Immigration Policy and the Survival of the Welfare State: A Political Economy Model (Preliminary Draft)

Edith Sand and Assaf Razin
Tel Aviv University, September 2006

Abstract

The paper develops an OLG political economy model of social security and migration. We assume a a pay-as-you-go (PAYG) social security system, which employs payroll taxes on the working young in order to finance a social-security benefit to the aged. Migration policy is endogenously determined by the conventional negative effect of migration on wages, and by strategic considerations. The current young would like to influence the old-young composition of next period voters through migration. The paper characterizes sub-game perfect Markov equilibria, for different patterns of fertility rates among native born and migrants. We demonstrate the political economy mechanism whereby migration strengthens the social security system in the presence of ageing.

1 Introduction

All over the world, declining population growth rates and rising life expectancy have strong impact on the social security system, as we know it. Due to decreased fertility rates and longer life expectancy, the EU population, in particular is ageing, leading to a likely fall in the working population in the 25 states from 303 million to 297 million by 2020. A smaller labour force means less economic growth and faltering social security system. In this context, migration is is regarded by many as necessary for the sustainability of the this system. The analytics of a p[olitical-economy equilibrium model in which migration and taxes interact, are yet to be worked out carefully.[1]

The analysis of the of the inter-generational and intra-generational aspects of the sustainability of social security has had a revival of sorts in recent time. Razin and Sadka (1999, 2004) make an argument that highlights the importance of migration in enlarging the labor force in OLG model with pay-as-you-go fiscal system, for current and future generations of native born . They consider an overlapping-generations model, where each generation lives for two periods. In each period a new generation with a contin-uum of individuals is born. Each individual possesses a one unit of labor-schooling time endowment in the first period, when young. There is a pay-as-you-go (PAYG) pension system, which employs payroll taxes (at a flat rate) on the working young in order to finance a uniform benefit (b) to the aged. in an infinite-horizon, overlapping generations economy, this net burden is perfectly consistent with a net gain to the native born population. The additional obligation of the fiscal system to pay pension benefits to the incoming migrants, when they retire, could be shifted forward, in effect, indefinitely. If, hypothetically, the world would come to a stop at a certain point of time in the future, the young generation at that point would bear the deferred cost of the present migration. But in an ever-lasting economy, the migrants,

[1]Immigration does not provide in itself a full-fledged long-term solution to falling birth rates and an ageing population, but it is one of the available tools within a broader policy mix.

On the sustainability of pay-as-you-go social security see also Hassler, Jose V. Rodriguez Mora, and Kjetil Storesletten and Fabrizio Zilliboti (2003), and THEODORE C. BERGSTROM

JOHN L. HARTMAN (2005).

by supplying work and helping the financing the pension benefit of period zero to native born retirees, are a boon to the host country population: old, young, and future generations.

Cooley and Soares (1999) constructed a general equilibrium model in which a pay-as-you-go social security system can be adopted and sustained as a political and economic equilibrium. Razin, Sadka and Swagell (2001, 2002) develop an OLG model where the extent of taxation and redistribution policy is generally determined as a political-economy equilibrium by a balance between those who gain from higher taxes/transfers and those who lose. In a stylized model of migration and human capital formation, we show – somewhat against the conventional wisdom – that low-skill immigration may lead to a lower tax burden and less redistribution than would be the case with no immigration, even though migrants (naturally) join the pro-tax/transfer coalition. The model captures two conflicting effects of migration on taxation and redistribution. On the one hand, migrants who are net beneficiaries of the welfare state will join forces with the low income native-born voters in favor of higher taxes and transfers. On the other hand, redistribution becomes more costly to the native-born as the migrants share the redistribution benefits with them. But the governments transfers, in this model, accrued uniformly to young and old. Thus the intra-generational transfer is the trigger for the so-called fiscal leakage from the median voter to the net beneficiaries of the welfare system, and not the inergenerational transfer as in a pure social ecurity model.

Our main purpose is to highlight the intergenerational-transfer mechanism through which migration can strengthen the social security system.n this paper we develop a political economy model of social security and migration. Like the rest of the literature, we employ an overlapping-generations model, where each generation lives for two periods. In each period a new generation with one representative individual is born. Thus we completely abstract from intra-generational income transfers. There is a pay-as-you-go (PAYG) social security system, which employs payroll taxes (at a flat rate) on the working young in order to finance a uniform benefit (b) to the aged.Our purpose is to explore how the role of migration in stregthening the social security system is influenced by the aging of the native-born population.The subgame-perfect Markov equilibrium is characterized for different patterns of population growth among native born and immigrants. [2]

[2]Prescott and Rios-Rull (2000) argue that a necessary feature for equilibrium is that beliefs about the behavior of other agents are rational. We argue that in stationary OLG environments this implies that any future generation in the same situation as the initial generation must do as well as the initial generation did in that situation. We conclude that the existing equilibrium concepts in the literature do not satisfy this condition. We then propose an alternative equilibrium concept, organizational equilibrium, that satisfies this condition. We show that equilibrium exists, it is unique, and it improves over autarky without achieving optimality. Moreover, the equilibrium can be readily found by solving a maximization program.

