Sequential Network Formation with Farsighted Agents

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Abstract

This paper studies a sequential model of network formation when agents are not myopic and have capacity constraints. We find that the equilibrium network configuration depends crucially on the rate of decreasing returns. The complete star and a hub-and-spoke type of architecture emerge for extreme values of decay. But the network might not be connected for intermediate levels of decay.

1 Introduction

Interpersonal networks, be it within one's kin, among neighbours or among colleagues, have been emphasized by varied studies to have a significant and persistent impact on economic and social outcomes of individuals. Evidence of the importance of informal networks has been documented in job search and employment outcomes (Granovetter 1974, Montgomery 1991, Holzer 1987,), consumer behaviour (Feick and Price 1987; Ellison and Fudenberg 1995), credit and insurance markets (McMillan and Woodruff 1999, Fafchamps and Lund 2001, Banerjee and Munshi 2003), adoption of new technologies (Conley and Udry 2001) and in sociological interactions as social norms, status attainment and ethnic segregation[3]. Examples of formal networks include trade networks (Lazerson 1993, Nishiguchi 1994) and R&D alliances among firms (Powell 1996, Delapierre and Mytelka 1998). The widespread impact of networks in economic and social environments motivated extensive theoretical research on the process of network formation[4].

This paper focuses on individual incentives for network formation by forward-looking agents with capacity constraints. In particular, we consider the capacity constraint of an agent as the maximum number of links he can initiate while forming the network. The underlying premise here is that forming links takes up resources in terms of time, money or effort. The notion of constraints is reasonable since link formation in a network is only one among many sources of economic benefits. Agents participating, say, in the labour market, can only

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[3]There exists a large literature in sociology on issues of family and kinship networks, ethnic networks and their effects on income, labour market participation, social cohesion and ethnic integration. See, among others, Coleman (1988), Burt (1992). See Wasserman and Faust (1994) for an introductory book on networks in sociology.

[4]The theoretical literature is discussed below.

invest upto a certain proportion of their time in their social networks due to opportunity costs of social investment. For example, consider a community of agents of the same ethnic group where agents participate in the formal labour market as well as in their ethnic network. They earn a market wage and get some non-market benefits from their ethnic network. The benefits from their network could be information about better job prospects, a favour done by a friend or simply dinner invitations from neighbours. In order to form friends, however, a person has to put effort in terms of time or money. Given the agent's participation in the labour market, he probably would not want to spend too much time on such effort and hence have a capacity on the number of network links he himself initiates. Of course, different agents might have different opportunity costs and hence invest different amounts in social networking. This heterogeneity in capacity might be potentially related to heterogeneities in agents' abilities. For example, agents could be either of a high or low type. Suppose, a high(low) type agent earns more(less) in the labour market and also provides higher(lower) benefits to his friends in the network. In our basic model, however, we focus on homogeneous agents with an extreme form of capacity constraint in that each can initiate only one link. The agent initiating the link bears the cost and this allows us to model the process as a non-cooperative game.

The first strand of literature on network formation follows Jackson and Wolinsky (1996) (henceforth JW). Their work is closely related to the literature on coalition formation in cooperative games (Myerson1977, Aumann and Myerson 1988) but the value of the network in JW depends on the exact structure of the network. JW focussed on individual incentives to form or sever links and highlighted the conflict between stable networks and socially efficient ones. The equilibrium concept used, however, is pairwise stability which takes into account the idea that a link is formed only by mutual consent but can be severed unilaterally. The notion of pairwise stability represents decisions of coalitions of exactly two players. Following JW, numerous papers have analysed communication networks or job contact networks using the notion of pairwise stability and strong stability.[5]

Bala and Goyal (2000, henceforth BG) on the other hand model network formation as a non cooperative game by introducing one-sided link costs. BG proposed a simultaneous move one-shot game where homogeneous agents form links to other agents at some exogenous cost which is entirely borne by the intiator. The benefit obtained is non-rival, e.g.the benefit could be access to some information of the partner linked to. They considered both one-way (only the initiator getting the benefit) and two-way (both agents in the link) flow of benefits with and without decay[6] and characterised the set of strict Nash networks for the different specifications. Following BG there have been a number

[5]For an excellent survey on the literature on various approaches to network formation see Jackson (2003). For papers using the notion of pairwise stability, refer to Dutta and Mutuswami (1997), Dutta, B., A. van den Nouweland and S. Tijs (1998), Johnson and Gilles (2000).

