Testable Implications of a Simple Moral Hazard Model of Mutual Insurance

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June 2003

Abstract: This paper examines the nonparametric testable implications of a simple moral hazard model of mutual insurance. We demonstrate that if the data only contains information related to agents' optimal effort choice then the testable implications are minimal. The budget constraint, fair prices and consumption smoothing are the only testable implications of the model. With additional information associated with the off-equilibrium effort level, there are data sets where these conditions are satisfied but the data cannot be rationalized by our model. Under both assumptions on the data, if the data can be rationalized then we construct a rationalizing utility function.

Key words: Insurance, moral hazard, non-parametric tests, testable implications.

1 Introduction

In this paper, we derive the testable implications of a simple moral hazard model of mutual insurance in a general equilibrium economy. Moreover, we characterize the utility functions that rationalize a data set consistent with the moral hazard model. The model described in this paper (suggested by Gottardi (1998)) is a simplified version of general equilibrium models with asymmetric information for example Prescott and Townsend (1984), Bennardo and Chiappori (1998).

Moral hazard refers to asymmetric information economies in which an agent can choose costly private actions that affect the probability of suffering a loss. These models are widely used in applied areas of microeconomics (see Stadler and Castrillo (1999), Chiappori (1998)). In most applications, the utility functions satisfy some parametric restrictions. This joint hypothesis does not allow an independent test of the moral hazard model. In this paper, our analysis is nonparametric, hence eliminating the specification error implicit in assuming that utility functions are members of some parametric family. Afriat (1967) initiated the method of constructing non-parametric tests for utility maximization. Afriat's theorem shows the equivalence of the existence of a concave monotonic continuous utility function that rationalizes the data, an axiom of revealed preference (a condition on observed variables only), and the existence of a solution to a system of linear inequalities (with utilities and marginal utilities as unknowns). Varian (1982,1983) takes this approach to derive nonparametric tests of consumer behavior under different assumptions (separability, homotheticity, the expected utility model). Brown and Mazkin (1996) derive nonparametric testable restrictions on the equilibrium manifold in a pure trade economy. They introduce the use of the Tarski-Seidenberg theorem and quantifier elimination to derive testable restrictions of the model. Snyder (1999) follows Brown and Matzkin's method to examine the testable restrictions of Pareto optimality in a public goods economy

In this paper, we assume that we observe a finite data set consisting of n observations. Each observation includes the variables: prices, consumption, income and probabilities. These observed values are assumed to be the values for the infinity of identical agents in every observed economy. We look for restrictions that will enable us to test the hypothesis that the data is consistent with the model. By assumption, the infinity of agents are identical across observations in the sense that they have the same Bernoulli utility function and the same effort costs in terms of utility. However, observations may differ in the possible values wealth can take, in the probabilities, and therefore also in the optimal choices of effort and consumption. The data may consist of cross sectional observations, for example different states in the United States. The Bernoulli utility function is the same for agents in all states but the risk differs across states.

One problem that arises when seeking to test moral hazard models is that if agents choose high effort, we may only observe the economic consequences associated with this effort choice. The data may contain no information about what would have happened if the agent had chosen the low effort. For example when agents choose high effort we may learn from the data the probability of outcomes contingent on high effort, but not the probability distribution contingent on low effort. We first explore the case where the economist can only observe variables related to the effort that agents optimally choose. Then we turn to the case where the economist also knows the probability distribution that corresponds to the unchosen low effort level.

When the data consists of variables relating only to the optimal effort level that the agents actually chose, that is endowment consumption bundles and probabilities and insurance prices associated with agents' optimal choices of effort, the nonparametric testable restrictions on the data are very weak. When the optimal choice is high effort, the budget constraint, fair prices and consumption smoothing are the only testable restrictions that can be derived from the moral hazard model. Hence, it is difficult to reject the model. Given a data set that satisfies these restrictions we show existence of a strictly concave utility function (a CRRA utility) that rationalizes the data.

To show that a data set is consistent with the model given a particular concave utility function, we need to find values for the unobserved effort costs and the low effort probabilities so that all the conditions for an equilibrium in the model are satisfied for all the observations. In an equilibrium where agents choose high effort the incentive compatibility constraint binds. Equilibrium consumption is therefore the point of intersection of the binding incentive compatibility constraint and the budget constraint. The budget constraint is observed and the observed consumption point is assumed to lie on it. The incentive compatibility constraint on the other hand, depends on unobserved utility function effort costs and low effort probabilities. The data can be rationalized if the unobserved can be chosen to make the incentive compatibility curve pass exactly through the observed consumption bundle. When the probability associated with low effort is not observed, this free parameter can be used to move the incentive compatibility curve to make it intersect with the budget line exactly in the observed consumption bundle

To be a little more precise, given data, if we pick some concave Bernoulli utility function, we are left with the difference between effort costs and the low effort probability as free variables. In order to rationalize the data with the utility we picked, we need to choose effort costs and probabilities such that the consumption bundles will solve the incentive compatibility constraint with equality, and choosing high effort will be better for the agent than choosing low effort and fully insuring herself. The incentive compatibility constraint and the condition for optimal choice of effort level imply a lower bound and an upper bound respectively on the probability of the good state contingent on low effort in every observation. Combining these bounds gives a necessary and sufficient condition for a concave utility function to rationalize a given data set.

The risk neutral utility function can rationalize any data satisfying the budget constraint, fair prices and the smoothing condition. The intuition behind this is that a risk neutral agent cares only about her expected consumption. Therefore, when the cost of high effort is not too large, high effort is preferred since it allows the agent to enjoy a higher expected consumption. Finding effort costs and probabilities so that the data is consistent with the model when the utility is risk neutral is therefore always possible. The risk neutral utility function is a special case of the family of CRRA utility functions. It satisfies the necessary and sufficient condition that we derive for a utility function to rationalize the data. We show that if the coefficient of relative risk aversion is small enough (the agent is not too risk averse) then CRRA utility also satisfies these necessary and sufficient conditions so that the utility function rationalizes the data as high effort moral hazard equilibria.

Surprisingly, if in addition to the observations with consumption smoothing, we have observations with full insurance then we can still find a utility function and effort costs to rationalize the data with the moral hazard model. Hence, the additional observations do not imply additional testable restrictions. Consider a data set with n consumption smoothing observations that satisfy the budget constraint and m observations with full insurance that also satisfy the budget constraint. We can interpret such data as generated by our model treating the n consumption smoothing observations as high effort moral hazard equilibria and the m full insurance observation as equilibria in which agents chose low effort and a full insurance contract. We show that any utility function that rationalizes the first n consumption smoothing observations also rationalizes the entire data set. To rationalize the data we first rationalize the high effort observations. Then for full insurance observations we choose the unobserved high effort probabilities in a way that will make the endowment bundle be the optimal choice of a high effort agent and this will always be worse than low effort and full insurance.

