Firm Learning, Market Selection, and Industry Productivity Dynamics

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Motivation

Questions:

- What are the mechanisms of industry productivity improvements.
- How are these mechanisms affected by competition and innovation policies.

- Ericson and Pakes (1995): a general framework for estimating models of firm entry, exit, investment and firm-specific uncertainty using Markov-perfect industry equilibrium.
- However, computational concerns limit the applicability of this framework to industry with very few firms (usually ≤ 10).
- Recent work by Weintraub, Benkard and Van Roy (2005):
  1. Propose a new solution concept "oblivious equilibrium" that dramatically reduces the computational complexity.
  2. Facilitate analysis of applied problems when there are a large number of firms.
  3. Prove an asymptotic result that "oblivious equilibrium" closely approximates Markov Perfect Equilibrium as the market size grows.

What I do:

- Extend previous applications of Ericson and Pakes (1995) to include:
  1. Two types of investment: R&D and physical capital
  2. R&D spill-over
- Apply the equilibrium concept of Weintraub, Benkard and Van Roy (2005).
- Estimate the model using firm-level Korean electric motor industry data.
- Evaluate effects of R&D subsidies policies.

Data

- Korean electric motor (SIC31101) industry from 1991-1996, which is collected by Korean Statistical Office.
- Nice features of the data
  1. detailed R&D expenditure on an annual basis.
  2. electric motor: a manufacturing industry with a long history and mature technology.
  3. industry incumbents usually engage in in-lab process innovation.

Stylized facts from Korean electric motor industry:

1. R&D intensity distribution is highly skewed, with considerable fraction of firms reporting zero R&D.
2. Probability of a single firm doing R&D is positively correlated with its size.
3. Productivity dispersion is large and its ranking is persistent.
4. A firm's R&D intensity affects the probability of its improvement.
5. Firms that are further behind the frontier have higher probability of improvement in its future efficiency.
6. The entry rate averages .34 and the exit rate averages .28 over the sample years.

Motivation for our modelling strategy.

Model Environment

- The industry consists of a number of heterogenous firms.
- Each firm's state $\omega \in \Omega$: knowledge capital x and physical capital k. $x \in \mathbb{X} = \{x^1 < x^2 < x^3 <, ...\}$ and $k \in \mathbb{K} = \{k^1 < k^2 < k^3 <, ...\}$.
- Industry state at t is denoted $s_t : \Omega \to \mathbb{R}_+$: where $s_t(\omega)$ is the mass of firms at state ω.

Sequence of Actions

- Production and demand function:
  1. Production function:

  $$q_t = exp(\bar{X}_t + x_t)(l_t^{\alpha}(k_t)^{1-\alpha})^{\gamma}$$

  Current industry frontier $\bar{X}_t$ is exogenously determined.
  2. Each firm produces a differentiated product and its demand is determined by:

  $$q_t = Q_t(p_t/P_t)^{\eta}$$

  where Q_t and P_t are industry aggregate output and price index.
- At the beginning of each period t, incumbent firms simultaneously set their prices p_t and labor l_t.

- Then, incumbent firms and potential entrants make their exit and entry decisions simultaneously.
  1. Incumbent firms privately observes an idiosyncratic random scrap value ϕ_t. If they exit, they get that value. If not, they make R&D and physical capital investment decisions.
  2. Potential entrants decide whether to enter next period based on current industry state s_t. Each entrant pays a fixed entry cost κ and takes its initial draw of x and k from a fixed distribution Φ_e.
- Incumbent firms and potential entrants' actions determine s_{t+1}.

Transition of State Variables

- The evolution of firm's knowledge capital x_t depends on: for $x_t = x^j \in \mathbb{X}$

$$x_{t+1} = \begin{cases} x^{j+1}, & \text{with probability } \frac{(1-\delta)D_t}{1+D_t}; \\ x^{j-1}, & \text{with probability } \frac{\delta}{1+D_t}; \\ x^{j}, & \text{with probability } \frac{1-\delta+\delta D_t}{1+D_t} \end{cases}$$

  where $D_t = [\frac{d_t}{k_{t+1}} + \theta(\sum_{x \geq x_t} \sum_k \frac{exp(x)s_t(x,k)}{N_t})]$ and δ is an idiosyncratic depreciation shock.