[6]i.e. the benefit exchanged between two agents may or may not decline (decay) with the distance between them.

of attempts to extend this basic model. Galeotti et al (2005) and Galeotti (2006) extend their model to include heterogeneity in values and costs in both one-way and two-way flow of benefits. The heterogeneity in their model is not partner specific. i.e. value and cost depends on the agent's own type.[7] Kannan et al (2007) follow BG and introduce costs of indirect links. Hence, the benefits from one's indirect contacts (friends e.g.) are not free, as was the case in BG. They look at different cost specifications and analyse how the cost structure affects the network. Hojman and Szeidl (2004) focus on the role of decay in network formation and highlight periphery-sponsorship as a robust feature of the equilibrium network in such settings. They however limit the access to information to a finite distance in the network.

All these papers analyse network formation as a simultaneous-move game and hence are myopic and static in nature. Watts (2001a), on the other hand, proposes a dynamic model of network formation where a finite set of agents can form and sever links. Pairs of players meet at random and decide on the link to maximise their myopic payoff. The game has an infinite horizon and she looks at the set of stable networks that emerge. Watts (2001b) focuses on a specific model with forward looking agents that results in circle networks. Derorian (2006) analyses a finite time sequential move game and shows a non-monotonic relationship between level of cost and the formation of a complete graph (specifically, a star). Also, in these models an agent can form as many links as she wants. There is no capacity constraint. Another point to note is that the formation of a link is unilateral i.e. it does not require consent from the agent being linked to. Of course, for a static, myopic game there is no reason for not accepting a link. But, when agents are forward looking an agent might reject a link for strategic reasons. The exact nature of such strategic concerns will be clear later.

In this paper we propose a sequential-move game of network formation with capacity constraints. Initially, we consider ex-ante identical agents and express the budget constraint in terms of the number of links each player is allowed to propose. In the basic model, we restrict the capacity to be one link that each agent is allowed to propose. Each agent proposes a link at each period by choosing one agent to link to. The responder can accept or reject the link. Whatever be the decision, the responder becomes the proposer in the next period and so on. The first proposer is chosen at random. The game ends when all agents have had the opportunity to propose a link. The sequential nature of our model is similar to that of the multiperson bargaining literature, the 'offer' here, being a social link. In a bargaining situation, different orders of play could be relevant. For example, it could be a pre-determined order as in Shaked (1986) or random proposers like Okada (1996). The order of play considered here is similar to that of Selten (1981) and Chatterjee et al (1993).

In our basic model with homogeneous agents, we find that the equilibrium network architecture is a single connected component whenever the rate of decreasing returns is very high or very low. In particular, it is a periphery-

[7]For more on heterogeneity, see Billand, Bravard and Sarangi (2006).

sponsored star or a mixed star when the extent of decreasing returns is close to zero. On the other hand, when there are very strong decreasing returns the equilibrium network is a single wheel with a local star. With an arbitrary level of decay, however, there could be two components in equilibrium. We compare our results with existing ones in the discussion.

The remainder of the paper proceeds as follows. Section 2 describes the basic model followed by the analysis in Section 3. Section 4 discusses welfare and efficiency issues. Section 5 extends the basic model to heterogeneous agents. Section 6 includes a discussion of assumptions and results. Section 7 concludes. All proofs are relegated to the appendices.

2 Model: Sequential formation, perfect foresight

2.1 The Environment

$N = \{1', 2', ...n'\}$ is the set of agents forming a network. The network at time 0 is empty. Agents are budget constrained. Assume that all agents have the same contraint and each can form only one link. An agent is randomly chosen to be the first proposer (relabeled agent 1). He proposes a link to some agent j'. The responder accepts or rejects the link. and becomes the next proposer (and hence relabeled agent 2). At any time t, label the agent proposing as agent t. If t proposes to $k' \neq t$, k' rejects or accepts. If k' had already proposed, then an agent is randomly chosen from the set of non proposers. Otherwise k' is the next proposer i.e. k' is relabelled as agent $t+1$. Links once formed are not allowed to be severed. The proposals stop once every agent has had the chance to propose. Hence the game continues for n periods. If i initiates a link and j accepts, they are linked and it is denoted by $g_{ij} = 1$.Collection of all such links make a graph g. Let g_{t-1} denote the graph at time t before agent t makes a decision. At n, g_n is formed and agents meet each other. The probability of meetings is decreasing in the distance between agents. Payoffs are realised after meeting.