Michele Boldrin, and Aldo Rustichini (2000) model PAYG social security systems as the outcome of majority voting within a OLG model with production. When voting, individuals make two choices: pay the elderly their pensions or default. which amount to promise themselves next period. Under general circumstances, there exist equilibria where pensions are voted into existence and maintained. Our analysis uncovers two reason for this. The traditional one relies on intergenerational trade and occurs at inefficient equilibria. A second reason relies on the monopoly power of the median voter. It occurs when a reduction in current savings induces a large enough increase in future return on

Migration policy is endogenously determined by the conventional negative effect of migration on wages, and by strategic considerations. The current young would like to influence the old-young composition of next period voters through migration. The paper characterizes sub-game perfect Markov equilibria, for different patterns of fertility rates among native born and migrants. We demonstrate the political economy mechanism whereby migration strengthens the social security system in the presence of ageing.

2 The model

The economy is populated by overlapping generations of identical individuals. Individuals live for two periods. When young, an individual work and makes the labor-leisure and consumption-savings decisions. When old, the individual retires, and receives transfer payments and the rental rate income from capital accumulation. The tax-transfer system is "pay as you go" where the government levies a flat tax on the young's wage income, which fully finances the transfer payments to the old in the same period.

The economy is small and open, but with immigration quotas. Immigrants enter the economy when young, and gain the right to vote only in the next period, when old. Except from having a higher population growth rate, immigrants have the same preferences and take part of the welfare system as the native born population.

The model is a political economy model where the political decisions re-

capital to compensate for the negative effect of the tax. We characterize the steady state and dynamic properties of these equilibria and study their welfare properties.

garding labor taxation and immigration quotas are taken through majority voting. The Markov perfect political equilibriums of the game feature a dynamic of repeated voting where individuals have a forward looking property, in the sense that they take into account the effect of their current voting on the next period voting decisions.

As a first step, we consider a simplified economy with no capital. Simplifying the model helps understanding the basic intuition of the political economic equilibrium, and emphasizes the changes derived by adding savings into the analysis. In addition, the extended model with capital enables the existence of an additional political equilibrium, which renders the equilibrium path less volatile.

2.1 The baseline model

The utility of individuals, which is assumed to be a logarithmic utility function[3], are given as follows:

$$U^y(w_t, \tau_t, b_{t+1}) = Log[w_t l_t(1-\tau_t) - \frac{l_t^{\Psi+1}}{\Psi+1}] + \beta Log[b_{t+1}] \tag{1}$$

$$U^o(b_t) = b_t \tag{2}$$

where U^y and U^o are the utility functions of young and old individuals, $\beta \in [0,1]$ is the discount factor, and $\Psi > 0$ is the labor supply elasticity with respect to the wage rate. The transfer payments to the old at period t, b_t, are financed by collecting a flat income tax rate, $\tau_t \in [0,1]$, from the young

[3] The utility function is characterized by the presence of no income-effects on the demand for leisure (Greenwood, Hercowitz, and Huffman (1988)).

individual's wage income at the same period, $w_t l_t$, where l_t denotes the hours worked.

Labor is used as the only input in production of a homogenous final good according to a linear production function:

$$Y_t = N_t \tag{3}$$

where Y_t is the output, and N_t is the labor supply in period t. We assume a competitive labor market. Due of constant return to scale, the wage rate is constant and equals one. A worker can be either native born or immigrant. The number of immigrants is limited to a ceiling rate as a percentage of the total number of individuals in the native born population, $\gamma \in [0, 1]$, which denotes the economy's immigration quotas[4]. The labor supply in period t is therefore denoted by:

$$N_t = L_t l_t (1 + \gamma_t) \tag{4}$$

where L_t is the number of working individuals in the native born population.

Immigrants have the same preferences as the native born population, but they have different population growth rate. The assumption is that the native born population has a lower population growth rate, $n \in [-1, 1]$, than that of the immigrant population, $m \in [-1, 1]$, so that, $n < m$. We assume that the immigrant's descendants are completely integrated into the economy and therefore have the same population growth rate as the native born population does. The number of native born individuals at period t can be written as

[4] The maximal ceiling rate percentage γ is at most one, which means that the number of immigrants cannot surpass the number of native born.

follows,

$$L_t = L_{t-1}(1+n) + \gamma_{t-1}L_{t-1}(1+m) \tag{5}$$

In addition, immigrants are also assumed to either contribute to or benefit from the welfare state in the same way as native born. Because the tax-transfers system redistributes income from the young to the old, the balanced government budget constraint implies:

$$b_{t+1}N_t = \tau_{t+1}w_{t+1}N_{t+1} \tag{6}$$

Re-arranging the expression yields:

$$b_{t+1} = \frac{\tau_{t+1}w_{t+1}l_{t+1}[(1+n)+\gamma_t(1+m)](1+\gamma_{t+1})}{(1+\gamma_t)} \tag{7}$$

