7

Monetary Policy and the Open Economy

All the models analyzed in earlier chapters assumed a closed economy: households and firms were not able to trade in goods or financial assets with agents located in other economies. This chapter relaxes that assumption by developing an open economy extension of the basic New Keynesian model analyzed in chapter 3. The framework introduces explicitly the exchange rate, the terms of trade, exports, and imports, as well as international financial markets. It also implies a distinction between the consumer price index—that includes the price of imported goods—and the price index for domestically produced goods. Such a framework can in principle be used to assess the implications of alternative monetary policy rules for an open economy. Because the framework nests as a limiting case the closed economy model of chapter 3, it allows the exploration of the extent to which the opening of the economy affects some of the conclusions regarding monetary policy obtained for the closed economy model: in particular, the desirability of a policy that seeks to stabilize inflation (see chapter 4). It is also worth analyzing what role, if any, the exchange rate plays in the optimal design of monetary policy and/or what is the measure of inflation that the central bank should seek to stabilize. Finally, the framework can be used to determine the implications of alternative simple rules, as was done in chapter 4 for the closed economy.

The analysis of a monetary open economy raises a number of issues that a modeler needs to confront, and which are absent from its closed economy counterpart. First, a choice needs to be made between the modelling of a "large" or "small" economy, i.e., between allowing or not, respectively, for repercussions in the rest of the world of developments (including policy decisions) in the economy being modelled. Second, the existence of two or more economies subject to imperfectly correlated shocks generates an incentive to trade in assets between residents of different countries in order to smooth their consumption over time. Hence, a decision must be made regarding the nature of international asset markets and, more specifically, the set of securities that can be traded in those markets, with

This chapter is based on Galí and Monacelli (2005), with the notation modified for consistency with earlier chapters. Section 7.3 on the transmission of monetary policy shocks contains original material.

possible assumptions ranging from financial autarky to complete markets. Third, one needs to make some assumption about firms' abilities to discriminate across countries in the price they charge for the goods they produce ("pricing to market" versus "law of one price"). Furthermore, whenever discrimination is possible and prices are not readjusted continuously, an assumption must be made regarding the currency in which the prices of exported goods are set ("local currency pricing," i.e., prices are set in the currency of the importing economy versus "producer currency pricing," i.e., prices are set in the currency of the producer's country). Other dimensions of open economy modelling that require some choices include the allowance or not of nontradeable goods, the existence of trading costs, the possibility of international policy coordination, and so on.

A comprehensive analysis of those different modelling dimensions and how they may affect the design of monetary policy would require a book of its own, thus it is clearly beyond the scope of this chapter. The modest objective here is to present an example of a monetary open economy model to illustrate some of the issues that emerge in the analysis of such economies and which are absent from their closed economy counterparts. In particular, a small open economy model is developed, with complete international financial markets, where the law of one price holds. Then, in the discussion of the model's policy implications and in the notes on the literature in section 7.6, there is reference made to a number of papers that adopt different assumptions and briefly discuss the extent to which this leads their findings to differ from those obtained here.

The framework below, originally developed in Galí and Monacelli (2005), models a small open economy as one among a continuum of (infinitesimally small) economies making up the world economy. For simplicity, and in order to focus on the issues brought about by the openness of the economy, the possible presence of either cost-push shocks or nominal wage rigidities is ignored. The assumptions on preferences and technology, combined with the Calvo price-setting structure and the assumption of complete financial markets, give rise to a highly tractable model and to simple and intuitive log-linearized equilibrium conditions. The latter can be reduced to a two-equation dynamical system consisting of a New Keynesian Phillips curve and a dynamic IS-type equation, whose structure is identical to the one derived in chapter 3 for the closed economy, though its coefficients depend on parameters that are specific to the open economy while the driving forces are a function of world variables (that are taken as exogenous to the small open economy). As in its closed economy counterpart, the two equations must be complemented with a description of how monetary policy is conducted.

After describing the model and deriving a simple representation of its equilibrium dynamics, section 7.3 analyzes the transmission of monetary policy shocks, emphasizing the role played by openness in that transmission. Section 7.4 turns to the issue of optimal monetary policy design, focusing on a particular case for which the flexible price allocation is efficient. Under the same assumptions it is straightforward to derive a second-order approximation to the consumer's utility, which can be used to evaluate alternative policy rules. Section 7.5 assesses the merits of two different Taylor-type rules, a policy that fully stabilizes the CPI, and an exchange rate peg. Section 7.6 concludes with a brief note on the related literature.

7.1 A Small Open Economy Model

The world economy is modelled as a continuum of small open economies represented by the unit interval. Since each economy is of measure zero, its performance does not have any impact on the rest of the world. Different economies are subject to imperfectly correlated productivity shocks, but it is assumed that they share identical preferences, technology, and market structure.

Next, the problem facing households and firms located in one such economy will be described in detail. Before doing so, a brief remark on notation is in order. Because the focus is on the behavior of a single economy and its interaction with the world economy, and in order to lighten the notation,variables are used *without* an i-index to refer to the small open economy being modelled. Variables with an $i \in [0, 1]$ subscript refer to economy i, one among the continuum of economies making up the world economy. Finally, variables with an *asterisk superscript* (*) correspond to the world economy as a whole.

7.1.1 Households

A typical small open economy is inhabited by a representative household who seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \, U(C_t, N_t) \tag{1}$$

where N_t denotes hours of labor, and C_t is a composite consumption index defined by

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} \, (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \, (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{2}$$

where $C_{H,t}$ is an index of consumption of domestic goods given by the constant elasticity of substitution (CES) function

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $j \in [0, 1]$ denotes the good variety.[1] $C_{F,t}$ is an index of imported goods given by

$$C_{F,t} \equiv \left(\int_0^1 (C_{i,t})^{\frac{\gamma-1}{\gamma}} \, di \right)^{\frac{\gamma}{\gamma-1}}$$

where $C_{i,t}$ is, in turn, an index of the quantity of goods imported from country i and consumed by domestic households. It is given by an analogous CES function

$$C_{i,t} \equiv \left(\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Note that parameter $\varepsilon > 1$ denotes the elasticity of substitution between varieties produced within any given country.[2] Parameter $\alpha \in [0, 1]$ can be interpreted as a measure of openness.[3] Parameter $\eta > 0$ measures the substitutability between domestic and foreign goods from the viewpoint of the domestic consumer, while γ measures the substitutability between goods produced in different foreign countries.