2.2 The Strategy

At any time t, the agent i who moves could have two decisions to make. First, he accepts or rejects if he has been proposed to at $t-1$ which is denoted by an action $a_t \in \{A, R\}$. If he is an isolated agent, then he has no such decision to make which is denoted by ϕ. The second decision is that of initiating a link to some agent $l_t \in N \backslash \{i\}$ or abstaining, denoted by ϕ_l. The history, in this case, is g_{t-1}, the graph formed till stage $t-1$ and the history of acceptance/rejections, i.e. history $h_t = \{a_\tau, l_\tau\}_{\tau=0}^{t-1}$.

Hence, a *strategy* of an agent i moving at time t is a mapping from the history at time t to the action set. We represent the strategy by $s_i : h_t \to \{A, R, \phi\} X \{\{\phi_l\} U N \backslash \{i\}\}$.

2.3 The Payoffs

We introduce some related definitions. Given a network g, define $d(i, j, g)$,the geodesic distance between i and j as the length of the shortest path between the two agents in g. If i and j are directly linked, $d(i, j, g) = 1$. By convention, if two agents are not connected then $d(i, j, g) = \infty$. Probability of i meeting j is $p_{ij}(g) = p^{d(i,j,g)-1}$, $p \in (0, 1)$. Also define $N_i^d(g) = \{j \neq i : g_{ij} = 1$ or $g_{ji} = 1\}$, i.e. the set of agents to whom i is directly linked. Let $\mu_i^d(g) = |N_i^d(g)|$. Similarly, define $N_i(g) = \{j :$ either $g_{ij} = 1$ or $g_{ji} = 1$ or $\exists j_1, j_2, ...j_k$ such that $g_{ij_1} = g_{j_1 j_2} = ... = g_{j_k j} = 1\}$ ie there exists a path connecting i and j.

Given a network g, the payoff to agent i in g , is given by

$$\pi_i(g) = \sum_{j \neq i} \{p^{d(i,j,g)-1}(u_j - c_u) - cI(g_{ij} = 1)\}$$

where u_j is the value of the benefit from j and c_u is the cost of providing it. Since agents can form only one link, the link formation cost is c only if i initiates a link and 0 otherwise. The benefit $u_j = u$ for all j implies agents are homogeneous. We will later consider situtations where $u_j = \theta_j$ where θ_j is the type of agent j. We will focus on the case of $u - c_u > 0$. We normalise $u - c_u = x = 1$.

Before we proceed to the analysis we introduce some standard network architectures. Given a graph g, a set $C \subset N$ is called a *component* of g if for pair of agents i and j in C, $j \in N_i(g)$ and there is no strict superset C' of C for which this is true. A component is *minimal* if C is no longer a component if $g_{ij} = 1$ is replaced by $g_{ij} = 0$. A network g is said to be *connected* if it has a unique component. A network is an *empty network* g^e if $N_i(g) = \{i\}$ and is a complete network g^c is $N_i(g) = N \backslash \{i\}$ for all $i \in N$. A *wheel network*, g^w(*or* just W) is one where agents are arranged as $i_1, i_2, ..., i_n$ with $g_{i_1 i_2} = g_{i_2 i_3} = ... = g_{i_n i_1} = 1$ and there are no other links. A *star network* g^s has a central agent i such that $N_j^d(g) = \{i\}$ for all $j \in N \backslash \{i\}$ and there are no other links. A star is *centre sponsored* if $g_{ij} = 1$ and $g_{ji} = 0$ for all j and *periphery-sponsored* if $g_{ij} = 0$ and $g_{ji} = 1$ for all j in g^s. A *mixed star* is a combination of the two. We call a graph a *wheel with local stars* if the graph contains both a wheel and a star. Let us denote it by WS_k, when the wheel has k agents and the local star has the $N - k$ spoke agents. A connected acyclic component with exactly one path is called a *chain.* Let us denote it by Ch_l, where the subindex is the length of the chain. Examples of these architectures are illustrated in Figure 1.