We also look for testable implications of the model when we assume the economist has information about the probabilities associated with both levels of effort (or the corresponding prices) in addition to the consumption bundles and endowments. Even if we have data consisting of high effort observations, we may be able to observe the probability of a good state if low effort were chosen. This information can be determined from past periods in which high effort was either not available or too costly so that the agents employed low effort. Or, it may be that people know the consequence of low effort, for example it may be clear that with low effort a bad state always occurs. Or it can be that the insurers know the low effort probabilities from research or experiments and this information is inferred by the economist from prices in an insurance contract that is available but not purchased.

When the data contains information about the probabilities associated with both effort levels the model is shown to be testable in the sense that we derive a set of necessary and sufficient linear inequalities that the data must satisfy to be consistent with the moral hazard model. In this case the budget constraint, fair prices and consumption smoothing are no longer sufficient for the data to be consistent with the model, stronger restrictions on the observable variables must hold in order for the data to be rationalized by some concave utility function. In this case where both probabilities are observed, the Bernoulli utility function and effort costs are the only unobserved. Since these need to be the same for all observations, we lose the freedom (that we have if low effort probability is unobserved) to move each incentive compatibility constraint to pass through the different consumptions bundles. In some data sets, the different probabilities and consumptions in different observations may not allow for a choice of one common utility function that generates incentive compatibility constraints that pass through the corresponding consumption points. There are data sets inconsistent with the model hence it is testable.

When the data can be rationalized, the rationalizing concave utility function can be constructed as in Afriat (1967), but we can no longer guaranty that there is a CRRA utility function that rationalizes the data. When we can observe the probabilities associated with both effort levels, it is possible to find data sets that satisfy the budget constraint, fair prices and the smoothing condition but for which it is impossible to find effort costs such that the data satisfy the incentive compatibility constraint.

Section 1 describes the moral hazard model and gives some previously known results about the equilibrium of the model. In part A of section 2 we derive the testable implications of the model when the data contain information related to the optimal effort choice, and we characterize the utility functions that rationalize a specific data set. In part B we consider data sets that contain information associated with both effort levels. We derive the linear equilibrium inequalities. We show that the system is nontrivial and that the restrictions are stronger than the ones derived earlier.

2 Model

In the model, there are infinitely many identical agents. There are two time periods $t = 0, 1$. There is one consumption good consumed at $t = 1$. Agents face risk, each agent's endowment (or wealth) is a random variable W that is realized at $t = 1$. Wealth can take one of two possible values $W_G > W_B > 0$. The difference $W_G - W_B$ can be thought of as a possible loss. For example, the bad state might be one where an accident happens to the agent. The random endowments of all agents are independent and identically distributed (IID). Since we have identical agents and we focus on symmetric equilibria, we drop the agent index in order to keep notation simple.

An agent can effect the distribution of wealth (or her chance to suffer a loss) by her choice of effort level. There are two effort levels available for each agent, $e = H$ is the high effort and $e = L$ is the low effort. Agents choose effort level at time $t = 0$. High effort can be interpreted as being more careful. The effort is costly to the agents in terms of utility. The costs of high effort and low effort are denoted V_H and V_L respectively. High effort costs more, $V_H > V_L$. The probability distribution for W depends on the agent's effort. Wealth W takes the large (good) value W_G with probability Π_e ($e = L$ or H), and the small (bad) value W_B with probability $1 - \Pi_e$. There is a higher probability for the good state if the agent chooses high effort, $1 > \Pi_H > \Pi_L > 0$.

This is a model of mutual insurance where an agent can insure by buying securities that pay a unit of consumption contingent on the realization of her random wealth, W. Assets are sold by an insurance company that serves as an intermediary. Every agent can trade with the insurance company only in her own securities. Insurance is assumed to be fair and the insurance market clears. Contracts are exclusive, the agents trade with one intermediary who observes their trades. Each Agent's preferences are represented by a Von Neumann-Morgenstern expected utility function separable in wealth and effort costs, $E[u(x)|e] - V_e$, where $u(x)$ is a concave increasing Bernoulli utility function. The expectation and the cost of effort V_e depend on the effort level e.

We will now characterize equilibrium in the model. The results in this section are well known from previous analysis (e.g.Bennardo and Chiappori (1998) and Gottardi (1998)), and included in this paper since our analysis is based on these results. Lemma 1 summarizes the characterization of equilibrium as a system of polynomial inequalities. This format will be most helpful for the study of testable implications in section 2.

First we consider the symmetric information case where the effort, e is observed.

Let P_{Ge} be the price at $t = 0$ of the asset delivering one unit of good in state G, if the effort level is e. And P_{Be} be the price at $t = 0$ of the asset delivering one unit of good in state B, if the effort level is e.

The agent's choice problem is:

$\max_{\{x,e\}} E[u(x)|e] - V_e$

Subject to:

$$X_G > 0, X_B > 0 \tag{1}$$

Budget constraint:

$$P_{Ge}[X_G - W_G] + P_{Be}[X_B - W_B] = 0 \tag{2}$$

The market clearing condition is:

$$\Pi_e[X_G - W_G] + [1 - \Pi_e][X_B - W_B] = 0 \tag{3}$$

This is the market clearing condition since by the law of large numbers, the fraction of individuals that get a good realization of W is Π_e, when all agents choose effort level e. Note that each agent trades her own securities, that is, the assets an agent buys will deliver a unit of good contingent on the realization of this agent's random endowment and not on other agents. Agents cannot trade in securities contingent on someone else's realization of endowments. The market clearing condition is written in expectation however, since there are an infinite number of agents, markets exactly clear, meaning that the insurance company will have zero profit.

If the parameters of the model are such that:

$$u(E(W|H)) - V_H > u(E(W|L) - V_L$$

the competitive equilibrium of the complete information model is:

$$X_G^* = X_B^* = E(W|H)$$

$$e^* = H$$

$$P_{Ge} = \Pi_e, P_{Be} = 1 - \Pi_e$$

Hence, in the complete information case there is full insurance, a Pareto efficient outcome.

Now consider the asymmetric information case where the choice of effort is unobserved. The insurer needs to impose the incentive compatibility constraints in order to be able to condition prices on the unobserved effort. Agents can buy assets at prices P_{Ge} and P_{Be} only if the amounts of assets they demand satisfy the incentive compatibility constraint for effort level e. According to this model agents choose an effort level H or L and consumption levels X_G and X_B (or purchases of state contingent assets) to maximize expected utility subject to an additional constraint, the incentive compatibility constraint:

$$\Pi_e U(X_G) + [1 - \Pi_e] U(X_B) - V_e \geq \Pi_{e'} U(X_G) + [1 - \Pi_{e'}] U(X_B) - Ve' \ (e \neq e\prime) \tag{4}$$

To solve this problem the agent can find the optimal consumption bundle contingent on her choosing effort level H, (X_{GH}, X_{BH}) and the optimal bundle contingent on her choosing effort level L, $(X_{GL},\ X_{BL})$. Then she chooses the effort level that gives her highest utility, and the corresponding optimal bundle.