Maximization of (1) is subject to a sequence of budget constraints of the form

$$\int_0^1 P_{H,t}(j)\, C_{H,t}(j)\, dj + \int_0^1 \int_0^1 P_{i,t}(j)\, C_{i,t}(j)\, dj\, di + E_t\{Q_{t,t+1} D_{t+1}\} \leq D_t + W_t N_t + T_t \quad (3)$$

for $t = 0, 1, 2, \ldots$ where $P_{H,t}(j)$ is the price of domestic variety j. $P_{i,t}(j)$ is the price of variety j imported from country i. D_{t+1} is the nominal payoff in period $t+1$ of the portfolio held at the end of period t (and which includes shares in firms), W_t is the nominal wage, and T_t denotes lump-sum transfers/taxes. The previous variables are all expressed in units of domestic currency. $Q_{t,t+1}$ is the stochastic discount factor for one-period-ahead nominal payoffs relevant to the domestic household. Assume that households have access to a complete set of contingent claims, traded internationally.

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[1] As discussed below, each country produces a continuum of differentiated goods, represented by the unit interval.

[2] Notice that it is irrelevant to think of integrals like the one in (2) as including or not the corresponding variable for the small economy being modelled, because its presence would have a negligible influence on the integral itself (in fact, each individual economy has a zero measure). The previous remark also applies to many other expressions involving integrals over the continuum of economies (i.e., over i) that the reader will encounter below.

[3] Equivalently, $1 - \alpha$ is a measure of the degree of home bias. Note that in the absence of some home bias the households in the small open economy would attach an infinitesimally small weight to local goods, and consumption expenditures would be allocated to imported goods (except for an infinitesimally small share allocated to domestic goods).

The optimal allocation of any given expenditure within each category of goods yields the demand functions

$$C_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \; ; \; C_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\varepsilon} C_{i,t} \tag{4}$$

for all $i, j \in [0, 1]$, where $P_{H,t} \equiv \left(\int_0^1 P_{H,t}(j)^{1-\varepsilon}\, dj\right)^{\frac{1}{1-\varepsilon}}$ is the *domestic* price index (i.e., an index of prices of domestically produced goods) and $P_{i,t} \equiv \left(\int_0^1 P_{i,t}(j)^{1-\varepsilon}\, dj\right)^{\frac{1}{1-\varepsilon}}$ is a price index for goods imported from country i (expressed in domestic currency) for all $i \in [0, 1]$. Combining the optimality conditions in (4) with the definitions of price and quantity indexes $P_{H,t}$, $C_{H,t}$, $P_{i,t}$, and $C_{i,t}$ yields $\int_0^1 P_{H,t}(j)\, C_{H,t}(j)\, dj = P_{H,t}\, C_{H,t}$ and $\int_0^1 P_{i,t}(j)\, C_{i,t}(j)\, dj = P_{i,t}\, C_{i,t}$.

Furthermore, the optimal allocation of expenditures on imported goods by country of origin implies

$$C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma} C_{F,t} \tag{5}$$

for all $i \in [0, 1]$ where $P_{F,t} \equiv \left(\int_0^1 P_{i,t}{}^{1-\gamma}\, di\right)^{\frac{1}{1-\gamma}}$ is the price index for *imported* goods, also expressed in domestic currency. Note that (5), together with the definitions of $P_{F,t}$ and $C_{F,t}$, implies that total expenditures on imported goods can be written as $\int_0^1 P_{i,t}\, C_{i,t}\, di = P_{F,t}\, C_{F,t}$.

Finally, the optimal allocation of expenditures between domestic and imported goods is given by

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \; ; \; C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{6}$$

where $P_t \equiv [(1-\alpha)\, (P_{H,t})^{1-\eta} + \alpha\, (P_{F,t})^{1-\eta}]^{\frac{1}{1-\eta}}$ is the CPI.[4] Note that under the assumption of $\eta = 1$ or, alternatively, when the price indexes for domestic and foreign goods are equal (as in the steady state described below), parameter α corresponds to the share of domestic consumption allocated to imported goods. It is also in this sense that α represents a natural index of openness.

Accordingly, total consumption expenditures by domestic households are given by $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_t C_t$. Thus, the period budget constraint can be rewritten as

$$P_t\, C_t + E_t\{Q_{t,t+1}\, D_{t+1}\} \leq D_t + W_t\, N_t + T_t. \tag{7}$$

[4] It is useful to notice, for future reference, that in the particular case of $\eta = 1$, the CPI takes the form $P_t = (P_{H,t})^{1-\alpha}(P_{F,t})^{\alpha}$, while the consumption index is given by $C_t = \frac{1}{(1-\alpha)^{(1-\alpha)}\alpha^{\alpha}} C_{H,t}{}^{1-\alpha}\, C_{F,t}{}^{\alpha}$.

As in previous chapters, the period utility function is specialized to be of the form $U(C, N) \equiv \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}$. Thus, the remaining optimality conditions for the household's problem can be rewritten as

$$C_t^{\sigma} \; N_t^{\varphi} = \frac{W_t}{P_t} \tag{8}$$

which is the standard intratemporal optimality condition. In order to derive the relevant intertemporal optimality condition note that the following relation must hold for the optimizing household in the small open economy

$$\frac{V_{t,t+1}}{P_t} \; C_t^{-\sigma} = \xi_{t,t+1} \; \beta \; C_{t+1}^{-\sigma} \; \frac{1}{P_{t+1}} \tag{9}$$

where $V_{t,t+1}$ is the period t price (in domestic currency) of an Arrow security, i.e., a one-period security that yields one unit of domestic currency if a specific state of nature is realized in period $t+1$, and nothing otherwise, and where $\xi_{t,t+1}$ is the probability of that state of nature being realized in $t+1$ (conditional on the state of nature at t). Variables C_{t+1} and P_{t+1} on the right side should be interpreted as representing the values taken by the consumption index and the CPI at $t+1$ conditional on the state of nature to which the Arrow security refers to being realized. Thus, the left side captures the utility loss resulting from the purchase of the Arrow security considered (with the corresponding reduction in consumption), whereas the right side measures the expected one-period-ahead utility gain from the additional consumption made possible by the (eventual) security payoff. If the consumer is optimizing the expected utility gain, it must exactly offset the current utility loss.

Given that the price of Arrow securities and the one-period stochastic discount factor are related by the equation $Q_{t,t+1} \equiv \frac{V_{t,t+1}}{\xi_{t,t+1}}$, (9) can be rewritten as[5]

$$\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) = Q_{t,t+1} \tag{10}$$

which is assumed to be satisfied for all possible states of nature at t and $t+1$.

Taking conditional expectations on both sides of (10) and rearranging terms, a conventional stochastic Euler equation can be derived

$$Q_t = \beta \; E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right\} \tag{11}$$

where $Q_t \equiv E_t\{Q_{t,t+1}\}$ denotes the price of a one-period discount bond paying off one unit of domestic currency in $t+1$.