3 Analysis: Homogeneous agents

The network formation game of our model is a finite horizon problem and the equilibrium concept is that of subgame perfect equilibrium. We assume that when a proposer of a link is indifferent between multiple agents, he chooses one randomly and with equal probability.

Assumption (A1): When indifferent between a set of agents to link to, the proposer randomises between them with equal probability.

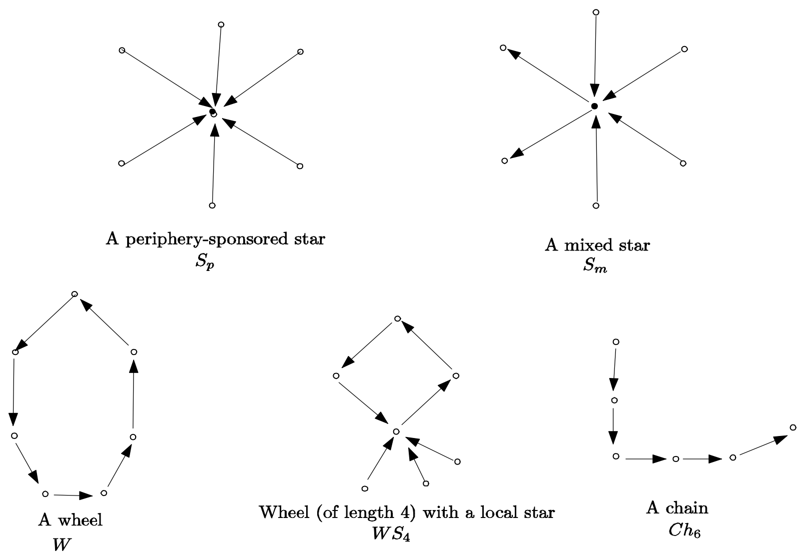

Figure 1: Network Architectures

Note that given **A1** rejections cannot occur on the equilibrium path since no agent i would incur the cost of intitating a link which will be rejected and yield no myopic benefit[8] Hence. the relevant history is the current graph (which influences the myopic payoff) and not the past actions since no player is moving twice. From hereon, we confine ourselves to strategies that depend only on the graph formed till the current stage.

First let $c_u < u$ so that agents always get net positive benefit from meeting some agent. Also assume $c < x = u - c_u = 1$. Note that the responder of time $t-1$ is the proposer at time t if he accepted the link. The agents in this model have perfect foresight and realise the implications of their strategies on future action by players. We use backward induction to analyse actions of each player.

Before we proceed let us introduce some further notation.

$\pi_t(g_{t-1})$: Payoff agent t is assured to get given the graph g_{t-1}.

$\pi_t^{myo}(j, g_{t-1})$: Payoff t receives from agents $k < t$, after $\{tj\}$ link is formed $= \sum_{j<t} p_{tj}(g_{t-1} + \{tj\}).x$. This is the myopic payoff since t does not take into account the payoff from meeting the future entrants.

$\pi_t^f(j, g_{t-1})$: Payoff t receives from all future entrants if he links to j. It is determined by the equilibrium strategies of future players given that the $t+1$ state will be the graph $g_t = g_{t-1} + \{tj\}$

[8]However, this could still be beneficial if this would lead some future agent to choose the agent i (whose proposal was rejected earlier) with higher probability. This cannot happen since (i) if indifferent, the future agent randomises and does not choose i with higher probability and (ii) if not indifferent, that agent i would have been chosen even if he had not made a rejected offer (for his "centrality").

$v_t(g_{t-1}) = \arg\max_{j<t} \pi_t^{myo}(j, g_{t-1})$

$\Pi_t(j, g_{t-1})$: Total payoff to agent t from linking to j, when the graph is g_{t-1}. Note that Π is determined by the equilibrium strategy of all other agents.

Note that $\pi_t(g_{t-1}) = \sum_{j<t} p_{tj}(g_{t-1}).x = 0$ if t is isolated and > 0 if t is linked. Also, we distinguish between $v_t(g_{t-1})$ when t is isolated and when he is not since the agent v that maximises $t's$ payoff in the 2 cases is potentially different. Let the optimal agent in the case when t is isolated and when he is not be $v_t^{iso}(g_{t-1})$ and $v_t^{con}(g_{t-1})$ respectively. By defining $\pi_t^{myo}(j, g_{t-1})$ and $\pi_t^f(j, g_{t-1})$ separately we will try to disentangle the two types of incentives an agent has when he considers linking to an agent j : (i) t wants to maximise his payoff from meeting all earlier entrants. but (ii) $t's$ choice would change the graph g_t and future entrants would link according to their equilibrium strategies which affects $t's$ payoff from agents $k > t$ through $d(t, k, g_n)$.