Definition 1. Given probabilities, $1 \geq \Pi_H > \Pi_L \geq 0$, endowments, $W_G > W_B > 0$ and costs of effort, $V_H > V_L$,

$$\{(P_{GH}, P_{BH}), (P_{GL}, P_{BL}), (X_{GH}, X_{BH}), (X_{GL}, X_{BL})\} \in R^8_+$$

is a high effort moral hazard equilibrium if:

1. For $e = L,\ H$ (X_{Ge}, X_{Be}) solves the agent's choice problem, the maximization of his expected utility subject to non-negativity, the budget and the incentive compatibility constraint.

2. High effort is preferred to low effort:

$$\Pi_H u(X_{GH}) + [1 - \Pi_H]u(X_{BH}) - V_H \geq \Pi_L u(X_{GL}) + [1 - \Pi_L]u(X_{BL}) - V_L$$

3. The market clearing condition (3) for $e = H, L$.

Similarly, we define a low effort equilibrium by changing the direction of the inequality (5).

Let us characterize the solution to the agent's problem for each effort level in the case

$$(P_{GH}, P_{BH}) \propto (\Pi_H, 1 - \Pi_H) \text{ and } (P_{GL}, P_{BL}) \propto (\Pi_L, 1 - \Pi_L)$$

(prices proportional to the probabilities). For the low effort level, L, the solution to the agent's choice problem for $e = L$ is the full insurance point

$$X_{GL} = X_{BL} = \Pi_L W_G + [1 - \Pi_L]W_B$$

(Point F in figure 1). The reason being that this point is feasible for the agent and being risk averse she prefers it to any other point on her budget line. For the high effort level, we will see that the solution to the agent's choice problem is the point of intersection between the high effort budget line and the incentive compatibility constraint when solved with equality (point X^* in figure1). The pairs $(X_G,\ X_B)$ in R^2_+ can be partitioned to a set for which the incentive compatibility constraint with $e = H$ holds (the shaded area), a set for which incentive compatibility with $e = L$ holds, and a set (curve) for which incentive compatibility holds with equality. (See figure1).

The budget lines are decreasing. The curve denoted in figure 1 with IC is the collection of points for which the incentive compatibility constraint holds with equality. On this curve, X_G is implicitly defined as a function of X_B by the incentive compatibility when solved with equality. This is an increasing function (not necessarily convex). All points on the 45^o line are such that low effort is preferred to high effort since when $X_G = X_B$ incentive compatibility reduces to $-V_H < -V_L$. If the set of feasible points in the $e = H$ problem is non-empty then there is a unique intersection of IC and the budget line. All points above IC are such that high effort is preferred. Since the agent has an increasing utility function, her optimal choice must be on the budget line. The point at which IC intersects the budget line second order stochastically dominates any other feasible point on the budget line and therefore it is the optimal choice for the agent.

An equilibrium in this model exists. Let $(P_{GH}, P_{BH}) = (\Pi_H, 1 - \Pi_H)$ and $(P_{GL}, P_{BL}) = (\Pi_L, 1-\Pi_L)$. If the point of intersection of IC and the budget line, X^* is preferred to the full insurance low effort level point, F, then there is a high effort moral hazard equilibrium (agents all choose high effort and consume X^*). Otherwise, there is a low effort equilibrium with full insurance consumption. The equilibrium is constrained Pareto optimal (under incentive compatibility constraint).

Lemma 1 *1. Given a strictly concave utility function , effort costs $V_H > V_L$, endowments $W_G > W_B > 0$ and probabilities $\Pi_H > \Pi_L$,*

$$\{(X_{GH}, X_{BH}), (X_{GL}, X_{BL}), (P_{GH}, P_{BH}), (P_{GL}, P_{BL})\} \in R^8_+$$

is a high effort moral hazard equilibrium if and only if the following conditions are satisfied:

C1. $\Pi_H = \frac{P_{GH}}{(P_{GH}+P_{BH})}$ *and* $\Pi_L = \frac{P_{GL}}{(P_{GL}+P_{BL})}$
C2. $X_{GL} = X_{BL} = \Pi_L W_G + (1 - \Pi_L) W_B$
C3. $P_{GH}(X_{GH} - W_G) + P_{BH}(X_{BH} - W_B) = 0$
C4. $V_H - V_L = (\Pi_H - \Pi_L)[u(X_{GH}) - u(X_{BH})]$
C5. $\Pi_L u(X_G H) + (1 - \Pi_L) u(X_B H) - u(\Pi_L W_G + (1 - \Pi_L) W_B) > 0$

2. These conditions imply consumption smoothing,

$$W^i_G > X^i_{GH} > X^i_{BH} > W^i_B \tag{5}$$

The first condition means that prices are actuarially fair. The second condition states that agents fully insure if they choose low effort. The third condition is the budget constraint. The fourth condition is the incentive compatibility constraint which holds with equality in equilibrium. The fifth condition states that high effort is preferred to low effort. The proof is straightforward.

3 Testable Implications

We now derive the testable implications of the model. Assume that we observe a finite data set, index the observations $i = 1..n$. We look for restrictions on the observed data that will enable us to test the hypothesis that the data is consistent with the model. The data is consistent with the model (we can rationalize the data) when there exists a concave utility function and effort costs (common to all observations) such that the observed data can be explained as equilibria of the model.

3.0.1 Implications with Data Associated with the Optimal Action

In this section, we will consider data sets that only contain information about the actions taken by agents. Suppose we have a data set consisting of n observations of consumption bundles, endowments, high effort probabilities and prices of the state contingent securities that agents face. Every economy from which we collect data has a large number of identical agents. There will be a fraction of agents who had a good realization of wealth and a fraction of agents who had a bad realization of wealth even though all agents chose the same effort level. Therefore, we are able to observe wealth and consumption in both good and bad states. We ask whether we can explain the data that we observe with the moral hazard model described in the previous section.

Definition 2 *We say that a concave increasing Bernoulli utility function $u(x)$ rationalizes the data*

$$\{(X^i_{GH}, X^i_{BH}), (W^i_G, W^i_B), (P^i_{GH}, P^i_{BH}), (\Pi^i_H)\} \in R^6_+, i = 1 \ldots n,$$

where $X^i_{GH} > X^i_{BH}$ and $W^i_G > W^i_B > 0$, as high effort moral hazard equilibria in the model if there exist probabilities of the good state (depending on the effort level L) Π^i_L such that $1 \geq \Pi^i_H > \Pi^i_L > 0$, and effort costs $V_H > V_L$ (independent of the i) so that

$$\begin{aligned}\{(X^i_{GH}, X^i_{BH}), (X^i_{GL} &= X^i_{BL} = \Pi^i_L W^i_G + (1 - \Pi^i_L) W^i_B), \\ (P^i_{GH}, P^i_{BH}), (P^i_{GL} &= \Pi^i_L, P^i_{BL} = (1 - \Pi^i_L))\}\end{aligned}$$

is a high effort moral hazard equilibrium for all i, for agents with utility function $u(x)$ and costs of effort costs V_H,V_L. We say that we can rationalize a data set if there exists a concave increasing utility function that rationalizes the data.

1 provides the testable implications result given the assumptions we maid on the observed data.

