Yue (2006), based on the fact that Argentina defaulted five times during the 1824-2005 period (see Carmen M. Reinhart & Savastano (2003)).

[4]While the number output grid points had minimal effect on the results, the number of asset grid points had negligible effect on the results presented in this paper.

Table 2: Moments in the benchmark economy with default

Moment	Data	Benchmark
$\sigma(c)/\sigma(y)$	1.10	1.15
$\rho(c,y)$	0.95	0.96
$E(r)$	10.25	3.48
$\sigma(r)$	5.58	3.23
$\rho(r,y)$	-0.88	-0.25
$\rho(tb,y)$	-0.64	-0.20
$\rho(tb,r)$	0.70	0.45
$prob(def),\%$	2.76	2.89

7 Numerical Results

Figure 5 plots the optimal asset holding of the borrower for the lowest and the highest output levels. The country with $B < 0$ is able to borrow much more in the high (rather than low) output state. This happens because in high output states the borrower is less likely to default and so cost of borrowing is low. That the borrower is less likely to default in high output states is the result of 1) the specific penalty function and 2) the output persistence.

Figure 5 plots the optimal assets holding in the complete markets economy. Note that there is much more lending in this economy. This is because repayment is tailored to every state and there is no default in equilibrium. Larger loans are now extended to the borrower at the cost of larger repayments in high output states.

Figure 3: Optimal asset holding in the economy with incomplete markets

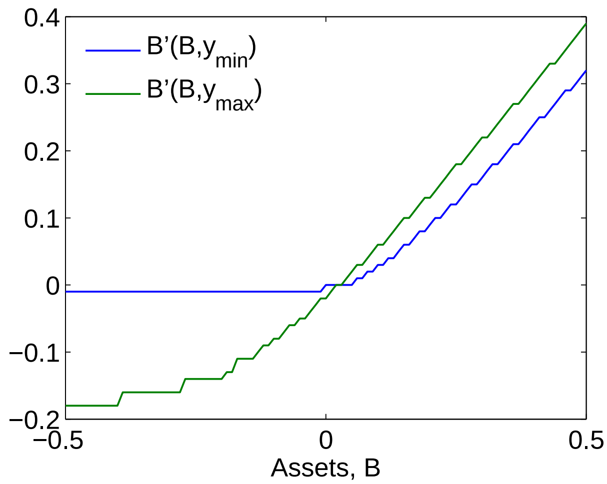
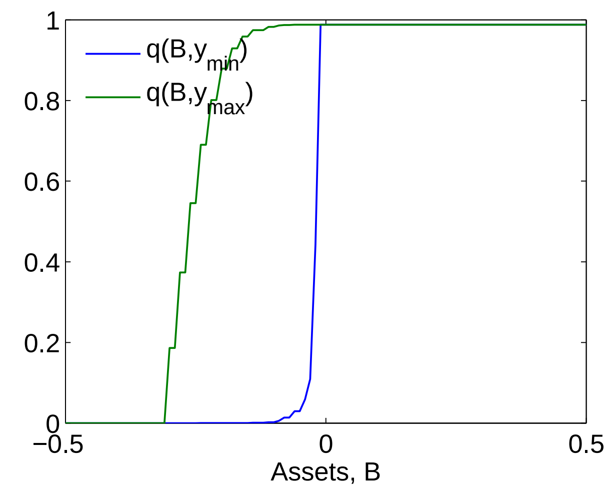


Figure 4: Optimal asset holding in the economy with incomplete markets



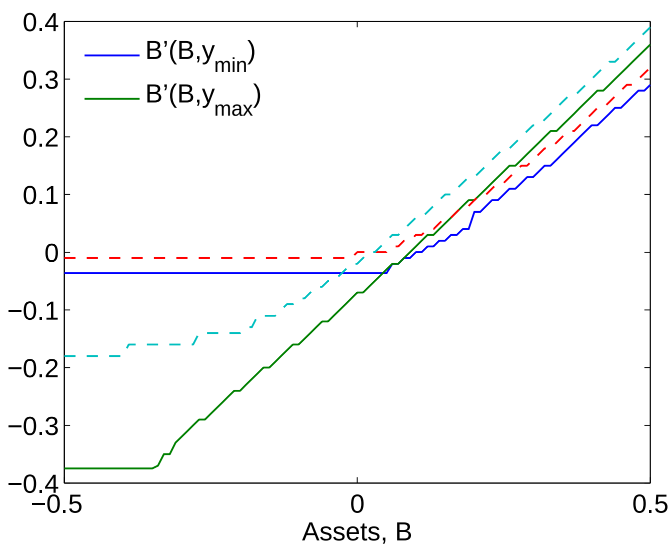
8 Measuring Welfare Improvement

It pays to study a simple example here. Consider a representative-agent two-period endowment economy with no access to international borrowing. In the first period endowment is non-stochastic and is normalized to unity. In