- Firms physical capital: $k_{t+1} = (1-\delta_c)k_t + i_t$, where i_t is the firm's investment (divestment). Adjustment cost for physical capital k_t to k_{t+1} is:

$$c(k_t, k_{t+1}) = c_a(i_t/k_t)^2 k_t - c_s i_t (i_t < 0)$$

Incumbent's Problem

- Incumbent firms choose p_t to maximize $\pi_t = p_t q_t - w_t l_t$ given individual state (x_t, k_t) and industry state s_t. static competition
- Given i.i.d. distributed scrap value ϕ_t, each incumbent establishment decides whether to stay or exit:

$$V(x_t, k_t; s_t) = \pi(x_t, k_t; s_t) + E[max\{V_c(x_t, k_t; s_t), \phi_t\}]$$

where the value of continuation $V_c(x_t, k_t; s_t)$ is defined as:

$$\begin{aligned} V_c(x_t, k_t; s_t) = max_{d_t, k_{t+1}}\{&-c_d d_t - i_t - c(k_t, k_{t+1}) \\ &+\beta E_{s_{t+1}}[V(x_{t+1}, k_{t+1}; s_{t+1})|x_t, d_t, s_t] \end{aligned}$$

where c_d is the cost of per unit R&D investment.

Potential Entrant's Problem

- Large number of potential entrants each period.
- Markov symmetric strategy for incumbent $a = \{d, k, \chi\}$.
- Potential entrants enter, denoted by $\epsilon = 1$, if:

$$V_e(s_t) \equiv \beta E_{s_{t+1}}[\int V(x_e, k_e, s_{t+1}) d\Phi^e | s_t] \geq \kappa$$

- The number of firms entering at industry state s_t is a Poisson random variable, with mean $M(s_t)$. poisson entry

Entry Model:

- Suppose there are N potential entrants, $v_N(i)$ is the expected present value for each entering firm if i firms enter simultaneously.
- Each potential entrant employs the same strategy, the condition for a mixed strategy Nash Equilibrium is:

$$\sum_{i=0}^{N-1} C_{N-1}^{i} p_N^{i} (1-p_N)^{N-1-i} v_N(i+1) = \kappa$$

- The equation has a unique solution $p_N^* \in (0,1)$, the number of firms entering is a binomial random variable Y_N with parameters (N, p_N^*). As $N \to \infty$, $Y_N \Rightarrow Z$, which is a poisson random variable with mean M.

return

Markov Perfect Equilibrium

- $V(x, k, s|a', a, \epsilon)$: the expected discounted payoffs of a firm (x, k) at industry state s who plays strategy a', while all rival establishments follow strategy a and potential entrants follow strategy ϵ.
- An industry equilibrium comprises value function $V(x, k; s)$, strategies a and ϵ, and markov transition kernel $T(s'|s)$ such that:
  1. incumbent: $V(x, k, s|a, a, \epsilon) \geq V(x, k, s|a', a, \epsilon)$, $\forall a'$
  2. entrants: $E_{s'}[\int V(x, k, s'|a, \epsilon) d\Phi^e|s] \leq \kappa$, with equality if $M(s) > 0$
  3. $T(s'|s)$ is consistent with firms' strategies.

Why I use Oblivious Equilibrium:

- Much lighter computational burden, making it feasible to estimate the model parameters.
- Close approximation to Markov Perfect Equilibrium as the market size grows.

asymptotic markov equilibrium

Oblivious Equilibrium

- Take strategies $a = (d, k, \chi)$ and ϵ, $s_{a,\epsilon} = lim_{t\to\infty} E[s_t]$ being the long run expected state of the industry if all agents follow a and ϵ. $\tilde{V}(x, k|a', s_{a,\epsilon})$ is the expected payoff of incumbent who follows a', under the assumption that its competitors' state will be equal to $s_{a,\epsilon}$ in all future periods.
- Oblivious equilibrium strategies a and ϵ satisfy:
  1. for an incumbent $\tilde{V}(x, k|a, s_{a,\epsilon}) \geq \tilde{V}(x, k|a', s_{a,\epsilon})$, $\forall a'$
  2. entrants satisfy zero profit condition such that $\beta \int \tilde{V}(x, k|s_{a,\epsilon}) d\Phi^e \leq \kappa$, with equality if $M > 0$
  3. $s_{a,\epsilon}$ is consistent with firms' strategies.