Theorem 3 *Given a data set $\{(X^i_{GH}, X^i_{BH}), (W^i_G, W^i_B), (P^i_{GH}, P^i_{BH}), (\Pi^i_H)\}$ $i = 1 \ldots n$ where $P^i_{GH} > 0$, $P^i_{BH} > 0$, $W^i_G > W^i_B \geq 0$, $X^i_{GH} > X^i_{BH} > 0$ the following conditions are equivalent:*

I. There exists a strictly concave, increasing, smooth utility function (from the family of CRRA) that rationalizes the data.

II. The data satisfies for all $i = 1 \ldots n$:

1. Budget constraint (2)

2. Fair prices:

$$\Pi^i_H = \frac{P^i_{GH}}{(P^i_{GH} + P^i_{BH})} \tag{6}$$

3. Consumption smoothing (6)

This theorem shows that the testable implications of the moral hazard model are minimal if we can only observe information related to the optimal effort choice. Any data set where prices are fair and where consumption bundles lie on the budget line and exhibit consumption smoothing can be rationalized with the model. Moreover, we can also rationalize it with a strictly concave smooth utility function. In fact, the proof shows that there is continuum of functions from the CRRA family with sufficiently low coefficients of relative risk aversion that rationalize the data. Note that each restriction in 2 depends on one observation only thus increasing a data set with more observations that satisfy these restrictions does not change the testability of the data at hand. However, it limits our choice of utility functions that can rationalize the data. Looking at the CRRA family, the more observations we have, the lower the coefficient of relative risk aversion must be in order to rationalize the data.

To prove this theorem we use two propositions. Proposition 1 gives a necessary and sufficient condition for a strictly concave utility function to rationalize the data. The result of proposition 1 is itself important, it allows us to verify for any given data set whether a particular utility function rationalizes the data. In proposition 2, for data sets where consumption bundles lie on the budget line and exhibit consumption smoothing, we show there exists a strictly concave CRRA utility function that rationalizes the data.

Proposition 1. Given a data set $\{(X^i_{GH}, X^i_{BH}), (W^i_G, W^i_B), (P^i_{GH}, P^i_{BH}), (\Pi^i_H)\}$ $i = 1 \dots n$ where

$P^i_{GH} > 0$, $P^i_{BH} > 0$, fair prices (7), consumption smoothing (6), and the budget constraint (2) hold a necessary and sufficient condition for a strictly concave Bernoulli utility function to rationalize the data as high effort moral hazard equilibria is that for every $i = 1 \dots n$,

$$\Pi^i_H - \min_j \{ \frac{\Pi^j_H [u(X^j_{GH}) - u(X^j_{BH})]}{[u(X^i_{GH}) - u(X^i_{BH})]} \} \leq \Pi^{i^*},$$

where Π^{i^*} is defined as the largest zero of the convex function:

$$f^i(\Pi) = \Pi u(X^i_{GH}) + (1 - \Pi) u(X^i_{BH}) - u(\Pi W^i_G + (1 - \Pi) W^i_B)$$

in the range $[0, \Pi^i_H]$.

Before proving this proposition let us consider the case of a linear utility function $u(x) = x$. Each observation in the data set consists of prices proportional to probabilities, endowment, W and consumption bundle, X that lies on the budget line to the right of W (see figure 2). The low effort budget must be steeper than the given high effort budget and pass through the endowment point W. The indifference curves are strait lines parallel to the budget lines. Incentive compatibility equality constraint in this case is a linear function,

$$V_H - V_L = (\Pi_H - \Pi_L)(X_{GH} - X_{BH}).$$

In the (X_B, X_G) plain, IC is parallel to the 45^o line. Effort levels and low effort probabilities are unobserved. All we need in order to rationalize the data as high effort moral hazard equilibria is to choose effort levels (the same for all observations) and low effort probabilities (or the slopes of the low effort budget lines), such that for every observation the IC line passes through the point X. If we can do so, we are guaranteed both that X is the optimal consumption in case of high effort and that the agent prefers high effort to low effort. The later is true since by incentive compatibility the low effort and the high effort indifference curves at X have the same utility value while the low effort indifference curve through W (the utility if low effort is chosen) gives lower utility. We show later that we can always find effort costs and low effort probabilities such that the resulting IC line will pass through X.

With non linear utility function, in order to rationalize the data we need to worry not only about IC passing through the point X, (high effort equilibrium consumption) but also that X will be preferred to F (low effort and full insurance). In order to rationalize the data we need to find effort levels and probabilities, Π_L^i, such that, for every observation both "high effort is preferred to low effort" and the incentive compatibility conditions are satisfied. We will see that the first condition implies an upper bound on Π_L^i. Intuitively if Π_L^i is close to Π_H^i then it is not worthwhile to pay the cost of high effort, and so low effort will be preferred. The second condition implies a lower bound on Π_L^i. We can rationalize the data if and only if for all i, the set of all possible Π_L^i is not empty (namely the upper bound is larger than the lower bound). This is the condition stated in the proposition. We derive this condition formally in the proof which is provided in the appendix.

In lemma 2 we show that the risk neutral utility function satisfies the conditions in proposition 1 and therefore rationalizes the data.

Lemma 4 *The risk neutral utility function, $u(x) = x$, rationalizes any data that satisfies the budget constraint and the smoothing condition.*

Proof. Proof. When $u(x) = x$,

$$f(\Pi_H^i) = \Pi_H^i X_{GH}^i + (1 - \Pi_H^i) X_{BH}^i - (\Pi_H^i W_G^i + (1 - \Pi_H^i) W_B^i) = 0$$

by the budget constraint. Therefore $i^* = \Pi_H^i$ and the sufficient condition from proposition 1 is satisfied. ■

Now that we have established that the risk neutral utility function rationalizes the data, we will use this to find a strictly concave utility function. The idea is that if the risk neutral utility function rationalizes the data, we must be able to find some strictly concave utility "sufficiently close" to the risk neutral utility function that will also rationalize the data. To formalize "sufficiently close" We will take a parameterized family of utility functions from which the risk neutral utility function is obtained for some value of the parameter. In particular we choose to work with the widely used CRRA utility functions family.

Let us look at the family

$$u(\gamma, x) = \frac{x^{1-\gamma^2}}{1-\gamma^2}$$

of CRRA utility functions. With this representation γ^2 is the coefficient of relative risk aversion. This family is well defined for all $\gamma \in (-1, 1)$. When

$\gamma = 0$ $u(0,x) = x$, the risk neutral utility. When $\gamma \neq 0$ $u(\gamma, x)$ is strictly concave and monotonically increasing. For this family of functions we show in lemma 3 in the appendix that $\Pi^*(\gamma)$ is continuous. Then we use the fact that the risk neutral utility rationalizes the data to find a continuum of functions with low enough coefficients of relative risk aversion that also rationalize the data. Intuitively, we need a low coefficient of relative risk aversion since the more risk averse the agent is the more likely she is to prefer low effort level together with a full insurance contract rather than the high effort partial insurance contract.