[5] Note that under complete markets a simple no room for arbitrage argument implies that the price of a one-period asset (or portfolio) yielding a random payoff D_{t+1} must be given by $\sum V_{t,t+1} D_{t+1}$ where the sum is over all possible $t+1$ states. Equivalently, that price can be written as $E_t\{ \frac{V_{t,t+1}}{\xi_{t,t+1}} D_{t+1} \}$. Thus, the one-period stochastic discount factor can be defined as $Q_{t,t+1} \equiv \frac{V_{t,t+1}}{\xi_{t,t+1}}$.

For future reference, recall that (8) and (11) can be respectively written in log-linearized form as

$$w_t - p_t = \sigma \; c_t + \varphi \; n_t$$
$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} \; (i_t - E_t\{\pi_{t+1}\} - \rho) \tag{12}$$

where lowercase letters denote the logs of the respective variables, $i_t \equiv -\log Q_t$ is the short term nominal rate, $\rho \equiv -\log \beta$ is the time discount rate, and $\pi_t \equiv p_t - p_{t-1}$ is CPI inflation (with $p_t \equiv \log P_t$).

7.1.1.1 Domestic Inflation, CPI Inflation, the Real Exchange Rate, and the Terms of Trade: Some Identities

Next, several assumptions and definitions are introduced, and a number of identities are derived that are extensively used below. *Bilateral terms of trade* between the domestic economy and country i is defined as $\mathcal{S}_{i,t} = \frac{P_{i,t}}{P_{H,t}}$, i.e., the price of country i's goods in terms of home goods. The *effective terms of trade* are thus given by

$$\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}}$$
$$= \left(\int_0^1 \mathcal{S}_{i,t}^{1-\gamma} \, di \right)^{\frac{1}{1-\gamma}}$$

which can be approximated (up to first order) around a symmetric steady state satisfying $\mathcal{S}_{i,t} = 1$ for all $i \in [0, 1]$ by

$$s_t = \int_0^1 s_{i,t} \; di \tag{13}$$

where $s_t \equiv \log \mathcal{S}_t = p_{F,t} - p_{H,t}$.

Similarly, log-linearization of the CPI formula around the same symmetric steady state yields

$$p_t \equiv (1 - \alpha) \; p_{H,t} + \alpha \; p_{F,t}$$
$$= p_{H,t} + \alpha \; s_t. \tag{14}$$

It is useful to note, for future reference, that (13) and (14) hold *exactly* when $\gamma = 1$ and $\eta = 1$, respectively.

It follows that *domestic inflation*, defined as the rate of change in the index of domestic goods prices, i.e., $\pi_{H,t} \equiv p_{H,t+1} - p_{H,t}$, and *CPI inflation* are linked according to the relation

$$\pi_t = \pi_{H,t} + \alpha \; \Delta s_t \tag{15}$$

which makes the gap between the two measures of inflation proportional to the percent change in the terms of trade, with the coefficient of proportionality given by the openness index α.

Assume that the *law of one price* holds for individual goods at all times (both for import and export prices), implying that $P_{i,t}(j) = \mathcal{E}_{i,t}\ P^i_{i,t}(j)$ for all $i, j \in [0, 1]$, where $\mathcal{E}_{i,t}$ is the bilateral nominal exchange rate (the price of country i's currency in terms of the domestic currency), and $P^i_{i,t}(j)$ is the price of country i's good j expressed in terms of its own currency. Plugging the previous assumption into the definition of $P_{i,t}$ yields $P_{i,t} = \mathcal{E}_{i,t}\ P^i_{i,t}$, where $P^i_{i,t} \equiv \left(\int_0^1 P^i_{i,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$ is country i's domestic price index. In turn, by substituting into the definition of $P_{F,t}$ and log-linearizing around the symmetric steady state,

$$\begin{aligned} p_{F,t} &= \int_0^1 (e_{i,t} + p^i_{i,t})\ di \\ &= e_t + p^*_t \end{aligned}$$

where $p^i_{i,t} \equiv \int_0^1 p^i_{i,t}(j)\ dj$ is the (log) domestic price index for country i (expressed in terms of its own currency), $e_t \equiv \int_0^1 e_{i,t}\ di$ is the (log) *effective nominal exchange rate*, and $p^*_t \equiv \int_0^1 p^i_{i,t}\ di$ is the (log) *world price index*. Notice that for the world as a whole, there is no distinction between CPI and domestic price level, nor between their corresponding inflation rates.

Combining the previous result with the definition of the terms of trade yields the expression

$$s_t = e_t + p^*_t - p_{H,t}. \tag{16}$$

Next, a relationship is derived between the terms of trade and the real exchange rate. First, the *bilateral real exchange rate* is defined with country i as $\mathcal{Q}_{i,t} \equiv \frac{\mathcal{E}_{i,t} P^i_t}{P_t}$, i.e., the ratio of the two countries' CPIs, both expressed in terms of domestic currency. Let $q_t \equiv \int_0^1 q_{i,t}\ di$ be the (log) *effective real exchange rate*, where $q_{i,t} \equiv \log \mathcal{Q}_{i,t}$. It follows that

$$\begin{aligned} q_t &= \int_0^1 (e_{i,t} + p^i_t - p_t)\ di \\ &= e_t + p^*_t - p_t \\ &= s_t + p_{H,t} - p_t \\ &= (1-\alpha)\ s_t \end{aligned}$$

where the last equality holds only up to a first-order approximation when $\eta \neq 1$.[6]

[6] The last equality can be derived by log-linearizing $\frac{P_t}{P_{H,t}} = [(1-\alpha) + \alpha\ \mathcal{S}_t^{1-\eta}]^{\frac{1}{1-\eta}}$ around a symmetric steady state, which yields $p_t - p_{H,t} = \alpha\ s_t$.

7.1.1.2 International Risk-Sharing

Under the assumption of complete markets for securities traded internationally, a condition analogous to (9) must also hold for the representative household in any other country, say country i

$$\frac{V_{t,t+1}}{\mathcal{E}_t^i P_t^i} \, (C_t^i)^{-\sigma} = \xi_{t,t+1} \beta \, (C_{t+1}^i)^{-\sigma} \, \frac{1}{\mathcal{E}_{t+1}^i P_{t+1}^i}$$

where the presence of the exchange rate terms reflects the fact that the security purchased by the country i's household has a price $V_{t,t+1}$ and a unit payoff expressed in the currency of the small open economy of reference, and hence, needs to be converted to country i's currency.