The labor-leisure decision of young individuals is made by maximizing their utility while taking the prices and policy choices as given:

$$l_t^{\Psi} = w_t(1-\tau_t) \tag{8}$$

Substituting for b_t , $b_{t+1}$and l_t in equations (7) and (8), the indirect utility functions of the young and old individuals from equations (1) and (2) can be written as:

$$\begin{aligned} U^y(w_t,\tau_t,\tau_{t+1},w_{t+1}) &= Log[\frac{\Psi}{\Psi+1}w_t l_t(1-\tau_t)] + \\ &\quad \beta Log[\frac{\tau_{t+1}w_{t+1}l_{t+1}[1+n+\gamma_{t+1}[(1+_t(1+m)](1+\gamma_{t+1})}{(1+\gamma_t)}] \end{aligned} \tag{9}$$

such that,

$$l_t^{\Psi} = w_t(1 - \tau_t) \tag{10}$$

$$l_{t+1}^{\Psi} = w_{t+1}(1 - \tau_{t+1}) \tag{11}$$

$$U^o(b_t) = \frac{\tau_t w_t l_t[(1+n) + \gamma_{t-1}(1+m)](1+\gamma_t)}{(1+\gamma_{t-1})} \tag{12}$$

Note that the old individual favors a maximal openness rate of the economy towards immigration, as more immigration increases the number of current workers, which raises the total amount of tax collected, and the transfer payments each old individual receives. Higher taxation increases the tax collected per hour of work but decreases the total amount of working hours. Thus, the old preferable tax rate is an interim solution which depends on Ψ and equals $\tau^* = \frac{\Psi}{\Psi+1}$.

The young individual favors a minimal tax rate as it has an overall effect of decreasing his wage income. Regarding immigration quotas, the young individual favors a maximal openness rate, since it increases next period transfer payments. There are two opposite forces that affect next period transfer payments: more immigrants in the current period, increases the number of workers in the next generation due to the immigrants' higher fertility rate, which leads to higher transfer payments in the next period. On the other hand, a higher openness rate at the current period, increases the number of current young who will be the next period's old. This effect decreases the amount of transfer each individual receives in the next period, since the same total amount of tax collected is redistributed over more individuals. Since we assumed that the immigrants fertility rate is higher than that of the native born, $m > n$, the first effect is stronger which leads the marginal increase in the openness rate to have an overall effect of increasing next period transfer payments.

The system's dynamic is affected by two component: the fact that the young individual in period t anticipates that his voting choice at present over the openness rate affects the amount of next period transfer payments and the fact that, as will be discussed in the next section, the next period's openness rate affects the identity of the next period decisive voter. Deriving the individual's preferences over both policy options, the definition of the political equilibrium of the economy can be introduced.

2.1.1 Political equilibrium

In this subgame-perfect Markov equilibrium has a "switching" strategy, in the sense that under the assumption that immigrants enter the country while young and gain the right to vote only in the next period when they are old, voters take into account the effect of admitting a certain number of immigrants on the composition of voters and their voting preferences in the next period. Moreover, when the number of young exceed the number of old in the population, the young decisive voter admits a limited number of immigrants, in order to change the decisive voter's identity from young to old in the next period.

The equilibrium path depends on the native born and immigrant's fertility rates. If the fertility rates of the native born and immigrants are both positive, there is a steady state with no taxation. If alternatively, the sum of the fertility rates is negative, there is also a steady state, but with a certain level of taxation and full openness to immigration. Otherwise, the sum of the fertility rates can be positive and the native born population's fertility rate negative. In this case, some degree of openness to immigration always prevails while there is an alternate period by period taxation policy, depending on the identity of the decisive voter. In a given period there is a certain level of taxation and full openness to immigration, while in the next there is no taxation and a more restrictive policy towards immigration.

The equilibrium concept being used is a Markov perfect equilibrium of perfect foresight, as developed by Krusell and Rios-Rull (1996). The Markov Perfect equilibrium definition is as follows:

Definition 1 *A Markov perfect political equilibrium is defined as a vector of policy decision rules,* $\Psi = (T, G)$*, where* $T : [0,1] \longrightarrow [0,1]$*, is the taxation policy rule,* $T(\gamma_{t-1})$*, and* $G : [0,1] \longrightarrow [0,1]$*, is the immigration quota policy rule,* $G(\gamma_{t-1})$*, such that the following functional equation holds:*

1. $\widehat{\Psi}(\gamma_{t-1}) = \arg\max_{\pi_t} V^i(\gamma_{t-1}, \pi_t, \pi_{t+1})$ *subject to* $\pi_{t+1} = \Psi(\gamma_t)$*, where* $\pi_t = (\tau_t, \gamma_t)$ *is defined as the vector of policy platform, and* V^i *is the indirect utility of the current decisive voter.*

2. The fixed-point condition requires that if next period policy outcome is derived by the vector of policy decision rules- Ψ*, the maximization of the indirect utility of the current decisive voter will reproduce the same law of motion,* $\widehat{\Psi}(\gamma_{t-1}) = \Psi(\gamma_{t-1})$*.*

The Markov perfect political equilibrium notion states that the expected policy function, which depends on the minimal present state variables, must be self-fulfilling. The policy variables which are the tax rate, τ_t, and the openness rate, γ_t, have to maximize the decisive voter's indirect utility function, while taking into account that next period decision rules depend on the state variable i.e. the current openness rate of the economy towards immigration.