Proposition 5 *For all data that satisfy the budget constraint, fair prices and the smoothing condition there exist strictly concave CRRA utility functions that rationalize the data.*

The proof of proposition 2 is provided in the appendix. So far, we have focused on data with consumption smoothing but not full insurance. We have rationalized the data as high effort moral hazard equilibria. Consider now a data set with n consumption smoothing observations $W_G^i > X_{GH}^i > X_{BH}^i > W_B^i$ that satisfy the budget constraint and m observations with full insurance $X_{GL}^j = X_{BL}^j$ that satisfy the budget constraint. Theorem 2 shows that any utility function and effort costs that rationalize the first n observations will rationalize the $n+m$ observations. Thus, the additional observations add no further testable restrictions. While we explain the first n observations as equilibria where high effort is the optimal choice, we may explain the additional m full insurance observations as resulting form a choice of low effort.

Theorem 6 *Given a data set* $\{(X_G^i, X_B^i), (W_G^i, W_B^i), (P_G^i, P_B^i), (\Pi^i)\}$ $i = 1 \ldots n + m$ *where* $P_G^i > 0$, $P_B^i \geq 0$, $\Pi^i = \frac{P_G^i}{P_G^i + P_B^i}$, $W_G^i > X_G^i > X_B^i > W_B^i > 0$ *for* $i = 1 ... n$ *and* $W_G^i > X_G^i = X_B^i > W_B^i > 0$ *for* $i = (n+1)...m$ *,that satisfies the budget constraint (2). Any strictly concave* $u(x)$ *that rationalizes the first n observations as high effort moral hazard equilibria rationalizes the given* $n+m$ *observations according to the model with the first n observations being high effort equilibria and the last* m *observations being low effort equilibria.*

The condition for $u(x)$ to rationalize the data is as in proposition 1 and the existence of such utility function follows from proposition 2. The proof is provided in the appendix.

have assumed in the analysis above that we can observe every possible variable related with the equilibrium effort choice, that is consumption bundles, endowments, prices and the probabilities of good and bad states contingent on the equilibrium effort choice. What if we observe less? If we only observe either market prices or the probabilities and not both, we use the result that prices must be proportional to the probabilities in order to determine what the unobserved variable must be. The results of this section still hold, only fair prices are no longer a testable restriction. If neither the probabilities nor the prices are observed but endowments are different than consumption bundles, prices and probabilities can be inferred from the need to satisfy the budget constraint, and this restriction would no longer be a testable implication, leaving consumption smoothing as the only testable implication. If we do not observe endowments or consumption but observe the rest of the variables, we are left with fair prices as the only implication since given one of these variables we would be able to pick the other in a way that satisfies the budget constraint and the smoothing condition. The results in this section suggest that we cannot hope for substantial testable implications unless we observed variables related with the off equilibrium effort choice. We discuss the implications of the model under this assumption in the next section.

3.1 Implications with Data on Probabilities Associated Both Effort Levels

A more restrictive set of assumptions consists of observing the probabilities of the states of the world (or asset prices) for both efforts (not only the optimal effort that agents choose). Even if the data consists of high effort observations, we may be able to observe the probability of a good state if low effort were to be used. It is possible to rationalize a data set when we can choose the unobserved utility function and effort costs in such a way that every incentive compatibility constraint, for every observation, passes exactly through the consumption point that lies on the budget line. Unlike the previous case, where the low effort probability was a free variable that could be used to move the incentive compatibility constraints to fit the different consumptions, in the case both probabilities are observed, we lose this freedom. The utility and effort costs need to be the same for all observations. It is no longer always possible to move the incentive compatibility constraint so that it will adjust to the different observations.

To find testable restrictions we follow the approach introduced in Brown and Matzkin (1996). The first step is to find a set of equilibrium inequalities, where the unknown values (utility levels and effort costs) are variables and the data are the coefficients, such that the data can be rationalized if and only if there is a solution to these inequalities. Then we appeal to the Tarski-Seidenberg theorem to prove that the equilibrium inequalities can be reduced to an equivalent finite family of polynomial inequalities in the coefficients of the system. This family contains all of the testable implications of the model. In general, the Tarski-Seidenberg algorithm does not terminate in polynomial time. We can prove that this system of inequalities in the data is nontrivial namely it is satisfied neither by all data sets nor by no data set. We show this by constructing an example where the data set is rationalized (there is a solution to the system) and an example of a data set that cannot be rationalized (there is no solution to the system). Further, we note that our family of inequalities is linear given a particular data set. Thus, checking testability amounts to solving a finite system of linear inequalities. This can be done in polynomial time. Theorem 3 derives the system of linear inequalities and shows the testability of the model.

Theorem 7 *Theorem 3. Given a data set* $\{(X^i_{GH}, X^i_{BH}), (W^i_G, W^i_B), (P^i_{GH}, P^i_{BH}), (\Pi^i_H)(\Pi^i_L)\}$

$i = 1 \ldots n$ *where* $0 \leq \Pi_L^i < \Pi_H^i = \frac{P_{GH}^i}{P_{GH}^i + P_{BH}^i}, X_{GH}^i > X_{BH}^i \geq 0$ *and* $W_G^i > W_B^i > 0$ *and* $P_{GH}^i > P_{BH}^i \geq 0$.

a. We can rationalize the data as high effort moral hazard equilibria with a strictly concave utility function if and only if there is a solution to the linear equilibrium inequalities:$\{U_{se}^i\}$ $i = 1 \ldots n$,■$\{M_{se}^i > 0\}$ $i = 1 \ldots n$ ■$V_H > V_L \geq 0$ *such that for all i the following conditions hold:*

C1-C3

C4'. $V_H - V_L = (\Pi_H^i - \Pi_L^i)[U_{GH}^i - U_{BH}^i]$

C5'. $\Pi_L^i U_{GH}^i + (1 - \Pi_L^i) U_{BH}^i - U_{GL}^i > 0$

C6. Strict concavity: $U_{se}^i - U_{s'e'}^j < M_{s'e'}^j (X_{se}^i - X_{s'e'}^j)$ *for all* $i, j = 1 \ldots n$, $e, e' \in \{H, L\}$, $s, s' \in \{G, B\}$.

b. The model is testable as in Brown and Matzkin that is consistent (equilibrium inequalities sometimes have a solution) and refutable (equilibrium inequalities sometimes do not have a solution).

Conditions C2 simply define the variables X_{GL}^i, X_B^i. Conditions C1-C5' are as in lemma 1 only replacing utility functions with a variable representing the value of the utility. Condition C6 assures that we have a concave function. This condition is a version of the Afriat inequalities. In this theorem we take the definition of high effort equilibrium to be one in which high effort is strictly preferred. Otherwise we need to take a weak inequality in C5' for the necessity statement. With a weakly concave utility function the conditions in part a of the lemma are sufficient. The proof of theorem 3 is provided in the appendix.

The testable implications that we can obtain when we have data on the probabilities (or prices) associated with both possible effort levels are more restrictive than those we had in the case where only information related to the optimal choice can be obtained.

4 Appendix

Proof of Proposition 1.

$\Longrightarrow$ (Sufficient) Suppose the data and the concave utility function $u(x)$ satisfy the conditions in the theorem we would like to show that $u(x)$ rationalizes the data as high effort moral hazard equilibria. By Lemma 1 and definition 1, it suffices to show that there exist probabilities of the good state, Π_L^i such that $1 \geq \Pi_H^i > \Pi_L^i \geq 0$ and there exist effort costs $V_H > V_L$ such that for all i conditions C1-C5 hold.