The previous relation can be written in terms of our small open economy's stochastic discount factor as

$$\beta \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \left(\frac{P_t^i}{P_{t+1}^i} \right) \left(\frac{\mathcal{E}_t^i}{\mathcal{E}_{t+1}^i} \right) = Q_{t,t+1}. \tag{17}$$

Combining (10) and (17), together with the definition for the real exchange rate definition gives

$$C_t = \vartheta_i \, C_t^i \, \mathcal{Q}_{i,t}{}^{\frac{1}{\sigma}} \tag{18}$$

for all t, and where ϑ_i is a constant that will generally depend on initial conditions regarding relative net asset positions. Henceforth, and without loss of generality, symmetric initial conditions are assumed (i.e., zero net foreign asset holdings and an ex-ante identical environment), in which case $\vartheta_i = \vartheta = 1$ for all i.

Taking logs on both sides of (18) and integrating over i yields

$$\begin{aligned} c_t &= c_t^* + \frac{1}{\sigma} \, q_t \qquad (19) \\ &= c_t^* + \left(\frac{1-\alpha}{\sigma} \right) s_t \end{aligned}$$

where $c_t^* \equiv \int_0^1 c_t^i \, di$ is the index for world consumption (in log terms), and where the second equality holds only up to a first-order approximation when $\eta \neq 1$. Thus, the assumption of complete markets at the international level leads to a simple relationship linking domestic consumption with world consumption and the terms of trade.

7.1.1.3 A Brief Detour: Uncovered Interest Parity and the Terms of Trade

Under the assumption of complete international financial markets, the equilibrium price (in terms of the small open economy's domestic currency) of a riskless bond denominated in country i's currency is given by $\mathcal{E}_{i,t} \; Q_t^i = E_t\{Q_{t,t+1} \, \mathcal{E}_{i,t+1}\}$, where Q_t^i is the price of the bond in terms of country i's currency. The previous pricing equation can be combined with the domestic bond pricing equation $Q_t = E_t\{Q_{t,t+1}\}$ to obtain a version of the uncovered interest-parity condition

$$E_t\{Q_{t,t+1}\ [\exp\{i_t\} - \exp\{i_t^*\}\ (\mathcal{E}_{i,t+1}/\mathcal{E}_{i,t})]\} = 0.$$

Log-linearizing around a perfect foresight steady state, and aggregating over i, yields the familiar expression

$$i_t = i_t^* + E_t\{\Delta e_{t+1}\}. \tag{20}$$

Combining the definition of the (log) terms of trade with (20) yields the stochastic difference equation

$$s_t = (i_t^* - E_t\{\pi_{t+1}^*\}) - (i_t - E_t\{\pi_{H,t+1}\}) + E_t\{s_{t+1}\}. \tag{21}$$

As shown in appendix 7.1, the terms of trade are pinned down uniquely in the perfect foresight steady state. That fact, combined with the assumption of stationarity in the model's driving forces and unit relative prices in the steady state, implies that $\lim_{T\to\infty} E_t\{s_T\} = 0$.[7] Hence, (21) can be solved forward to obtain

$$s_t = E_t\left\{\sum_{k=0}^{\infty} [(i_{t+k}^* - \pi_{t+k+1}^*) - (i_{t+k} - \pi_{H,t+k+1})]\right\} \tag{22}$$

i.e., the terms of trade are a function of current and anticipated real interest rate differentials.

It must be pointed out that while equations (21) and (22) provide a convenient (and intuitive) way of representing the connection between terms of trade and interest rate differentials, they do not constitute an additional independent equilibrium condition. In particular, it is easy to check that (21) can be derived by combining the consumption Euler equations for both the domestic and world economies with the risk sharing condition (19) and equation (15).

Next, attention is turned to the supply side of the economy.

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[7] The assumption regarding the steady state implies that the real interest rate differential will revert to a zero mean. More generally, the real interest rate differential will revert to a constant mean, as long as the terms of trade are stationary in first differences. That would be the case if, say, the technology parameter had a unit root or a different average rate of growth relative to the rest of the world. Those cases would have persistent real interest rate differentials.

7.1.2 Firms

7.1.2.1 Technology

A typical firm in the home economy produces a differentiated good with a linear technology represented by the production function

$$Y_t(j) = A_t\, N_t(j)$$

where $a_t \equiv \log A_t$ follows the AR(1) process $a_t = \rho_a\ a_{t-1} + \varepsilon_t$, and where $j \in [0, 1]$ is a firm-specific index.[8]

Hence, the real marginal cost (expressed in terms of domestic prices) will be common across domestic firms and given by

$$mc_t = -\nu + w_t - p_{H,t} - a_t$$

where $\nu \equiv -\log(1-\tau)$, with τ being an employment subsidy whose role is discussed later in more detail.

7.1.2.2 Price Setting

As in the basic model of chapter 3, it is assumed that firms set prices in a staggered fashion. In particular, a measure $1-\theta$ of (randomly selected) firms sets new prices each period, with an individual firm's probability of reoptimizing in any given period being independent of the time elapsed since it last reset its price. As shown in chapter 3, the optimal price-setting strategy for the typical firm resetting its price in period t can be approximated by the (log-linear) rule

$$\overline{p}_{H,t} = \mu + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k\ E_t\{mc_{t+k} + p_{H,t+k}\} \tag{23}$$

where $\overline{p}_{H,t}$ denotes the log of newly set domestic prices, and $\mu \equiv \log \frac{\varepsilon}{\varepsilon-1}$ is the log of the (gross) markup in the steady state (or, equivalently, the equilibrium markup in the flexible price economy).[9]

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[8] An extension of the analysis to the case of decreasing returns considered in chapter 3 is straightforward. In order to keep the notation as simple as possible the analysis here is restricted to the case of constant returns.

[9] $\overline{p}_{H,t}$ is used to denote newly set prices instead of p_t^* (used in chapter 3), because in this chapter letters with an asterisk refer to world economy variables.

7.2 Equilibrium

7.2.1 Aggregate Demand and Output Determination

7.2.1.1 Consumption and Output in the Small Open Economy

Goods market clearing in the home economy requires

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C_{H,t}^i(j)\, di = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} \times \left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^i}\right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i}\right)^{-\eta} C_t^i\, di\right] \tag{24}$$

for all $j \in [0, 1]$ and all t, where $C_{H,t}^i(j)$ denotes country i's demand for good j produced in the home economy. Notice that the second equality has made use of (5) and (6) together with the assumption of symmetric preferences across countries, which implies $C_{H,t}^i(j) = \alpha\big(\frac{P_{H,t}(j)}{P_{H,t}}\big)^{-\varepsilon}\big(\frac{P_{H,t}}{\mathcal{E}_{i,t}P_{F,t}^i}\big)^{-\gamma}\big(\frac{P_{F,t}^i}{P_t^i}\big)^{-\eta} C_t^i$.