Given a set of parameters, compute the equilibrium:

1. Initial guess of the mean number of entrants M
2. Initial guess of the average long run industry structure s_0
3. Solve the monopolistic competition equilibrium aggregate price P given s_0.
4. Solve the incumbent's maximization problem and recover their optimal investment policy and exit policy: $a = (d, k, \chi)$ given s_0.
5. Construct the transition matrix $T_{x,k,\chi}$ using the optimal policies. The long run average industry structure is calculated as $s_1 = M(I - T_{x,k,\chi})^{-1}\Phi^e$.
6. If $|s_0 - s_1|$ is not close enough, go back to step (2).
7. Check the free-entry condition of potential entrants. If it doesn't hold, go back to step (1).

OE has Asymptotic Markov Equilibrium Property:

$$lim_{\mathrm{I}\to\infty} E_{\tilde{a},\tilde{\epsilon}}[sup_{a'} V^{\mathrm{I}}(\omega, s_t^{\mathrm{I}}|a', \tilde{a}, \tilde{\epsilon}) - V^{\mathrm{I}}(\omega, s_t^{\mathrm{I}}|\tilde{a}, \tilde{\epsilon})] = 0$$

if given: $\mu_t \in \bar{\mathbb{S}} = \{\sum_\omega \mu(\omega) = 1\}$ and $\bar{\mathbb{S}}_z = \{\mu \in \bar{\mathbb{S}} | \forall \omega > z, \mu(\omega) = 0\}$.

- $sup_{\omega \in \Omega, s \in \mathbb{S}} \ \pi^{\mathrm{I}}(\omega, s(\omega)) = O(\mathrm{I})$.
- For all increasing sequences $\{\mathrm{I}_k \in \mathbb{I} | k \in \mathbb{N}\}$, with $n(\mathrm{I}_k) = o(\mathrm{I}_k)$, $\omega > z$ and $\mu \in \bar{\mathbb{S}}_z$,

$$lim_{k\to\infty} \pi^{\mathrm{I}_k}(\omega, \mu, n(\mathrm{I}_k)) = \infty$$

- $sup_{\mathrm{I} \in \mathbb{I}, \omega \in \Omega, \mu \in \bar{\mathbb{S}}} \left| \frac{dln\pi^{\mathrm{I}}(\omega, \mu, n)}{dln(n)} \right| < \infty$.

where $s_t = n_t \mu_t$.

- Profit function takes the form:

$$\pi(\nu, \mu(\omega), n) = \frac{\mathrm{I}}{n}(1 + \frac{1}{\eta})\alpha\gamma[(1 + \frac{1}{\eta})\alpha\gamma - 1]\frac{\nu^{\sigma}}{\sum \mu(\omega)\nu^{\sigma}}$$

where $\nu = exp(x)k^{(1-\alpha)\gamma}$ and $\sigma = \frac{1+\eta}{\eta-(1+\eta)\alpha\gamma}$

- All three assumptions are satisfied.
- Finally, let $g(\omega') = max_{\mathrm{I}\in\mathbb{I},\omega\in\Omega,\mu\in\bar{\mathbb{S}}}|\frac{dln\pi^{\mathrm{I}}(\omega,\mu,n)}{d\mu(\omega')}|$ and $\tilde{\omega}^{\mathrm{I}}$ is randomly sampled from all incumbents while industry state is distributed according to its invariant distribution.
- A Light-Tail assumption needs: for all $\epsilon > 0$, $\exists z$ such that:

$$E[g(\tilde{\omega}^{\mathrm{I}})\mathbf{1}_{\tilde{\omega}^{\mathrm{I}}>z}] \leq \epsilon.$$

for all market size I.
This final assumption could be evaluated by simulation in practice.

Estimation of Model Preliminaries

- Set of parameters to be estimated:
  1. production function and competition: (α, γ, η)
  2. knowledge production: θ, δ
  3. exit and investment: (c_d, c_s, c_a, u_b)

First step: use static competition to recover (γ, η)

- Define the empirical production function:

$$Q_{it} = L_{it}^{\alpha_{ilt}} K_{it}^{\alpha_{ikt}} exp(\alpha_0 + \bar{X}_t + x_{it} + u_{it})$$

- Not restrict α_{lt} and α_{kt} to be constant overtime, however, the returns to scale $\gamma = \sum_{j=l,k} \alpha_{ijt}$
- Write the above equation in logarithm terms:

$$q_{it} = \alpha_0 + \alpha_{ilt} l_{it} + \alpha_{ikt} k_{it} + \bar{X}_t + x_{it} + u_{it}$$

- Need to control for well-known problems: unobserved price bias and sample selection/simultaneity bias.
- Unobserved price bias: use Hall (1990) and Klette and Griliches (1996)'s method to incorporate the information of demand function.
  ▸ klette
- Sample selection/simultaneity bias: apply the two-step procedure of Olley and Pakes (1996). In addition, we need to control for the R&D inputs.
  ▸ olley-pakes

- Production function close to constant returns to scale.
- Mark-up averages 1.209 in the electric motor industry.
- The coefficient on capital drops slightly from OLS to OP, indicating both capital and productivity is positively correlated with lagged R&D investment.