Condition C3 is given in the theorem. We know $\Pi_H^i = \frac{P_{GH}^i}{P_{GH}^i + P_{BH}^i}$. Define $\Pi_L^i = P_{GL}^i$, and define

$$X_{GL}^i = X_{BL}^i = \Pi_L^i W_G^i + (1 - \Pi_L^i) W_B^i,$$

for Π_L^i yet to be determined. This gives us conditions C1 and C2. Let V_L=0. It is left to show that we can find probabilities of the good state for low effort, Π_L^i such that $1 \geq \Pi_H^i > \Pi_L^i \geq 0$ and high effort cost $V_H > 0$ such that for all

i conditions C4 (incentive compatibility satisfied with equality) and C5 (effort H preferred to effort L) hold. Let

$$V_H = \min_j \{\Pi_H^j [u(X_{GH}^j) - u(X_{BH}^j)]\},$$

$V_H > 0$. Let

$$\Pi_L^i = \Pi_H^i - \frac{V_H}{u(X_{GH}^i) - u(X_{BH}^i)}$$

for all i. First we show that the low effort probabilities are well defined, that is $\Pi_L^i \in [0, \Pi_H^1)$.

$$\min_j \{\frac{\Pi_H^j [u(X_{GH}^j) - u(X_{BH}^j)]}{[u(X_{GH}^i) - u(X_{BH}^i)]}\} > 0$$

therefore $\Pi_L^i < \Pi_H^i$.

$$\underset{j}{min} \{\frac{\Pi_H^j [u(X_{GH}^j) - u(X_{BH}^j)]}{[u(X_{GH}^i) - u(X_{BH}^i)]} \leq \Pi_H^i \frac{\Pi_H^i [u(X_{GH}^i) - u(X_{BH}^i)]}{[u(X_{GH}^i) - u(X_{BH}^i)]} = \Pi_H^i$$

therefore $\Pi_L^i \geq 0$. Hence, the probabilities are well defined.

Rearranging the definitions of Π_L^i we see that C4, the incentive compatibility condition

$$V_H = (\Pi_H^i - \Pi_L^i)[u(X_{GH}^i) - u(X_{BH}^i)]$$

n is true for all i. It is left to verify that condition C5 (high effort is preferred to low effort) is satisfied for every observation.

The condition is for all i,

$$f^i(\Pi_L^i) \equiv \Pi_L^i u(X_{GH}^i) + (1 - \Pi_L^i) u(X_{BH}^i) - u(\Pi_L^i W_G^i + (1 - \Pi_L^i) W_B^i) \geq 0.$$

We defined Π^{i^*} to be the largest zero of the function $f^i(\Pi)$ in the range $[0, \Pi_H^i]$. Note that $f^i(\Pi)$ is a continuous convex function that is positive at $\Pi = 0$ (from monotonicity of $u(x)$ and the assumption $W_B^i < X_{BH}^i$). Also $f^i(\Pi)$ is non-positive at $\Pi = \Pi_H^i$ (from concavity of $u(x)$ and the budget constraint). Therefore $f^i(\Pi)$ obtains a zero in $(0, \Pi_H^i]$. If $f^i(\Pi_H^i) = 0$ then $\Pi^{i^*} = \Pi_H^i$ and if $f^i(\Pi_H^i) < 0$ then by convexity the function $f^i(\Pi)$ has a unique zero in the segment, Π^{i^*}. In the range $[0, \Pi^{i^*}]$, $f^i(\Pi_L^i) < 0$, high effort is preferred to low effort. In the range $(\Pi^{i^*}, \Pi_H^i]$ $f^i(\Pi_L^i) < 0$, low effort is preferred. We only need to check $\Pi_L^i \leq \Pi^{i^*}$. But

$$\Pi_L^i = \Pi_H^i - \frac{V_H}{u(X_{GH}^i) - u(X_{BH}^i)} = \Pi_H^i - \min_j \{\frac{\Pi_H^j [u(X_{GH}^j) - u(X_{BH}^j)]}{[u(X_{GH}^i) - (X_{BH}^i)]}\} \leq \Pi^{i^*}.$$

This holds for all i by the assumption of the theorem. Hence, when

$$\Pi_H^i - \min_j \{\frac{\Pi_H^j [u(X_{GH}^j) - u(X_{BH}^j)]}{[u(X_{GH}^i) - (X_{BH}^i)]}\} \leq \Pi^{i^*} \tag{7}$$

for every $i = 1 \ldots n$ we can find probabilities and effort costs such that $u(x)$ rationalizes the data as high effort moral hazard equilibria.

$\Longleftarrow$ (necessary) Suppose a strictly concave utility function, $u(x)$ rationalizes the data as high effort moral hazard equilibria. By the definitions and lemma 1 we know that there exist probabilities of the good state Π_L^i such that $1 \geq \Pi_H^i > \Pi_L^i \geq 0$ and effort costs $V_H > V_L \geq 0$ such that together with the data these satisfy, for all i, the conditions C1-C5. We need to show that for every i () holds..

Condition C4, the incentive compatibility condition, can also be written as

$$\Pi_L^i = \Pi_H^i - \frac{(V_H - V_L)}{u(X_{GH}^i) - u(X_{BH}^i)}.$$

The probability being non negative gives us the inequality $\Pi_H^i - \frac{(V_H - V_L)}{u(X_{GH}^i) - u(X_{BH}^i)} \geq 0$ for every i. Plugging the incentive compatibility for observation 1:

$$V_H - V_L = (\Pi_H^i - \Pi_L^i)[u(X_{GH}^1) - (X_{BH}^1)]$$

these non negativity conditions become:

for every i,

$$\Pi_H^i - \frac{(\Pi_H^i - \Pi_L^i)[u(X_{GH}^1) - (X_{BH}^1)]}{u(X_{GH}^i) - u(X_{BH}^i)} \geq 0$$

or

for every i,

$$(\Pi_H^1 - \Pi_L^1) \leq \frac{\Pi_H^i[u(X_{GH}^i) - u(X_{BH}^i)]}{u(X_{GH}^1) - u(X_{BH}^1)}$$

or

$$(\Pi_H^1 - \Pi_L^1) \leq \min_j \frac{\Pi_H^j[u(X_{GH}^j) - u(X_{BH}^j)]}{u(X_{GH}^1) - u(X_{BH}^1)}.$$

We use this inequality to obtain that for every i

$$\Pi_L^i = \Pi_H^i - (V_H - V_L)/[u(X_{GH}^i) - u(X_{BH}^i)] =$$

$$\Pi_H^i - (\Pi_H^1 - \Pi_L^1)\frac{u(X_{GH}^1) - u(X_{BH}^1)}{u(X_{GH}^i) - u(X_{BH}^i)} \geq$$

$$\Pi_H^i - \min_j \{\frac{\Pi_H^j[u(X_{GH}^j) - u(X_{BH}^j)]}{u(X_{GH}^i) - u(X_{BH}^i)}\}.$$