It should be noted that the state variable does not only influence the decisive voter's indirect utility but also the distribution of next period voters. As already mentioned, it is assumed that immigrants gain the right to vote only in the second period. Thus, the next period ratio of old to young voters in the native born population, denoted by u_{t+1}, is given by:

$$u_{t+1} = \frac{(1+\gamma_t)}{(1+n)+\gamma_t(1+m)} \tag{13}$$

Assuming that in case of a tie the old will be the decisive, the condition, $u_{t+1} < 1$, assures a majority of young individuals in the next period, while the condition, $u_{t+1} \geq 1$, assures a majority of old individuals. Therefore, the state variable of the economy, affects the next period ratio of young to old voters, u_{t+1}, which sets the profile of the next period decisive voter.

The Markov Perfect political equilibrium of the baseline model and its possible equilibrium paths, which depend on the fertility rates of the native born and immigrant populations, can be formalized as follows:

Proposition 2 *There exists an equilibrium with the following feature :*

$$T(\gamma_{t-1}) = \begin{cases} \tau_t = 0 & if \quad u_t(\gamma_{t-1}) < 1 \\ \tau_t = \frac{\Psi}{\Psi+1} & otherwise \end{cases} \tag{14}$$

$$G(\gamma_{t-1}) = \begin{cases} \gamma_t = -\frac{n}{m} & if \quad u_t(\gamma_{t-1}) < 1 \\ \gamma_t = 1 & otherwise \end{cases} \tag{15}$$

Where $[0,1] \ni \gamma_t$. *Under the assumption that the native born fertility rate is lower than that of immigrant, there are three possible equilibrium paths,* *depending on the fertility rates of the native born and immigrant populations:*

Proposition 3 *1. if $n > 0$, there is no taxation.*

2. if $m < -n$, there is full openness to immigration and a positive level of taxation.

3. if $n < 0$ and $m > -n$, there is an alternate taxation policy where some level of immigration always prevails: in periods where the decisive voter is old, the economy is fully opened to immigration and there is a positive level of taxation; and in periods where the decisive voter is young, there is no taxation and there is some restrictions on immigration.

The proposition is proved in the appendix. The intuition of the equilibrium decision rules is as follows. When the decisive voter is old, $u_t \geq 1$, the chosen openness rate is the maximal one, while the optimal tax rate is an interior solution. On the other hand, when the decisive voter is young, $u_t < 1$, the solution is less straightforward as the young voter's indirect utility function depends on the next period policy variables. While the optimal tax rate is always minimal, the openness rate is chosen strategically by the young voter, in order to change the identity of the decisive voter in the next period from young voter to old. This is due to the fact that in the next period, when the current young becomes old, if the old voter will be the decisive he will set the maximal level of transfer payments which the old voters receives.

There are three possible equilibrium paths depending on the fertility rates of the native born and immigrant populations:

The first equilibrium path is the one where the fertility rates of the native born and immigrant populations are positive, n, $m > 0$. In this case, there is a corner solution, where there are no openness rate values, which can change the identity of the next period decisive voter. This is due to the fact that for any not negative value of openness rate, the number of next period young voters exceeds the number of next period old voters. Therefore, since the decisive voter in the next period is young, who sets zero labor tax; no benefits will be paid to the old. Thus, the young will be indifferent regarding the level of openness rate as it does not influences his present income, or next period decisive voter's identity. As a result, the equilibrium path is a path where in every period there is a majority of young voters who set no taxation[5].

The second and the third equilibrium paths are characterized by a "switching" strategy of the young voters, which changes the identity of the next period decisive voter. Even though both equilibrium paths are characterized by a "switching" strategy of the young voters, the equilibrium paths differ. In the second equilibrium path, where the native born and immigrant population fertility rates are characterized by, $m + n < 0$, when there is a majority of old who set a maximal openness rate, the number of next period old voters exceeds the number of next period young voters. Thus, along the equilibrium path a majority of old will prevail, which will set a positive level of tax and a maximal level of openness rate in every period[6].

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[5]It should be noted that even if in the first period there is a majority of old voters, who admits the maximal number of immigrants, from the next period and on there will be a majority of young voters.

[6]In this case also, if in the first period there is a majority of young voters, who admit a certain number of immigrants in order to change the decisive voter in the next period from young to old, than from the next period and on there will always be a majority of

In the third equilibrium path, where the native born and immigrant populations fertility rate are characterized by, $n < 0$, and $m + n > 0$, the equilibrium path is characterized by an alternate taxation policy where some level of immigration always prevails. The reason is that when there is a majority of old who set a maximal openness rate, the number of next period old voters does not exceed the number of next period young voters. Therefore, when the decisive voter is old, there is a certain level of tax and a maximal level of openness rate, which induces a young decisive voter in the next period. When the decisive voter is young, he votes for a minimal tax rate, and votes strategically for a certain level of openness rate which changes the identity of the next period decisive voter to an old voter. This creates a cycling effect of an alternate taxation policy, with a certain level of immigration, depending on the identity of the decisive voter.

2.2 The extended model with capital

In this section, the same basic assumptions regarding the model feature are presented. The only exception is that young individuals are enable to save. The savings of the young which generates next period aggregate capital are being used as a factor of production, in a constant return to scale production function.

The utility of individuals are as before a logarithmic utility function:

$$U^y(w_t, \tau_t, s_t, r_{t+1}, b_{t+1}) = Log(w_t l_t(1-\tau_t) - s_t - \frac{l_t^{\Psi+1}}{\Psi+1}) + \beta Log(b_{t+1} + (1+r_{t+1})s_t) \tag{16}$$

old voters.

$$U^o(b_t) = b_t + (1 + r_t)s_{t-1} \tag{17}$$

where r_t is the interest rate, and s_t is the savings of the young at period t.