Table 3: Data (Argentina, 1980:1 - 2001:4) and Model Moments

Moment	Data	Benchmark	Cont. Bond
$\sigma(c)/\sigma(y)$	1.10	1.15	0.76
$\rho(c,y)$	0.95	0.96	0.89
$E(r)$	10.25	3.48	1.38
$\sigma(r)$	5.58	3.23	1.19
$\rho(r,y)$	-0.88	-0.25	0.03
$\rho(tb,y)$	-0.64	-0.20	0.55
$\rho(tb,r)$	0.70	0.45	0.45
$prob(def),\%$	3.76	2.89	0.89

Figure 5: Optimal asset holding in the economy with incomplete markets



the second period the agent receives endowment $1+e$ with probability π and $1-e$ with probability $1-\pi$. The agent ranks different consumption streams according to $u(c_1) + \beta E(u(c_2))$. In the equilibrium the agent's welfare is

$W = u(1) + \beta[\pi u(1+e) + (1-\pi)u(1-e)]$. With access to the complete financial markets with risk neutral intermediaries discounting profits at rate β^{-1} the agent is able to maintain a constant consumption stream $c_t = c^* := 1 + \beta e(2\pi - 1)/(1+\beta), \forall t$.

Lucas's (1987) measure, denoted by λ, is a solution to[5]

$$u(\lambda) + \beta[\pi u(\lambda(1+e)) + (1-\pi)u(\lambda(1-e))] = (1+\beta)u(c^*). \tag{10}$$

With $u(c) = c^{1-\gamma}/(1-\gamma)$, the Lucas's cost of fluctuations is

$$\lambda = \left[\frac{(1+\beta)u(c^*)}{W}\right]^{1/(1-\gamma)}. \tag{11}$$

On the other hand, we would like to have a "present discounted value" type of measure. The answer to this question, denoted by t, is

$$u(1+t) + \beta[\pi u(1+e) + (1-\pi)u(1-e)] = (1+\beta)u(c^*). \tag{12}$$

Even if λ is small, the preset discounted value measure may be large and, in fact, has to be approximately equal to $\lambda/(1-\beta) >> \lambda$.

There exists still another way to measure the welfare improvement. If the financial intermediaries are the initiators of the alternative regime, I can compute the new rate of return at which the agent would be indifferent between the two environments. Thus consider financial intermediaries that allow trading the full set of Arrow securities and require the rate return equal to $R > \beta^{-1}$. Then the agent's consumption would be $c_t = c^R := 1 + e(2\pi - 1)/(R+1), \forall t$. Assuming that $\pi < 0.5$, that is that the agent is a borrower, higher interest rate R implies lower consumption c^R. Now we want to find such R that the agent is indifferent between the financial autarky and

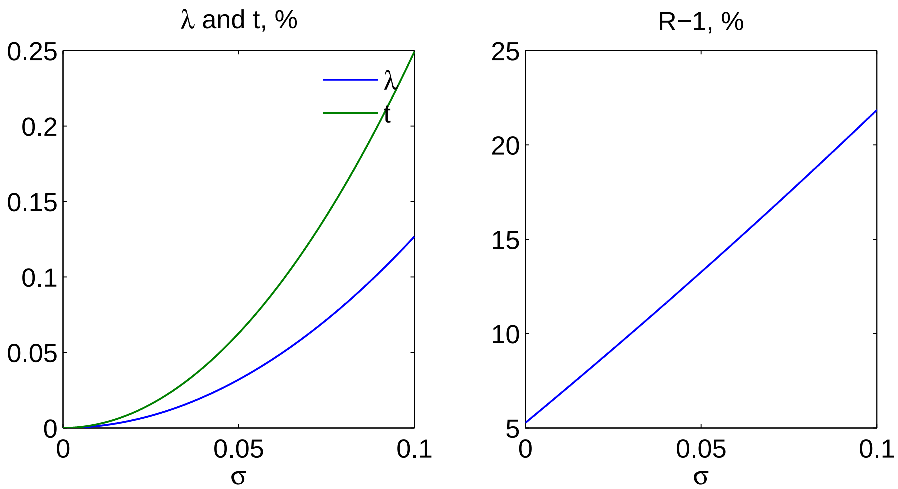
[5]Barlevy (2003) argues that stabilization (or any other policy for this matter) policy may also improve the growth rate, in which case the cost of the business cycles would be much larger.

the complete markets with more demanding creditors:

$$W = (1 + \beta)u(c^R). \tag{13}$$

The difference in returns, $R - 1/\beta$, is the desired measure of gain to financial intermediaries. It shows by how much financial intermediaries could increase their return by moving to a better financial system. The next plot reports the different welfare cost measures as a function of second period income volatility $\sigma_y = 2e\sqrt{\pi(1-\pi)}$. The parameters assumed for the following illustration are $\gamma = 0.5, \beta = 0.95, \pi = 2/3$.