We will now show that $\Pi_L^i \leq \Pi^{i^*}$ and by this obtain the required inequality. Condition C5 tells us that high effort is preferred to low effort,

$$f^i(\Pi_L^i) \equiv \Pi_L^i u(X_{GH}^i) + (1 - \Pi_L^i)u(X_{BH}^i) - u(\Pi_L^i W_G^i + (1 - \Pi_L^i)W_B^i) \geq 0.$$

Π^{i^*} is defined as the largest zero of the function $f^i(\Pi)$ in the range $[0, \Pi_H^i]$. In the range $[0, \Pi^{i^*}]$, $f^i(\Pi_H^i) \geq 0$, high effort is preferred to low effort. In the range $(\Pi^{i^*}, \Pi_H^i]$, $f(\Pi_H^i) < 0$, low effort is preferred. Since high effort is preferred to low effort $\Pi_L^i \leq \Pi^{i^*}$. Combining this with the previous result we see that when the data is rationalized as high effort moral hazard equilibria

$$\Pi_H^i - \min_j \{ \frac{\Pi_H^j [u(X_{GH}^j) - u(X_{BH}^j)]}{u(X_{GH}^i) - u(X_{BH}^i)} \} \leq \Pi^{i^*}.$$

for every $i = 1 \ldots n$.

Lemma 8 *The function $\Pi^{i^*}(\gamma)$ is a continuous*

proof of lemma 3.

The function $\Pi^{i^*}(\gamma)$is defined as the zero of the function:

$$f^i(\gamma, \Pi) = \Pi u(\gamma, X_{GH}^i) + (1 - \Pi) u(\gamma, X_{BH}^i) - u(\gamma, \Pi W_G^i + (1 - \Pi) W_B^i) = 0$$

in $[0, \Pi_H^i]$. For all γ in $(-1, 1)$, $f^i(\gamma, 0) > 0$, $f^i(\gamma, \Pi_H^i) \leq 0$ and $f^i(\gamma, \Pi^i)$ is continuous. Therefore we know that there exists a zero of the function in $(0, \Pi_H^i]$. The slope of $f^i(\gamma, \Pi)$ for a given γ is:

$$f_\Pi^i(\gamma, \Pi) = \frac{(X_{GH}^i)^{1-\gamma^2} - (X_{BH}^i)^{1-\gamma^2}}{1 - \gamma^2} - [\Pi W_G^i + (1 - \Pi) W_B^i]^{-\gamma 2} \times (W_G^i - W_B^i).$$

$$f_\Pi^i(\gamma, \Pi) = X_{GH}^i - X_{BH}^i - (W_G^i - W_B^i) < 0$$

for all Π by the smoothing condition. In particular $f_\Pi^i(0, 2) < 0$. The function $f_\Pi^i(0, \Pi)$ is a continuous function of Π in (-1,1) and therefore there exists an open neighborhood of $\gamma = 0$, $B_\varepsilon(0) \subset$(-1,1) such that for all γ in this neighborhood $f_\Pi^i(\gamma, 2) < 0$. The function $f^i(\gamma, \Pi)$ is a convex function of Π so $f_\Pi^i(\gamma, \Pi)$ is increasing in Π and therefore $f_\Pi^i(\gamma, \Pi) < f_\Pi^i(\gamma, 2) < 0$ for all $\gamma \in B_\varepsilon(0)$ and $\Pi \in (0, 2)$. We conclude that for all $\gamma \in B_\varepsilon(0)$ and $\Pi \in (0, 2)$, $f^i(\gamma, \Pi)$ is a strictly decreasing function of Π and has a unique zero (that lies in the segment $(0, \Pi_H^i]$). The function $f^i(\gamma, \Pi^i)$ is a C^1 function on the open neighborhood $B_\varepsilon(0) \times (0, 2)$ of $(0, \Pi_H^i)$. We know from the budget constraint that $f^i(\gamma, \Pi_H^i) = 0$. Differentiating the function $f^i(\gamma, \Pi)$ with respect to Π and evaluating at $(0, \Pi_H^i)$ we obtain that

$$f_\Pi^i(0, \Pi_H^i) = X_{GH}^i - X_{BH}^i - (W_G^i - W_B^i) \neq 0$$

from the consumption smoothing condition. Therefore, we can apply the implicit function theorem to find that there exist neighborhoods N_γ and N_Π of zero and Π_H^i, $N_\gamma \times N_\Pi \subset B_\varepsilon(0) \times (0, 2)$ such that $f^i(\gamma, \Pi) = 0$ has a unique solution in Π for all $(\gamma, \Pi^i) \in N_\gamma \times N_\Pi$. Furthermore the function $\phi : N_\gamma \to N_\Pi$ that uniquely defines this solution, is itself a C^1 function, in particular it is continuous. As we saw earlier, for all $\gamma \in B_\varepsilon(0)$ and $\Pi \in (0, 2)$, $f^i(\gamma, \Pi^i)$ has a unique zero that lies in the segment $(0, \Pi_H^i]$ we called it $\Pi^{i^*}(\gamma)$. So it must be then, that $\Pi^{i^*}(\gamma) \equiv \phi(\gamma)$ on N_γ. We have proved that $\Pi^{i^*}(\gamma)$ is a continuous function of γ.

Proof of proposition 2. Let us look at the family of CRRA utility functions, $u(\gamma, x) = \frac{x^{1-\gamma^2}}{1-\gamma^2}$.

The necessary and sufficient condition for $u(\gamma, x)$ to rationalize the data is

$$\Pi_H^i - \min_j \{ \frac{\Pi_H^j [u(\gamma, X_{GH}^j) - u(\gamma, X_{BH}^j)]}{[u(\gamma, X_{GH}^i) - u(\gamma, X_{BH}^i)]} \} - \Pi^{i^*}(\gamma) \leq 0.$$

As we obtained in proposition 2 this condition holds with a strict inequality for the linear utility function, that is for $\gamma = 0$. $u(\gamma, x)$ is a continuous function of γ, $u(\gamma, X_{GH}^i) \neq u(\gamma, X_{BH}^i)$ and $\Pi^{i^*}(\gamma)$ is continuous on an open set containing zero (by lemma 3). Therefore, the left-hand side of the inequality is a continuous function of γ. Hence there is a neighborhood of $\gamma = 0$, $B_\varepsilon(0)$ for which the inequality holds. For all γ in this neighborhood, $u(\gamma, x)$ is a strictly concave utility function that rationalizes the data.

Proof of Theorem 1. I$\Longrightarrow$II. If the data is rationalized as high effort moral hazard equilibria, then by lemma 1 the budget constraint and the fair prices condition and consumption smoothing must hold for all observations.

II$\Longrightarrow$I. Follows immediately from proposition 2.