The production function is a Cobb-Douglas production function which is assumed to use both labor and capital as its factors of production:

$$Y_t = N_t^{1-a} K_t^{\alpha} \tag{18}$$

where K_t is the aggregate amount of capital and N_t is defined as in the previous section. The wage rate and interest rate are determined by the marginal productivity conditions (capital is assumed to depreciate completely at the end of the period):

$$w_t = (1 - a)(1 + \gamma_t)^{-a} l_t^{-a} k_t^{\alpha} \tag{19}$$

$$r_t = \alpha(1 + \gamma_t)^{1-a} l_t^{1-a} k_t^{\alpha-1} - 1 \tag{20}$$

where k_t is capital in per capita term. The balanced government budget constraint is derived as in the previous section:

$$b_{t+1} = \frac{\tau_{t+1} w_{t+1} l_{t+1}[(1 + n) + \gamma_t(1 + m)](1 + \gamma_{t+1})}{(1 + \gamma_t)} \tag{21}$$

The saving-consumption decision of young individuals are made by maximizing their utility while taking the prices and policy choices as given, and the labor-leisure decision is given as in the previous section:

$$s_t = \frac{1}{1 + \beta} \left(\beta \frac{\Psi}{\Psi + 1} w_t l_t (1 - \tau_t) - \frac{b_{t+1}}{1 + r_{t+1}} \right) \tag{22}$$

$$l_t^{\Psi} = w_t(1 - \tau_t) \tag{23}$$

The market clearing condition requires that the net domestic saving generates net domestic investment:

$$s_t = k_{t+1}\left(\frac{1+n+\gamma_t(1+m)}{(1+\gamma_t)}\right) \tag{24}$$

Solving for b_{t+1} from equations (20) and (22), and substituting b_{t+1} in equations (14) , the young's utility function can be written as follows:

$$\begin{aligned} U^y(w_t,\tau_t,r_{t+1},\tau_{t+1}) &= Log\left(\frac{\beta}{1+\beta}\frac{\Psi}{\Psi+1}w_t l_t(1-\tau_t)(1+\beta f(\tau_{t+1}))\right) \\ &\quad +\beta Log\left(\frac{\beta}{1+\beta}\frac{\Psi}{\Psi+1}w_t l_t(1-\tau_t)(1+\beta f(\tau_{t+1}))(1+r_{t+1})\right) \end{aligned} \tag{25}$$

where $f(\tau_{t+1}) = \frac{\frac{1-\alpha}{\alpha}\frac{1}{1+\beta}\tau_{t+1}}{1-\frac{1-\alpha}{\alpha}\frac{1}{1+\beta}\tau_{t+1}}$, such that,

$$k_{t+1} = \frac{\beta}{1+\beta}\frac{\Psi}{\Psi+1}\frac{(1+\gamma_t)w_t l_t(1-\tau_t)(1-f(\tau_{t+1}))}{1+n+\gamma_t(1+m)} \tag{26}$$

$$l_t^{\Psi} = w_t(1-\tau_t) \tag{27}$$

$$l_{t+1}^{\Psi} = w_{t+1}(1-\tau_{t+1}) \tag{28}$$

and substituting b_t from equation (19) and k_t from equation (22), in equations (2), the old's utility function can be written as follows:

$$\begin{aligned} U^o(\gamma_{t-1},k_t,w_t,r_t,\tau_t) &= \frac{\tau_t w_t l_t[(1+n)+\gamma_{t-1}(1+m)](1+\gamma_t)}{(1+\gamma_{t-1})} + \\ &\quad (1+r_t)k_t\left(\frac{1+n+\gamma_{t-1}(1+m)}{(1+\gamma_{t-1})}\right) \end{aligned} \tag{29}$$

As in the previous analysis, the old individual favors a positive level of tax rate ($\tau^* = \frac{\Psi}{\Psi+1}$), and a maximal level of openness rate of the economy towards immigration.

The direct effects of the current policy variables on the young utility function, on the other hand, differs from the previous analysis, since there is another channel of influence through the savings of the young. The young individual favors a minimal tax rate as it decreases its present net wage income, more that it increases next period interest rate due to the decrease in its present savings. Regarding the immigration quotas, there are two opposite effects: The openness rate decreases the present wage income as there are more working individuals, but increases next period interest rate. The latter results from two effects of the openness rate which decreases the next period capital per capita: on the one hand more workers lowers the wage rate, which decreases the amount of savings; on the other hand, the more open the economy is, the lower is next period capital per capita, since there are more present individual who save, but even more individuals next period because of the higher immigrants' fertility rate. These opposite effects may lead to an interior solution for the preferable openness rate, denoted by γ^*[7].

The dynamic of the system is influenced, as in the simplified model, not only by the direct effects of the current policy variables on the next period utility of the young but also by the expected effects of the current policy variables on next period policy variables. Nevertheless, the state variables of the extended model include, in addition to the openness rate of the previous period, another state variable, the capital per capita at the beginning of the

[7] The explicit value of γ^*, is given in the appendix.

period. This change, add to the model another Markov perfect equilibrium, in addition to a very similar equilibrium as in the simplified model. Next section will present the two equilibrium profiles and analyze the effect of the additional state variable which creates another possible Markov Perfect equilibrium.