TABLE 5: PRODUCTION FUNCTION PARAMETERS
(STANDARD ERRORS IN PARENTHESES)

	OLS	OP
β_γ	0.828*	0.808*
	(0.0074)	(0.027)
β_η	0.200*	0.173*
	(0.047)	(0.059)
implied γ	1.035	0.979
implied mark-up	1.250	1.209

*significant at the 5% level

Plant level productivity can be recovered by:

$$\widehat{lnTFP}_{it} = (\tilde{v}_{it} - \hat{\beta_\gamma} k_{it} - \hat{\beta_\eta} q_{It}) \frac{\hat{\eta}}{\hat{\eta}+1} \qquad (1)$$

Estimation of Structural Parameters

The second step: recover $(\theta, c_a, c_s, c_d, \delta, u_b)$ jointly using simulated method of moments.

- Denote the set of data moments as Γ^d, which is a 11 by 1 vector.
- For a given set of parameters Θ, the industry equilibrium is solved and optimal policy functions (d^*, k^*, χ^*) are generated.
- Initialized from the solved oblivious equilibrium, use the optimal policy functions to simulate the path for each firm.
- Calculate $\Gamma^s(\Theta)$ for simulation s. The simulated moments are defined as:

$$\Gamma^S(\Theta) = \frac{1}{S}\sum_{s=1}^{S}\Gamma^s(\Theta)$$

- The MSM estimate $\hat{\Theta}$ minimizes the weighted distance between data and simulated moments:

$$L(\Theta) = min_{\Theta}[\Gamma^{d} - \Gamma^{S}(\Theta)]' W [\Gamma^{d} - \Gamma^{S}(\Theta)]$$

- Use an unbalanced panel by selecting firms existing in the electric motor industry at year 1991 and following them over the years until 1996.
- Entrants in subsequent years are only used to non-parametrically estimate the initial state distribution Φ^{e}.
- Parameters are taken from previous literature: $\beta = 0.95$ and $\delta_c = 0.069$. (Cooper and Haltiwanger (2006)).
- Parameters directly identified from data:
  1. Mean market size over the sample year $I = 528,941$ mil won.
  2. Parameter α is calculated as the average of α_{ilt}, $\alpha = 0.65$.
  3. Mean wage rate over the sample year $w = 11.15$ mil won/year.

Table 6: Key Data Moments

R&D Investment and Productivity Improvement	
fraction of R&D performers	14%
R&D intensity (R&D/Value-added)	0.013
mean relative productivity	0.48
std relative productivity level	0.23
fraction of productivity staying same or improving (lower 50% firms)	72%
fraction of productivity staying same or improving (higher 50% firms)	52%
Physical Capital Investment	
mean investment ratio ($\frac{i}{k}_{it}$)	.19
spike rate of positive investment ($\geq$ 30%)	21%
fraction of observations with positive investment	60%
corr between productivity shock and investment	30%
Firm Turnover	
mean exit rate	18%

Estimation Results

- The objective function $L(\Theta)$ is non-smooth and with many local optima. So I apply a recent estimation method proposed by Chernozhukov and Hong (2005).

  -Define a Quasi-posterior of Θ using functions of integral transformations of the criterion function $L(\Theta)$, which can be computed using Markov Chain Monte Carlo.

- I use their inference procedures to calculate confidence intervals from the quantiles of the Quasi-posterior distribution.