Plugging (24) into the definition of aggregate domestic output $Y_t \equiv \big[\int_0^1 Y_t(j)^{1-\frac{1}{\varepsilon}}\, dj\big]^{\frac{\varepsilon}{\varepsilon-1}}$ yields

$$\begin{aligned} Y_t &= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^i}\right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i}\right)^{-\eta} C_t^i\, di \\ &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha)\, C_t + \alpha \int_0^1 \left(\frac{\mathcal{E}_{i,t} P_{F,t}^i}{P_{H,t}}\right)^{\gamma-\eta} \mathcal{Q}_{i,t}^{\eta}\, C_t^i\, di\right] \\ &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \left[(1-\alpha) + \alpha \int_0^1 \left(\mathcal{S}_t^i\, \mathcal{S}_{i,t}\right)^{\gamma-\eta}\, \mathcal{Q}_{i,t}^{\eta-\frac{1}{\sigma}}\, di\right] \end{aligned} \tag{25}$$

where the last equality follows from (18), and where $\mathcal{S}_t^i$ denotes the effective terms of trade for country i, while $\mathcal{S}_{i,t}$ denotes the bilateral terms of trade between the home economy and country i. Notice that in the particular case of $\sigma = \eta = \gamma = 1$ the previous condition can be written exactly as[10]

$$Y_t = C_t\, \mathcal{S}_t^{\alpha}. \tag{26}$$

[10] Here one must use the fact that under the assumption $\eta = 1$, the CPI takes the form $P_t = (P_{H,t})^{1-\alpha}(P_{F,t})^{\alpha}$, thus implying $\frac{P_t}{P_{H,t}} = \big(\frac{P_{F,t}}{P_{H,t}}\big)^{\alpha} = \mathcal{S}_t^{\alpha}$.

More generally, and recalling that $\int_0^1 s_t^i \, di = 0$, the following first-order log-linear approximation to (25) is derived around the symmetric steady state

$$y_t = c_t + \alpha\gamma \, s_t + \alpha \left(\eta - \frac{1}{\sigma} \right) q_t$$
$$= c_t + \frac{\alpha\omega}{\sigma} \, s_t \tag{27}$$

where $\omega \equiv \sigma\gamma + (1-\alpha)\,(\sigma\eta - 1)$. Notice that $\sigma = \eta = \gamma = 1$ implies $\omega = 1$.

A condition analogous to the one above will hold for all countries. Thus, for a *generic country* i it can be rewritten as $y_t^i = c_t^i + \frac{\alpha\omega}{\sigma} \, s_t^i$. By aggregating over all countries, a world market clearing condition can be derived as

$$y_t^* \equiv \int_0^1 y_t^i \, di$$
$$= \int_0^1 c_t^i \, di \equiv c_t^* \tag{28}$$

where y_t^* and c_t^* are indexes for world output and consumption (in log terms), and where the main equality follows, once again, from the fact that $\int_0^1 s_t^i \, di = 0$.

Combining (27) with (19) and (28) yields

$$y_t = y_t^* + \frac{1}{\sigma_\alpha} \, s_t \tag{29}$$

where $\sigma_\alpha \equiv \frac{\sigma}{1+\alpha(\omega-1)} > 0$.

Finally, combining (27) with Euler equation (12) gives

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma} \, (i_t - E_t\{\pi_{t+1}\} - \rho) - \frac{\alpha\omega}{\sigma} \, E_t\{\Delta s_{t+1}\}$$
$$= E_t\{y_{t+1}\} - \frac{1}{\sigma} \, (i_t - E_t\{\pi_{H,t+1}\} - \rho) - \frac{\alpha\Theta}{\sigma} \, E_t\{\Delta s_{t+1}\}$$
$$= E_t\{y_{t+1}\} - \frac{1}{\sigma_\alpha} \, (i_t - E_t\{\pi_{H,t+1}\} - \rho) + \alpha\Theta \, E_t\{\Delta y_{t+1}^*\} \tag{30}$$

where $\Theta \equiv (\sigma\gamma - 1) + (1-\alpha)(\sigma\eta - 1) = \omega - 1$. Note that, in general, the degree of openness influences the sensitivity of output to any given change in the domestic real rate $i_t - E_t\{\pi_{H,t+1}\}$, given world output. In particular, if $\Theta > 0$ (i.e., for relatively high values of η and γ), an increase in openness raises that sensitivity (i.e., σ_α is smaller). The reason is the direct negative effect of an increase in the real rate on aggregate demand and output is amplified by the induced real appreciation (and the consequent switch of expenditure toward foreign goods). This will be partly offset by any increase in CPI inflation relative to domestic inflation induced by the expected real depreciation, which would dampen the change in the consumption-based real rate $i_t - E_t\{\pi_{t+1}\}$—which is the one ultimately relevant for aggregate demand—relative to $i_t - E_t\{\pi_{H,t+1}\}$.

7.2.1.2 The Trade Balance

Let $nx_t \equiv \left(\frac{1}{Y}\right)\left(Y_t - \frac{P_t}{P_{H,t}} C_t\right)$ denote net exports in terms of domestic output, expressed as a fraction of steady state output Y. In the particular case of $\sigma = \eta = \gamma = 1$, it follows from (25) that $P_{H,t} Y_t = P_t C_t$ for all t, thus implying a balanced trade at all times. More generally, a first-order approximation yields $nx_t = y_t - c_t - \alpha\, s_t$, which combined with (27) implies a simple relation between net exports and the terms of trade

$$nx_t = \alpha \left(\frac{\omega}{\sigma} - 1 \right) s_t . \tag{31}$$

Again, in the special case of $\sigma = \eta = \gamma = 1$, $nx_t = 0$ for all t, though the latter property will also hold for any configuration of those parameters satisfying $\sigma(\gamma - 1) + (1 - \alpha)(\sigma\eta - 1) = 0$. More generally, the sign of the relationship between the terms of trade and net exports is ambiguous, depending on the relative size of σ, γ, and η.