Figure 6: Different measures of welfare cost



While a hypothetical example, figure 6 reveals several properties of the cost measures. First, the agent should receive a transfer approximately equal to $(1 + \beta)\lambda$ times current income. Second, the financial intermediaries could earn up to 22% higher return if using the best financial contracts. (Partially, the reason this measure is so large is because the amount of borrowing in the economy is small; thus, higher rates do not have a large effect on the borrowing economy.) This also implies that in this environment fluctuations in the intermediaries' cost of funds has little effect on the borrower's welfare.

8.1 Welfare Costs in The Calibrated Economy

Figure 7: Optimal values in the different economies

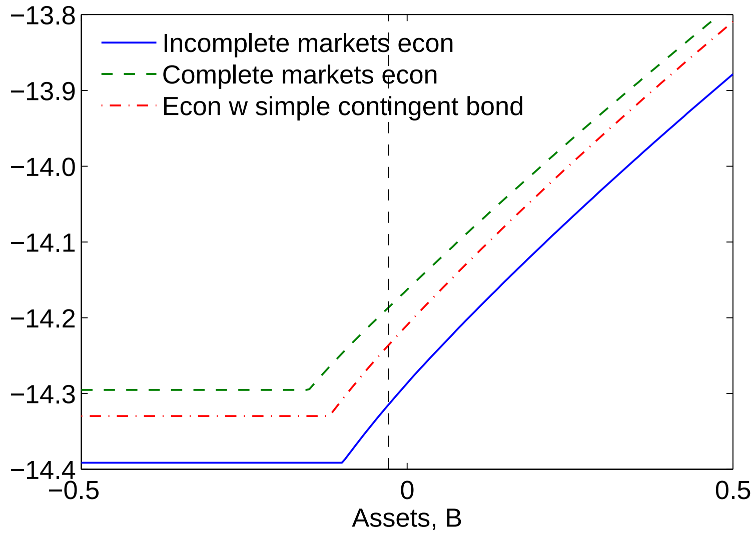


Figure 7 plots optimal values under the competing specifications of the financial markets. The transformed relative values $\lambda_{cm} = [V_{cm}(B)/V(B)]^{1/(1-\gamma)}$ and $\lambda_{cm} = [V_{cb}(B)/V(B)]^{1/(1-\gamma)}$ give us the Lucas measure of welfare improvement. The horizontal differences between the values give us the "present discount value" type of measure: $\Delta B_{cm} = B' - B : V(B') = V_{cm}(B)$ and $\Delta B_{cb} = B' - B : V(B') = V_{cb}(B)$. Finally we can compute the value function for the incomplete markets setting using a larger creditor's discount factor β_c. I could be increasing β_c until I match either V_{cm} or V_{cb} – denote these levels by β_{cm} and β_{cb}. The necessary increase in the discount factor can be interpreted as the possible increase in the rate of return to the financial intermediaries from better financial contracts. Thus, the last measure of welfare improvement is $\Delta r_{cm} = 1/\beta_c - 1/\beta_{cm}$ and $\Delta r_{cb} = 1/\beta_c - 1/\beta_{cb}$. All of these measures are reported in table 4.

All of these measures indicate possibilities for economically significant

Table 4: Welfare Improvement Measures

Economy	λ	ΔB	Δr
Complete markets	1.00%	0.15E(y)	>5%
Contingent bond	0.67%	0.10E(y)	>5%

welfare improvements. The last measure also suggests that exogenous movements in the world interest rate (creditors' required rate of return) have little effect on the economy's welfare.[6] On the other hand, access to the different financial markets has a significant effect on welfare.

9 Conclusions

In this paper I analyze a quantitative model of international lending with default and indexed debt. I show that there exists a large gain to using optimal financial contracts. The welfare improvement is equivalent to a 1% permanent increase in consumption or a 15% of GDP financial aid. However, optimal financial contract requires a significant amount of information to be specified. To address this question I study more simple financial arrangements. In particular I analyze the economy that can issue an indexed bond with two indexation states. Being able to issue such bond, an economy like Argentina can recover approximately 2/3 of the maximal possible gain. This simple financial improvement is equivalent to a 0.67% permanent increase in consumption and a 10% of GDP financial aid. Is 10% of GDP a significant amount? – In my view yes, as it constitutes 1/3 of the amount that was granted (forgiven in fact) to Argentina after its default in 2001.

It is possible to view the problem of the financial innovation from the

[6] **?** argue that reducing the volatility of the world interest rate could reduce volatility of the Argentina's output by approximately 25%.

perspective of the financial intermediaries. From this standpoint of view the financial intermediaries could earn more than 5% higher return on their investment had they used the simple indexed bond instead of the un-contingent debt. This cost is significantly larger than a 5 basis point origination fee typically charged by the financial institutions in the U.S.

As a final remark, the results in this study suggest that while the limited commitment framework allows us explaining emerging markets business cycles, it is necessary to contemplate other models/frictions which could explain why such large gains as found in this paper remain unexploited.

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