Table 7: Dynamic Parameter Estimates

	Point Estimate	90% confidence interval
c_d	.8901	[.7323, 1.0207]
θ	.5382	[.2749, .8622]
c_a	.1394	[.1140, .1673]
c_s	.9216	[.8471, .9939]
u_b	7067	[6058, 8148]
δ	.6618	[.6508, .6808]

Table 8: Model Fit

	Data Moments	Simulated Moments
R&D Investment and Productivity Dynamics		
fraction of R&D performers	14%	13%
R&D intensity (R&D/Value-added)	0.013	0.013
mean productivity level (TFP)	.48	.47
std productivity level (TFP)	.23	.27
fraction of productivity same/improving (lower 50% firms)	72%	64%
fraction of productivity same/improving (higher 50% firms)	52%	48%
Physical Capital Investment		
mean investment ratio ($\frac{i}{k}_{it}$)	.19	.18
spike rate of positive investment ($\geq$ 30%)	21%	22%
fraction of observations with positive investment	60%	63%
corr between productivity shock and investment	30%	30%
Firm Turnover		
mean exit rate	18%	17%

Simulated moments are generated with the point estimates of dynamic parameters, with 50 simulations.

Evaluation of Point Estimates

- c_s: Bloom (2006) gets a resale cost of about 0.42 using Compustat data. Cooper and Haltiwanger (2005) reports the resale price to be 0.975. Our value is within this range.
- c_a: Cooper and Haltiwanger (2005) reports a value of 0.225 while not controlling for fixed cost and 0.025 while controlling for fixed cost.
- u_b: The unconditional mean of scrap value is 3533 mil won, which is twice the industry average profit. The entry cost implied from free-entry condition is 6259 mil won.
- δ: Depreciation on firm's relative position is the combination of two forces: improvement of frontier and idiosyncratic depreciation of private knowledge. The rate of decay of knowledge ranges from 0.04 to .39 from various studies.

- Two lines of recent empirical literature that identifies two separate channels.
- First, market selection mechanism:
  1. Resource reallocation through entry and exit.
     Theoretical: Hopenhayn (1992), Melitz (2003)
     Empirical: Pavnick (2002),Aw, Chung and Roberts (2003)
  2. Resource reallocation through capital/labor adjustment.
     Empirical: Cooper, Haltiwanger and Power (1997), Davies and Haltiwanger (1992)

Assumption: firm level productivity is exogenously evolving.

return

- Second, firm learning mechanism:
  Firms' endogenous technological improvement through R&D effort and within-industry technological spill-over.
  1. Theoretical: Reinganum (1981), Spence (1984), Aghion et al (2001), (2005)
  2. Empirical: Jaffe (1986), Griliches (1998), Bloom, Schankerman and Van Reenen (2004)

Assumption: no entry and exit, no factor reallocation.

return

Why model technological learning and market selection in an imperfectly competitive setting?

- The intensity of product market competition directly affects the market selection channel.
  Theoretical: Asplund and Nocke (2005)
  Empirical: Syverson (2004)
- Firm's learning effort is endogenously shaped by the level of product market competition as well as pressure from potential entrants.
  Market competition: Reinganum (1981), Spence (1984)
  Entry pressure: Aghion, Blundell, Griffith, Howitt and Prantl (2006)

return

Patterns of R&D Investment

- Considerable fraction of firms (86% on average) reporting zero R&D.
- Probability of a single establishment doing R&D is positively correlated with its size.
- Among R&D performers, R&D intensity is weakly negatively correlated with either its labor or capital size.
- R&D intensity distribution is highly skewed.

Patterns of Productivity Dynamics

- Productivity dispersion is very large.
- Ranking of establishment-level productivity is quite persistent.
- Improvement of the industry productivity distribution from year 1991 to year 1996 indicates the effect of both firm learning and market selection.

Define weighted industry productivity as: $ln(TFP_t = \overline{ln(TFP_{it})} + \sum_i \Delta s_{it} \Delta ln(TFP_{it})$.

TABLE 2: DECOMPOSITION OF PRODUCTIVITY

year	$\overline{ln(TFP_{it})}$	$\sum_i \Delta s_{it} \Delta ln(TFP_{it})$
1991	0.920	0.324
1992	1.015	0.293
1993	1.014	0.344
1994	0.981	0.388
1995	0.970	0.471
1996	1.031	0.493

Firm's Learning Effort and Productivity

- A firm's R&D intensity affects the probability of it's improvement in relative productivity position.
- Firms that are further behind the frontier have higher probability of improvement in its future efficiency.