7.2.2 The Supply Side: Marginal Cost and Inflation Dynamics

7.2.2.1 Aggregate Output and Employment

Let $Y_t \equiv \left[\int_0^1 Y_t(j)^{1-\frac{1}{\varepsilon}}\, dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$ represent an index for aggregate domestic output, analogous to the one introduced for consumption. As in chapter 3, one can derive an approximate aggregate production function relating the previous index to aggregate employment. Hence, notice that

$$N_t \equiv \int_0^1 N_t(j)\, dj = \frac{Y_t}{A_t} \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} dj.$$

As shown in chapter 3, however, variations in $d_t \equiv \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} dj$ around the perfect foresight steady state are of second order. Thus, and up to a first-order approximation, the following relationship between aggregate output and employment holds as

$$y_t = a_t + n_t . \tag{32}$$

7.2.2.2 Marginal Cost and Inflation Dynamics in the Small Open Economy

As was shown in chapter 3, the (log-linearized) optimal price-setting condition (23) can be combined with the (log linearized) difference equation describing the evolution of domestic prices (as a function of newly set prices) to yield an equation determining domestic inflation as a function of deviations of marginal cost from its steady state value

$$\pi_{H,t} = \beta \; E_t\{\pi_{H,t+1}\} + \lambda \; \widehat{mc}_t \tag{33}$$

where $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$. Thus, relationship (33) does not depend on any of the parameters that characterize the open economy. On the other hand, the determination of real marginal cost as a function of domestic output in the open economy differs somewhat from that in the closed economy, due to the existence of a wedge between output and consumption, and between domestic and consumer prices. Thus, in the present model,

$$\begin{aligned} mc_t &= -\nu + (w_t - p_{H,t}) - a_t \\ &= -\nu + (w_t - p_t) + (p_t - p_{H,t}) - a_t \\ &= -\nu + \sigma \; c_t + \varphi \; n_t + \alpha \; s_t - a_t \\ &= -\nu + \sigma \; y_t^* + \varphi \; y_t + s_t - (1+\varphi) \; a_t \end{aligned} \tag{34}$$

where the last equality makes use of (19) and (32). Thus, it can be seen that the marginal cost is increasing in the terms of trade and world output. Both variables end up influencing the real wage through the wealth effect on labor supply resulting from their impact on domestic consumption. In addition, changes in the terms of trade have a direct effect on the product wage for any given consumption wage. The influence of technology (through its direct effect on labor productivity) and of domestic output (through its effect on employment and, hence, the real wage for given output) is analogous to that observed in the closed economy.

Finally, using (29) to substitute for s_t, the previous expression for the real marginal cost in terms of domestic output and productivity, as well as world output, can be rewritten as

$$mc_t = -\nu + (\sigma_\alpha + \varphi) \;\; y_t + (\sigma - \sigma_\alpha) \;\; y_t^* - (1+\varphi) \; a_t. \tag{35}$$

Generally, in the open economy, a change in domestic output has an effect on marginal cost through its impact on employment (captured by φ) and the terms of trade (captured by σ_α, which is a function of the degree of openness and the substitutability between domestic and foreign goods). World output, on the other hand, affects marginal cost through its effect on consumption (and, hence, the real wage as captured by σ) and the terms of trade (captured by σ_α). Note that the sign of its impact on marginal cost is ambiguous. Under the assumption of $\Theta > 0$ (i.e., high substitutability among goods produced in different countries), $\sigma > \sigma_\alpha$, implying that an increase in world output raises the marginal cost. This is so because in that case the size of the real appreciation needed to absorb the change in relative supplies is small with its negative effects on marginal cost more than offset by the positive effect from a higher real wage. Notice that in the special cases $\alpha = 0$ and/or $\sigma = \eta = \gamma = 1$, which imply $\sigma = \sigma_\alpha$, the domestic real marginal cost is completely insulated from movements in foreign output.

How does the degree of openness affect the sensitivity of marginal cost and inflation to changes in domestic and world output? Note also that, under the same assumption of high substitutability ($\Theta > 0$) considered above, an increase in openness reduces the impact of a change in domestic output on marginal cost (and, hence, on inflation), for it lowers the size of the required adjustment in the terms of trade. By the same token, it raises the positive impact of a change in world output on marginal cost by limiting the size of the associated variation in the terms of trade and, hence, its countervailing effect.

Finally, and for future reference, note that under flexible prices, $mc_t = -\mu$ for all t. Thus, the natural level of output in the open economy is given by

$$y_t^n = \Gamma_0 + \Gamma_a \, a_t + \Gamma_* \, y_t^* \tag{36}$$

where $\Gamma_0 \equiv \frac{\upsilon - \mu}{\sigma_\alpha + \varphi}$, $\Gamma_a \equiv \frac{1+\varphi}{\sigma_\alpha + \varphi} > 0$, and $\Gamma_* \equiv -\frac{\alpha \Theta \, \sigma_\alpha}{\sigma_\alpha + \varphi}$. Note that the sign of the effect of world output on the domestic natural output is ambiguous, depending on the sign of the effect of the former on domestic marginal cost, which in turn depends on the relative importance of the terms of trade effect discussed above.

7.2.3 Equilibrium Dynamics: A Canonical Representation

In this section the linearized equilibrium dynamics for the small open economy is shown to have a representation in terms of output gap and domestic inflation analogous to its closed economy counterpart.

Let $\widetilde{y}_t \equiv y_t - y_t^n$ denote the domestic output gap. Given (35) and the fact that y_t^* is invariant to domestic developments, it follows that the domestic real marginal cost and the output gap are related according to

$$\widehat{mc}_t = (\sigma_\alpha + \varphi) \; \widetilde{y}_t.$$

Combining the previous expression with (33) the following version of the New Keynesian Phillips curve for the open economy can be derived

$$\pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa_\alpha \; \widetilde{y}_t \tag{37}$$

where $\kappa_\alpha \equiv \lambda \, (\sigma_\alpha + \varphi)$. Notice that for $\alpha = 0$ (or $\sigma = \eta = \gamma = 1$) the slope coefficient is given by $\lambda \, (\sigma + \varphi)$ as in the standard, closed economy New Keynesian Phillips curve. More generally, note that the form of the inflation equation for the open economy corresponds to that of the closed economy, at least as far as domestic inflation is concerned. The degree of openness α affects the dynamics of inflation only through its influence on the slope of the NKPC, i.e., the size of the inflation response to any given variation in the output gap. If $\Theta > 0$ (which obtains for "high" values of η and γ, i.e., under high substitutability of goods produced in different countries), an increase in openness lowers σ_α, dampening the real depreciation induced by an increase in domestic output and, as a result, the effect of the latter on marginal cost and inflation.

Using (30) it is straightforward to derive a version of the so-called dynamic IS equation for the open economy in terms of the output gap

$$\widetilde{y}_t = E_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma_\alpha}\,(i_t - E_t\{\pi_{H,t+1}\} - r_t^n) \tag{38}$$

where

$$r_t^n \equiv \rho - \sigma_\alpha \Gamma_a (1-\rho_a)\, a_t + \frac{\alpha \Theta \sigma_\alpha \varphi}{\sigma_\alpha + \varphi}\, E_t\{\Delta y_{t+1}^*\} \tag{39}$$

is the small open economy's natural rate of interest.