2.2.1 Political equilibrium

There are two Markov perfect equilibriums in the extended model with capital. One of them can be characterized similarly to the previous section, by a "switching" strategy. The main feature of this equilibrium is that when the decisive voter is young, he admits a limited number of immigrants in order to change the decisive voter's identity from young to old in the next period. The only element that is supplemented is the savings, which have only a quantitative effect on the equilibrium path and do not influence the strategy the game.

The extension of the model including capital reveals an additional possible Markov perfect equilibrium, where there is an additional state variable, the capital per capita, which plays a crucial role of influencing the policy variables. The equilibrium is characterized by a range of values for the capital per capita state variable, for which the tax rate has an interior solution and is a function of this state variable and the openness rate is maximal. Meaning that for a range of state variables values, the young do not engage in a "switching" strategy, admitting a limited number of immigrants, in order to change the next period decisive voter's identity from young to old. On the contrary, he admits a maximal amount of immigrant, which renders a majority of young every period.

The equilibrium paths of the first equilibrium described, depend on the native born and immigrant's fertility rates as before, but are also influenced by the amount of initial capital per capita the economy is endowed with. The higher the initial amount of capital per capita, the higher is the amount of capital accumulated every period (influencing the equilibrium path in a quantitative way only).

The equilibrium paths of the second equilibrium described, depends qualitative not only upon the fertility rates but also on the initial capital per capita the economy in endowed with. If the fertility rates of the native born and immigrants are both positive, than for a range of initial capital per capita, the tax rate is positive and is a function of the capital per capita state variable, while the openness rate is maximal. For other values of initial capital per capita, there is no taxation and a more restrictive policy towards immigration for at least a few periods. If alternatively, the sum of the fertility rates is negative, for every value of initial capital, the tax rate is fixed and positive, and the openness rate is maximal. Otherwise, the sum of the fertility rates can be positive and the native born population's fertility rate negative. In this case, for a range of initial capital per capita, the tax rate is positive and is a function of the capital per capita state variable, while the openness rate is maximal. For other values of initial capital per capita, there is an alternate period by period taxation policy with some degree of immigration restrictions, at least for a few periods.

The Markov Perfect equilibrium definition of the extended case including capital is as follows:

Definition 4 *A Markov perfect political equilibrium is defined as a vector of policy decision rules, $\Psi = (T, G)$, and private decision rule, S, where $T : [0,1] \longrightarrow [0,1]$, is the taxation policy rule, $\tau_t = T(\gamma_{t-1}, k_t)$, and $G : [0,1] \longrightarrow [0,1]$, is the immigration quota policy rule, $\gamma_t = G(\gamma_{t-1}, k_t)$, and $S : [0,\infty) -> [0,\infty)$, is the saving decision rule, $k_{t+1} = S(\pi_t, k_t)$, such that the following functional equation holds:*

1. $\widehat{\Psi}(\gamma_{t-1}, k_t) = \arg\max_{\pi_t} V^i(\gamma_{t-1}, \pi_t, \pi_{t+1})$ *subject to* $\pi_{t+1} = \Psi(\gamma_t, S(\pi_t, k_t))$, *where* $\pi_t = (\tau_t, \gamma_t)$ *is defined as the vector of policy platform, and* V^i *is the indirect utility of the current decisive voter.*

2. $S(\pi_t, k_t) = \frac{\beta}{1+\beta}\frac{\Psi}{\Psi+1}\frac{(1+\gamma_t)w_t l_t(1-\tau_t)(1-f(\tau_{t+1}))}{1+n+\gamma_t(1+m)}$, *with* $\tau_{t+1} = T(\gamma_t, S(\pi_t, k_t))$.

2. The fixed-point condition requires that if next period policy outcome is derived by the vector of policy decision rules- Ψ, *the maximization of the indirect utility of the current decisive voter subject to the law of motion of capital, will reproduce the same law of motion,* $\widehat{\Psi}(\gamma_{t-1}) = \Psi(\gamma_{t-1})$.

The policy variables which are the tax rate, τ_t, and the openness rate, γ_t, have to maximize the decisive voter's indirect utility function, while taking into account the law of motion of capital and the fact that next period decision rules depend on the state variables i.e. the current period openness rate and next period capital per capita.

The first Markov Perfect equilibrium of the extended model and its possible equilibrium paths can be formalized as follows:

Proposition 5 *There exists an equilibrium with the following feature :*

$$T(\gamma_{t-1}) = \begin{cases} \tau_t = 0 & if \quad u_t(\gamma_{t-1}) < 1 \\ \tau_t = \frac{\Psi}{1+\Psi} & otherwise \end{cases} \tag{30}$$

$$G(\gamma_{t-1}) = \begin{cases} \gamma_t = \gamma^* & if \quad u_t(\gamma_{t-1}) < 1 \\ \gamma_t = 1 & otherwise \end{cases} \tag{31}$$

$$S(\pi_t, k_t) = \begin{cases} \frac{\beta}{1+\beta}\frac{\Psi}{\Psi+1}\frac{(1+\gamma_t)w_t l_t(1-\tau_t)(1-f(\frac{\Psi}{1+\Psi}))}{1+n+\gamma_t(1+m)} & if \quad u_t(\gamma_{t-1}) < 1 \\ \frac{\beta}{1+\beta}\frac{\Psi}{\Psi+1}\frac{(1+\gamma_t)w_t l_t(1-\tau_t)(1-f(0))}{1+n+\gamma_t(1+m)} & otherwise \end{cases} \tag{32}$$

where γ^* is uniquely defined and is given in the appendix. There are three possible equilibrium paths, depending on the fertility rates and on the initial amount of capital per capita the economy is endowed with, as in the previous section:

Proposition 6 *1. if $n > 0$, there is no taxation.*

2. if $m < -n$, there is full openness to immigration and a positive level of taxation.