Proof of theorem 2. Let $u(x)$ be a strictly concave utility that rationalizes the first n observations as high effort moral hazard equilibria with probabilities Π_L^i, $i = 1..n$ associated with low effort and effort costs $V_H > V_L$. In order for this utility function and effort levels to rationalize the additional m full insurance observations there need to exist (X_{GH}^i, X_{BH}^i), and $1 \geq \Pi_H^i > \Pi_L^i$ for $i = (n+1)...m$ such that low effort is preferred to high effort. That is

$$\Pi_H^i u(X_{GH}^i) + (1 - \Pi_H^i) u(X_{BH}^i) - V_H \leq u(\Pi_L^i W_G^i + (1 - \Pi_L^i) W_B^i) - V_L$$

where $(X_{GH}^i,\ X_{BH}^i)$ is the best consumption choice of the agent if she were to choose high effort.

For each $i = (n+1)...m$ we consider two possible cases.

Case 1: Suppose

$$\frac{V_H - V_L}{u(W_G^i) - u(W_B^i)} < 1 - \Pi_L^i$$

.

Let

$$\Pi_H^i = \Pi_L^i + \frac{V_H - V_L}{u(W_G^i) - u(W_B^i)}$$

then $1 > \Pi_H^i > \Pi_L^i$. Let $(X_{GH}^i, X_{BH}^i) = (W_G^i, W_B^i)$. It is now the case that both the budget constraint for high effort,

$$\Pi_{GL}^i (W_G^i - W_G^i) + (1 - \Pi_{GL}^i)(W_B^i - W_B^i) = 0,$$

and the incentive compatibility constraint

$$V_H - V_L = (\Pi_H^i - \Pi_L^i)[u(W_G^i) - u(W_B^i)]$$

are satisfied. Hence, (W^i_{GH}, W^i_{Bh}) is the best choice in the case where high effort is chosen. It is also the case that low effort is strictly better than high effort since given the incentive compatibility constraint, this is equivalent to

$$\Pi^i_L u(W^i_{GH}) + (1-\Pi^i_L)u(W^i_{BH}) - u(\Pi^i_L W^i_G + (1-\Pi^i_L)W^i_B) < 0,$$

which is true by strict concavity of $u(x)$. Notice that both the condition that the probability is less than 1 and that low effort is preferred are satisfied with strict inequality when $(X^i_{GH}, X^i_{BH}) = (W^i_G, W^i_B)$. Therefore we can take a very small perturbation of the endowments

$$(X^i_{GH}, X^i_{BH}) = (W^i_G - \varepsilon, W^i_B + \delta)$$

so that the budget constraint will still hold with equality, the probability Π^i_{GL} will still be well defined, and low effort will still be preferred. The perturbed endowments satisfy the smoothing condition.

Case 2: Suppose

$$\frac{V_H - V_L}{u(W^i_G) - u(W^i_B)} \geq 1 - \Pi^i_L.$$

In this case, the cost of high effort is so high that for any probability, Π^i_H, low effort is preferred to high effort. Take some $1 > \Pi^i_H > \Pi^i_L$. Given the assumption on effort costs the incentive compatibility constraint does not hold for the endowment bundle or for any consumption-smoothing bundle on the budget line. To see this we rearrange the incentive compatibility constraint to get:

$$(\Pi^i_H - \Pi^i_L)[u(X^i_{GH}) - u(X^i_{BH})] \geq V_H - V_L$$

but from the assumption in case 1and by $1 \geq \Pi^i_H$

$$V_H - V_L > (1-\Pi^i_L)[u(W^i_G) - u(W^i_B)] \geq (\Pi^i_H - \Pi^i_L)[u(W^i_G) - u(W^i_B)] \geq$$

$$(\Pi^i_H - \Pi^i_L)[u(X^i_{GH}) - u(X^i_{BH})]$$

by consumption smoothing.

Therefore incentive compatibility cannot hold.

Since none of the consumption smoothing bundles satisfy incentive compatibility, if the agent chooses high effort only bundles on the budget line to the left of the endowment may be feasible (bundles with higher variance: $X^i_{BH} < W^i_B$). But these bundles give lower expected utility then the endowment bundle when high is chosen (by second order stochastic dominance). Low effort and no trade give higher utility than high effort and no trade as we saw above. Low effort and full insurance is strictly better than low effort and no trade by concavity. Therefore low effort and full insurance is strictly preferred to the best an agent can do if she chooses high effort. Thus in this case, low effort is preferred to high effort. We conclude that the observation at hand is consistent with a low effort equilibrium given the utility and effort costs that rationalize the first n observations of the data.

Proof of theorem 3.

a. Suppose we can rationalize the data with a strictly concave increasing differentiable utility function $u(x)$. Then let $U^i_{se} = u(X^i_{se})$ and $M^i_{se} = u'(X^i_{se})$. The first five conditions are implied by the model (see lemma 1). The sixth condition follows from the strict concavity of u.

If there is a solution to the system, then by condition C6 and the construction in Afriat's theorem, there is a concave utility function such that $U^i_{se} = u(X^i_{se})$ and $M^i_{se} = u'(X^i_{se})$. By Chiappori and Rochet (1987) we can take a strictly concave smooth function by using convolution and making a small perturbation. Note that conditions C1-C3 are conditions on the observed variables only. Condition C4' is an equality condition that defines the difference between effort costs. Conditions C5 and C6 are strict inequalities and so will hold after a small enough perturbation of the utility levels. The data satisfies all the conditions for high effort moral hazard equilibrium given the strictly concave utility constructed and therefore this utility function rationalizes the data.

b. Having a system of equilibrium inequalities we now show that there are possible data sets for which there is no solution to the system that is, there are data sets that cannot be rationalized by any concave utility function.

For the condition C4' to be satisfied for all i it must be that for any i, j:

$$(\Pi^i_H - \Pi^i_L)[U^i_{GH} - U^i_{BH})] = (\Pi^j_H - \Pi^j_L)[U^j_{GH} - U^j_{BH}]$$

or

$$\frac{\Pi^i_H - \Pi^i_L}{\Pi^j_H - \Pi^j_L} = \frac{U^j_{GH} - U^j_{BH}}{U^i_{GH} - U^i_{BH}}$$

For any data with $X^j_{GH} > X^i_{GH} > X^i_{BH} > X^j_{BH}$ it must be that $U^j_{GH} > U^i_{GH} > U^i_{BH} > U^j_{BH}$, therefore

$$\frac{U^j_{GH} - U^j_{BH}}{U^i_{GH} - U^i_{BH}} > 1$$

. But it is possible to have the probabilities satisfy

$$\frac{\Pi^i_H - \Pi^i_L}{\Pi^j_H - \Pi^j_L} < 1,$$

hence the condition

$$\frac{\Pi^i_H - \Pi^i_L}{\Pi^j_H - \Pi^j_L} = \frac{U^j_{GH} - U^j_{BH}}{U^i_{GH} - U^i_{BH}}$$

cannot hold. For such a data set, there is no solution to the equilibrium inequalities. Note that we can find an example as described here while still satisfying fair prices, the budget and smoothing conditions so the restrictions in this theorem are more restrictive than those in theorem 1. It is clearly the case that the system of equilibrium inequalities sometimes has a solution. To find data for which the system has a solution we can simply take any data of consumption wealth and high effort prices that satisfy the smoothing and the budget conditions, and use the result in propositions 2 or 3 to find probabilities associated with the low effort.

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