Thus, it is seen that the small open economy's equilibrium is characterized by a forward looking IS-type equation similar to that found in the closed economy. Two differences can be pointed out, however. First, as discussed above, the degree of openness influences the sensitivity of the output gap to interest rate changes. Second, openness generally makes the natural interest rate depend on expected world output growth, in addition to domestic productivity.

7.3 Equilibrium Dynamics under an Interest Rate Rule

Next, the equilibrium response of our small open economy to a variety of shocks is analyzed. In so doing, it is assumed that the monetary authority follows an interest rate rule of the form already assumed in chapter 3, namely

$$i_t = \rho + \phi_\pi\, \pi_{H,t} + \phi_y\, \widetilde{y}_t + v_t \tag{40}$$

where v_t is an exogenous component, and where ϕ_π and ϕ_y are non-negative coefficients chosen by the monetary authority.

Combining (37), (38), and (40), the equilibrium dynamics for the output gap and domestic inflation can be represented by means of the system of difference equations

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_{H,t} \end{bmatrix} = \mathbf{A}_\alpha \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_\alpha\,(\widehat{r}_t^n - v_t) \tag{41}$$

where $\widehat{r}_t^n \equiv r_t^n - \rho$, and

$$\mathbf{A}_\alpha \equiv \Omega_\alpha \begin{bmatrix} \sigma_\alpha & 1-\beta\phi_\pi \\ \sigma_\alpha \kappa_\alpha & \kappa_\alpha + \beta(\sigma_\alpha + \phi_y) \end{bmatrix};\ \mathbf{B}_T \equiv \Omega_\alpha \begin{bmatrix} 1 \\ \kappa_\alpha \end{bmatrix}$$

with $\Omega_\alpha \equiv \frac{1}{\sigma_\alpha + \phi_y + \kappa_\alpha \phi_\pi}$. Note that the previous system takes the same form as the one analyzed in chapter 3 for the closed economy, with the only difference lying in the fact that some of the coefficients are a function of the "open economy parameters" α, η, and γ, and that $\widehat{r}_t^n$ is now given by (39). In particular, the condition for a locally unique stationary equilibrium under rule (40) takes the same form as shown in chapter 3, namely

$$\kappa_\alpha\ (\phi_\pi - 1) + (1 - \beta)\ \phi_y > 0, \tag{42}$$

which is assumed to hold for the remainder of this section.

Section 7.3.1 uses the previous framework to examine the economy's response to an exogenous monetary policy shock, i.e., an exogenous change in v_t. Given the isomorphism with the closed economy model of chapter 4, many of the results derived there can be exploited.

The analysis of the effects of a technology shock (or a change in world output), which is not pursued below, goes along the same lines as in chapter 3. First, one should determine the implications of the shock considered for the natural interest rate $\widehat{r}_t^n$ and then proceed to solve for the equilibrium response of the output gap and domestic inflation exactly as done below for the case of a monetary policy shock, given the symmetry with which v_t and $\widehat{r}_t^n$ enter the equilibrium conditions.[11]

7.3.1 The Effects of a Monetary Policy Shock

Assume that the exogenous component of the interest rate v_t follows an AR(1) process

$$v_t = \rho_v\ v_{t-1} + \varepsilon_t^v$$

where $\rho_v \in [0, 1)$.

The natural rate of interest is not affected by a monetary policy shock so $\widehat{r}_t^n = 0$ for all t for the purposes of this exercise. As in chapter 3, let us guess that the solution takes the form $\widetilde{y}_t = \psi_{yv}\ v_t$ and $\pi_t = \psi_{\pi v}\ v_t$, where ψ_{yv} and $\psi_{\pi v}$ are coefficients to be determined. Imposing the guessed solution on (37) and (38) and using the method of undetermined coefficients,

$$\begin{aligned} y_t &= \widetilde{y}_t \\ &= -(1 - \beta\rho_v)\Lambda_v\ v_t \end{aligned}$$

and

$$\pi_{H,t} = -\ \kappa_\alpha \Lambda_v\ v_t$$

where $\Lambda_v \equiv \frac{1}{(1-\beta\rho_v)[\sigma_\alpha(1-\rho_v)+\phi_y]+\kappa_\alpha(\phi_\pi-\rho_v)}$. It can be easily shown that as long as (42) is satisfied, $\Lambda_v > 0$. Hence, as in the closed economy, an exogenous increase in the interest rate leads to a persistent decline in output and inflation. The size

[11] Of course, as in chapter 3, it must be taken into account that a technology shock or a shock to world output also leads to a variation in the natural output level, thus breaking the identity between output and the output gap.

of the effect of the shock relative to the closed economy benchmark depends on the values taken by a number of parameters. More specifically, if the degree of substitutability among goods produced in different countries is high (i.e., if η and γ are high, then $\omega > 1$) then Λ_v can be shown to be increasing in the degree of openness, thus implying that a given monetary policy shock will have a larger impact in the small open economy than in its closed economy counterpart.

Using interest rate rule (40) can determine the response of the nominal rate, taking into account the central bank's endogenous reaction to changes in inflation and the output gap

$$i_t = \left[1 - \Lambda_v(\phi_\pi \kappa_\alpha + \phi_y(1 - \beta\rho_v))\right] v_t.$$

Note that as in the closed economy model, the full response of the nominal rate may be positive or negative, depending on parameter values. The response of the real interest rate (expressed in terms of domestic goods) is given by

$$\begin{aligned} r_t &= i_t - E_t\{\pi_{H,t+1}\} \\ &= \left[1 - \Lambda_v((\phi_\pi - \rho_v)\kappa_\alpha + \phi_y(1 - \beta\rho_v))\right] v_t \end{aligned}$$

which can be shown to increase when v_t rises (because the term in square brackets is unambiguously positive).

Using (29) can uncover the response of the terms of trade to the monetary policy shock

$$\begin{aligned} s_t &= \sigma_\alpha y_t \\ &= -\sigma_\alpha(1 - \beta\rho_v)\Lambda_v \, v_t. \end{aligned}$$

The change in the nominal exchange rate is given in turn by

$$\begin{aligned} \Delta e_t &= \Delta s_t + \pi_{H,t} \\ &= -\sigma_\alpha(1 - \beta\rho_v)\Lambda_v \, \Delta v_t - \kappa_\alpha\Lambda_v \, v_t. \end{aligned}$$

Thus, a monetary policy contraction leads to an improvement in the terms of trade (i.e., a decrease in the relative price of foreign goods) and a nominal exchange rate appreciation.