3. if $n < 0$ and $m > -n$, there is an alternate taxation policy where some level of immigration always prevails: in periods where the decisive voter is old, the economy is fully opened to immigration and there is a positive level of taxation; and in periods where the decisive voter is young, there is no taxation and there is some restrictions on immigration.

The intuition is the same as before. However, in this case the equilibrium paths are influenced also by the initial capital per capita in the following way: the higher the initial capital per capita, the higher is the amount of savings in every period as it increases the young's wage income. As the decision rules do not depend on the capital per capita state variable, it does not influence the strategy of the game or the equilibrium paths of the game in a qualitative way.

The additional Markov Perfect equilibrium of the extended model and its possible equilibrium paths can be formalized as follows:

Proposition 7 *Under several conditions on the parameters of the model, which are specified in the appendix, there exists another equilibrium with the following feature:*

$$T(\gamma_{t-1},k_t)=\begin{cases}\begin{cases}\tau^*(k_t) & if \quad k_t\in[(\frac{c}{F(\overline{\tau}\,)})^{\frac{1}{x}},(c)^{\frac{1}{x}}]\\ 0 & otherwise\end{cases} & if \quad u_t(\gamma_{t-1})<1\\ \frac{\Psi}{1+\Psi} & otherwise\end{cases} \tag{33}$$

$$G(\gamma_{t-1},k_t)=\begin{cases}\begin{cases}1 & if \quad k_t\in[(\frac{c}{F(\overline{\tau}\,)})^{\frac{1}{x}},(c)^{\frac{1}{x}}]\\ \gamma^* & otherwise\end{cases} & if \quad u_t(\gamma_{t-1})<1\\ 1 & otherwise\end{cases} \tag{34}$$

$$S(\pi_t,k_t)=\begin{cases}\frac{\beta}{1+\beta}\frac{\Psi}{\Psi+1}\frac{(1+\gamma_t)w_tl_t(1-\tau_t)(1-f(\tau^*(k_t)))}{1+n+\gamma_t(1+m)} & if \quad k_t\in[(\frac{c}{F(\overline{\tau}\,)})^{\frac{1}{x}},(c)^{\frac{1}{x}}]\\ \frac{\beta}{1+\beta}\frac{\Psi}{\Psi+1}\frac{(1+\gamma_t)w_tl_t(1-\tau_t)(1-f(\frac{\Psi}{1+\Psi}))}{1+n+\gamma_t(1+m)} & otherwise \qquad if \quad u_t(\gamma_t\\ \frac{\beta}{1+\beta}\frac{\Psi}{\Psi+1}\frac{(1+\gamma_t)w_tl_t(1-\tau_t)(1-f(\tau^*(k_t)))}{1+n+\gamma_t(1+m)} & if \quad k_t\in[y(\frac{c}{F(\overline{\tau}\,)})^{\frac{1}{x}},y\,(c)^{\frac{1}{x}}]\\ \frac{\beta}{1+\beta}\frac{\Psi}{\Psi+1}\frac{(1+\gamma_t)w_tl_t(1-\tau_t)(1-f(0))}{1+n+\gamma_t(1+m)} & otherwise \qquad other\end{cases} \tag{35}$$

where $x=1+\frac{(1+\Psi)}{\Psi+\alpha}$, $y=\left(1-\frac{\Psi}{1+\Psi}\right)^{\frac{\alpha}{\Psi+\alpha}-1}$, $\overline{\tau}=\frac{\Psi(1+\beta)+\alpha}{\Psi(1+\beta)+\alpha+\beta}$, $F(\tau)=(1+\frac{1-\alpha}{\alpha}\tau)^{1+\beta}(1-\tau)^{\frac{\alpha(1-\alpha)}{\alpha+\Psi}}$, *$c$ is a positive constant of integration and $\tau^*(k_t)$ is an implicit function of the tax rate, defined in the appendix, which is decreasing in k_t. There are few possible equilibrium paths, depending on the fertility rates of the native born and immigrant populations and on the initial capital the economy is endowed with:*

Proposition 8 *1. if* $n > 0$ *and the initial capital is in the range* $[(c)^{x}, (\frac{c}{F(\overline{\tau})})^{x}]$, *there is a positive tax rate which depends on the capital per capita state variable and full openness to immigration. If the initial capital is not in this range, there are at least few period in which there is no taxation and a some restriction on immigration.*

2. if $m < -n$, *there is fix and positive tax rate, and a full openness to immigration.*