Table 4: Fraction of Improvement for Different Quantiles

quantile	improving	non-improving	R&D intensity
1	0.398	0.602	0.009
2	0.320	0.680	0.015
3	0.254	0.746	0.015
4	0.162	0.838	0.012

- In a static Nash Equilibrium, normalized industry price index $\hat{P}_t = P_t exp(\bar{X}_t)$ is determined by the industry state s_t:

$$\hat{P}_t = \mathrm{I}^{1-\alpha\gamma}\left(\frac{w_t}{(1+1/\eta)\alpha\gamma}\right)^{\alpha\gamma}\left(\sum s_t(\nu)\nu^{\sigma}\right)^{-\frac{1}{\sigma}}$$

  where w_t is the industry wage rate and I is aggregate market size, which I take as constant overtime. $\nu = exp(x_t)k_t^{(1-\alpha)\gamma}$ and $\sigma = \frac{1+\eta}{\eta-(1+\eta)\alpha\gamma}$.

- Firm's maximized profit $\pi^*(k_t, x_t; s_t)$ is:

$$\pi(\nu, s_t(\nu)) = \mathrm{I}(1+\frac{1}{\eta})\alpha\gamma[(1+\frac{1}{\eta})\alpha\gamma - 1]\frac{\nu^{\sigma}}{\sum s_t(\nu)\nu^{\sigma}}$$

return

- Using firm's profit maximization condition for labor, it's straightforward to show that:

$$\alpha_{ilt} = \frac{\eta}{1+\eta} s_{ilt} \quad (2)$$

where s_{ilt} is the expenditure share of labor. Then:

$$q_{it} = \alpha_0 + \frac{\eta}{1+\eta} s_{ilt}(l_{it} - k_{it}) + \gamma k_{it} + \bar{X}_t + x_{it} + u_{it} \quad (3)$$

- Use demand side $Q_{it} = Q_t(P_{it}/P_t)^{\eta}$, we can express the deflated log value-added $\tilde{r}_{it}$ as:

$$\tilde{r}_{it} = s_{ilt}(l_{it} - k_{it}) + \frac{(1+\eta)\gamma}{\eta} k_{it} - \frac{1}{\eta} q_{It} + \frac{(1+\eta)}{\eta}(\alpha_0 + \bar{X}_t + x_{it} + u_{it})$$

where q_{It} is the weighted industry output.

return

- The general estimation equation is given by:

$$\tilde{v}_{it} = \beta_0 + \beta_\gamma k_{it} + \beta_\eta q_{It} + (\tilde{\varpi}_{it}) + \tilde{u}_{it} \quad (4)$$

where $\tilde{v}_{it} = \tilde{r}_{it} - s_{ilt}(l_{it} - k_{it})$, $\tilde{\varpi}_{it} = \frac{(1+\eta)}{\eta}(x_{it} + \bar{X}_t)$, $\tilde{u}_{it} = \frac{(1+\eta)}{\eta} u_{it}$, $\beta_\eta = -\frac{1}{\eta}$ and $\beta_\gamma = \frac{(1+\eta)\gamma}{\eta}$.

- Two sources of inconsistent estimates using OLS:
  1. Self-selection generated by exit leads to a negative bias in the capital coefficient.
  2. D_{it-1} depends on the previous period state x_{it-1} as well as k_{it}, so k_{it} and x_{it} are possibly correlated.
  3. Apply the two-step procedure of Olley and Pakes (1996).

- The first stage regression with a set of year dummies:

$$\tilde{v}_{it} = \beta_0 + \beta_\eta q_{It} + \varphi(i_{it}, k_{it}) + \tilde{u}_{it} \quad (5)$$

where $\varphi(i_{it}, k_{it}) = \beta_\gamma k_{it} + h_t(i_{it}, k_{it})$.

- The second stage equation is defined as:

$$\tilde{v}_{it} - \hat{\beta}_0 - \hat{\beta}_\eta q_{It} = \beta_\gamma k_{it} + g(\varphi_{it-1} - \beta_\gamma k_{it-1}, D_{it-1}, \underline{x}_{it}) + e_{it} \quad (6)$$

- Comments:
  1. $\underline{x}_{it}(k_{it})$ makes the firm indifferent between continuing and exiting.
  2. Use the first order condition of our dynamic problem to represent D_{it-1} as a function of x_{it-1} and k_{it}.

- Consequently, we can instead write the second stage equation as a fully nonlinear function of $\varphi_{t-1} - \beta_\gamma k_{it-1}$ and k_{it}:

$$\tilde{v}_{it} - \hat{\beta}_0 - \hat{\beta}_\eta q_{It} = \tilde{g}(\varphi_{it-1} - \beta_\gamma k_{it-1}, k_{it}) + e_{it} \quad (7)$$

return