Note that, in the long run, the terms of trade revert back to their original level in response to the monetary policy shock, while the (log) levels of both domestic prices and the nominal exchange rate experience a permanent change of size $-\frac{\kappa_\alpha\Lambda_v}{1-\rho_v}$ (given an initial shock of size normalized to unity).

Hence, the exchange rate will overshoot its long-run level in response to the monetary policy shock, if and only if,

$$\sigma_\alpha(1 - \beta\rho_v)(1 - \rho_v) > \kappa_\alpha\rho_v$$

which requires that the shock is not too persistent. It can be easily shown that the previous condition corresponds to that for an increase in the nominal interest rate in response to a positive v_t shock. Note that, in that case, the subsequent exchange rate depreciation required by the interest parity condition (20) leads to an initial overshooting.

7.4 Optimal Monetary Policy: A Special Case

This section derives and characterizes the optimal monetary policy for the small open economy described above, as well as the implications of that policy for a number of macroeconomic variables. The analysis, which follows closely that of Galí and Monacelli (2005), is restricted to a special case for which a second-order approximation to the welfare of the representative consumer can be easily derived analytically. Its conclusions should thus not be taken as applying to a more general environment. Instead, this exercise is presented as an illustration of the approach to optimal monetary design to an open economy.

Let us take as a benchmark the basic New Keynesian model developed in chapter 3. As discussed in that chapter, under the assumption of a constant employment subsidy τ that neutralizes the distortion associated with firms' market power, the optimal monetary policy is the one that replicates the flexible price equilibrium allocation. The intuition for that result is straightforward: With the subsidy in place, there is only one effective distortion left in the economy, namely, sticky prices. By stabilizing markups at their "frictionless" level, nominal rigidities cease to be binding, since firms do not feel any desire to adjust prices. By construction, the resulting equilibrium allocation is efficient, and the price level remains constant.

In an open economy—and as noted, among others, by Corsetti and Pesenti (2001)—there is an additional factor that distorts the incentives of the monetary authority beyond the presence of market power: the possibility of influencing the terms of trade in a way beneficial to domestic consumers. This possibility is a consequence of the imperfect substitutability between domestic and foreign goods, combined with sticky prices (that render monetary policy non-neutral). As shown below, and as discussed by Benigno and Benigno (2003) in the context of a two-country model, the introduction of an employment subsidy that exactly offsets the market power distortion is not sufficient to render the flexible price equilibrium allocation optimal, for, at the margin, the monetary authority would have an incentive to deviate from it to improve the terms of trade.

For the special parameter configuration $\sigma = \eta = \gamma = 1$ the employment subsidy that exactly offsets the combined effects of market power and the terms of trade distortions can be derived analytically, thus rendering the flexible price equilibrium allocation optimal. That result, in turn, rules out the existence of an average inflation (or deflation) bias and allows the focus on policies consistent with zero average inflation in a way analogous to the analysis for the closed economy found in chapter 4. Perhaps not surprisingly, and as shown below, the policy that maximizes welfare in that case requires that domestic inflation be fully stabilized, while allowing the nominal exchange rate (and, as a result, CPI inflation) to adjust as needed in order to replicate the response of the terms of trade that would be obtained under flexible prices.

One may wonder to what extent the optimality of strict domestic inflation targeting is specific to the special case considered here or whether it carries over to a more general case. The optimal policy analysis undertaken in Faia and Monacelli (2007), using a model nearly identical to the one considered here, suggests that while the optimal policy involves some variation in the domestic price level, the latter is almost negligible from a quantitative point of view, thus making strict domestic inflation targeting a good approximation to the optimal policy (or at least conditional on the productivity shocks considered here). Using a different approach, de Paoli (2006) reaches a similar conclusion, except when an (implausibly) high elasticity of substitution is assumed.[12] But even in the latter case, the losses that arise from following a domestic inflation targeting policy are negligible.[13] More generally, it is clear that there are several channels in the open economy that may potentially render a strict domestic inflation policy suboptimal, including a nonunitary elasticity of substitution, local currency pricing, incomplete financial markets, and so on, all of which are unrelated to the sources of policy tradeoffs that may potentially arise in the closed economy. The quantitative significance of the effects of those channels (individually or jointly) still needs to be explored in the literature, and its analysis is clearly beyond the scope of this chapter.

With that consideration in mind, let us next turn to the analysis of the optimal policy in the special case mentioned above.

7.4.1 The Efficient Allocation and Its Decentralization

Let us first characterize the optimal allocation from the viewpoint of a social planner facing the same resource constraints to which the small open economy is subject in equilibrium (in relation to the rest of the world), given the assumption of complete markets. In that case, the optimal allocation must maximize $U(C_t, N_t)$ subject to (i) the technological constraint $Y_t = A_t N_t$, (ii) a consumption/output possibilities set implicit in the international risk-sharing conditions (18), and (iii) the market clearing condition (25).

[12] Those results are conditional on productivity shocks being the driving force. Not surprisingly, in the presence of cost-push shocks of the kind considered in chapter 5, stabilizing domestic inflation is not optimal (as in the closed economy).

[13] In solving the optimal policy problem for the general case, de Paoli (2006) adopts the linear–quadratic approach originally developed in Benigno and Woodford (2005), which replaces the linear terms in the approximation to the households' welfare losses using a second-order approximation to the equilibrium conditions. Faia and Monacelli (2007) solve for the Ramsey policy using the original nonlinear equilibrium conditions as constraints of the policy problem.

Furthermore, the clearing of the labor market in steady state implies

$$C^{\sigma}\left(\frac{Y}{A}\right)^{\varphi} = \frac{W}{P}$$
$$= A\,\frac{1-\frac{1}{\varepsilon}}{(1-\tau)}\,\frac{P_H}{P}$$
$$= A\,\frac{1-\frac{1}{\varepsilon}}{(1-\tau)}\,\frac{1}{h(\mathcal{S})}$$

which, when combined with the sharing condition above, yields

$$Y = A^{\frac{1+\varphi}{\varphi}}\left(\frac{1-\frac{1}{\varepsilon}}{(1-\tau)\,(Y^{*})^{\sigma}\,\mathcal{S}}\right)^{\frac{1}{\varphi}}. \tag{47}$$

Notice that, conditional on A and Y^{*}, (46) and (47) constitute a system of two equations in Y and S with a unique solution given by

$$Y = Y^{*} = A^{\frac{1+\varphi}{\sigma+\varphi}}\left(\frac{1-\frac{1}{\varepsilon}}{1-\tau}\right)^{\frac{1}{\sigma+\varphi}}$$

and

$$\mathcal{S} = 1$$

which in turn must imply $S_i = 1$ for all i.

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