3. if $n < 0$ *and* $m > -n$, *the decisive voter is young and the initial there capital is in the range* $[(\frac{c}{F(\overline{\tau})})^{\frac{1}{x}}, (c)^{\frac{1}{x}}]$, *or the decisive voter is old and initial there capital is in the range* $[y(\frac{c}{F(\overline{\tau})})^{\frac{1}{x}}, y\,(c)^{\frac{1}{x}}]$, *there is a positive tax rate which depends on the capital per capita state variable and full openness to immigration. If the initial is not in this range, there are at least few period in which there is an alternate taxation policy where some level of immigration always prevails.*

The proposition is proved in the appendix. The additional equilibrium in the extended model is characterized by a decision rule of the young decisive voter which does not change next period decisive voter's identity, for a range of values of the capital per capita state variable. In these range, the tax rate is positive and depends on the amount of capital per capita, while the openness rate is maximal. This strategy is optimal for the young decisive voter instead of the "switching" strategy, which changes the next period decisive voter's identity from young to old voter. The switching strategy is optimal every period, separately, given the amount of capital per capita at the beginning of the period, but not necessarily from a dynamic looking approach, when capital is involved. The reason is that this strategy does not take into account that the next period state variable is influenced by the future policy variables. The higher is next period tax rate the lower is the current savings of the young, since the tax rate influences next period transfer payments and the incentive to save. Thus, the savings are not chosen in a way that maximizes the young life-time utility function, and for some parameter, the optimal strategy of the young is a decision rule for the tax rate, which is not increasing in the amount of capital per capita, and a maximal rate of openness.

This strategy is equilibrium from the following reason. The tax rate is not increasing in the amount of capital per capita. If we assume differently, meaning that the tax rate was increasing in k, than the higher was next period capital per capita, the higher was next period tax rate and so the future transfer payments. The higher the transfer payments next period, the lower were the savings of the young as it increases its future income. Since the savings of the young generates next period capital per capita, it cannot be equilibrium.

Regarding the openness rate, the higher the openness rate is the lower is the next period capital per capita. This is caused by two different effects which decrease the amount of next period capital per capita. From the one hand, more workers lowers the wage rate, which decreases the amount of savings; on the other hand, the more open the economy is, the lower is next period capital per capita, since there are more present individual who save, but even more individuals next period because of the higher immigrants' fertility rate. Thus, the higher the openness rate is, the lower is next period amount of capital per capita, which increases next period tax rate (according to the tax rate decision rule ($\tau^*(k_t)$)), and so next period transfer payments. This additional positive effect of the openness rate on the indirect utility of the young, increases the preferable openness rate of the young under the described tax rate decision rule ($\tau^*(k_t)$). Thus, the optimal openness rate is the maximal openness rate, if the decision rule of the tax rate depends on the amount of capital per capita.

There are several possible equilibrium paths depending on the fertility rates and the amount of capital per capita. The possible paths are a combination of the previous analysis regarding the fertility rates, with another element influencing the equilibrium paths, the amount of capital per capita:

1. The fertility rates of the native born and immigrant populations are positive, n, $m > 0$. In this case, for any not negative value of openness rate, the number of next period young voters exceeds the number of next period old voters, which means that the decisive voter is always young. Therefore, if the capital per capita is in the range $[(\frac{c}{F(\overline{\tau})})^{\frac{1}{x}}, (c)^{\frac{1}{x}}]$, than the optimal strategy of the young is always, a positive tax rate which depends on the capital per capita state variable, and a maximal openness rate. Otherwise the initial capital is not in the relevant range, and therefore the tax rate is as in the previous analysis, a minimal tax rate and a positive openness rate. It should be noted that as the capital evolves, there could be a period where the amount of capital per capita enter the range $[(\frac{c}{F(\overline{\tau})})^{\frac{1}{x}}, (c)^{\frac{1}{x}}]$, and from then on the optimal strategy is the previous one.

2. If the sum of the fertility rates is negative, than as before the number old voters always exceeds the number of young voters, which means that the decisive voter is always young. In that case the equilibrium path is the same

as in the previous case: the optimal strategy of the old is a certain amount of tax rate, and a maximal level of openness.

3. If the sum of the fertility rates are positive, but the native born fertility rates is negative, there are two possible equilibrium paths. If the capital per capita state variable is in the relevant range[8], the optimal strategy of the young is a positive tax rate which depends on the capital per capita state variable, and a maximal level of openness. Otherwise there is in the previous equilibrium, an alternate taxation policy, with a certain level of immigration. It should be noted that as the capital evolves, there could be a period where the capital per capita enter the relevant range, and from then on the optimal strategy is the previous one.

3 Appendix

4 References

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[8]The relevant range for the capital per capita is as follows: If the decisive voter is young, and the capital per capita is in the range $[(c)^x, (\frac{c}{F(\overline{\tau})})^x]$, the optimal decision rules are $\pi_t = (\tau(k), 1)$, from that period and on. If the decisive voter is old and the capital per capita is in the range $[y\,(c)^x, y(\frac{c}{F(\overline{\tau})})^x]$, than next period decisive voter is young and the next period capital per capita is in the range $[(c)^x, (\frac{c}{F(\overline{\tau})})^x]$. Thus, in that case also, the optimal decision rules are $\pi_t = (\tau(k), 1)$ from that period and on.

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