Note that the payoffs are functions of p_{tj} which in turn is a function of distances between agents $d(t, j, g)$. In particular, the final payoff to agent t will be functions of $d(t, j, g_n)$ where g_n is determined by the equilibrium strategy profile of all n agents. So, when deciding whom to link to, t considers both current and future payoffs and links to a j such that $\Pi_t(j, g_{t-1})$ is maximised.

Before we state the main propositions of this section, note that due to the assumption of $u > c_u$ no agent would reject a proposal unless the graph is such that following a rejection of a link he has a positive probability of being chosen as the centre of a star by the following agents. In other words, if the graph is such that this incentive is not at work, an agent would accept a proposed link since he is not incurring any cost and the net benefit from the link is $u - c_u = 1$ which is positive.

We first give a few Lemmas which will be used in the following analysis.

Also, denote the graph at the beginning of time t, i.e. before t makes a decision, by g_{t-1}.

Lemma 1 *If $t \in C' \subseteq g_{t-1}$, then C' must be a chain.*

Proof: Suppose not. Let agents $\{k_0, ...t\} \in C'$.Since C' is not a chain there must be an agent $k_1, k_0 < k_1 < t$ who proposed to agent $j < k_1$.Let k_1 be the first such agent. Hence $k_1 + 1$ is isolated at time $k_1 + 1$ and since $k_1 + 1 \in C'$, he connects to $j < k_1 + 1$ in equilibrium which implies that $k_1 + 2$ is isolated and so on. Continuing the argument it implies that t is isolated. Hence C' must be a chain.

Let there be $l+1$ agents in the chain i.e.the length of the chain is l. (distance between t and the farthest agent $\in C'$).

Lemma 2 *If $W_l, W_{l'} \in h_{t-1}, l > l'$, then agent t if isolated will not choose $j \in W_{l'}$.*

Proof: This obtains directly from comparing the payoffs of an isolated agent. First note that in equilibrium, if $s_t^* = j < t$, then $s_{t+1}^* = j < t+1$. Also, in this case, $v_{t+1}^{iso} = v_t^{iso}$. This implies that

$$\pi_t(s_t = j \in W_l) = [1 + 2p + 2p^2 + ... + 2p^{\frac{l-1}{2}}] + (N-t)p$$

Since the payoff is increasing in the size of the wheel l, the lemma follows.

Lemma 3 *If agents $\{k+1, k+2, ..., k+l_1\}$ form a wheel, then agents $\{k+l_1+1, ..., k+l_1+l_2\}$ will not form a wheel for $l_1 > l_2$.*

Lemma 4 says that at any time t, if $W_{l_1} \in g_{t-1}$, then agents $j \geq t$ will not form a wheel of length $l_2 < l_1$.

Proof: Suppose not. Let $l_1 \geq l_2 + 1$ and let agents $\{k+1, k+2, ..., k+l_1\}$ form W_{l_1} and agents $\{k+l_1+1, ..., k+l_1+l_2\}$ form W_{l_2}. Also let x be the first agent such that $s_x^* = j < x$. By Lemma 3, we know that $s_x^* = j \in g_{x-1} \backslash W_{l_2}$. This implies that

$$\pi_{j \in W_{l_2}}(s_j^* = j+1) = \pi_j^{myo} + \pi_j^f = 2 + 2p + 2p^2 + ... + 2p^{\frac{l-3}{2}} + [0]$$

Now consider agent $k + l_1 + l_2$. This agent is connected to a chain $\{k+l_1+1, ...\}$. He can link to $k+l_1+1$ and form W_{l_2} or link to some $j \in W_{l_1}$. His payoff from $s^* = j \in W_{l_1}$ is

$$\begin{aligned}
\pi_{k+l_1+l_2}(s^* &= j \in W_{l_1}) = 1 + p + p^2 + ... + p^{l_2-2} + 1 + 2p + 2p^2 + ... + 2p^{\frac{l_1-1}{2}} + \pi^f \\
&\geq p + p^2 + ... + p^{l_2-2} + 2 + 2p + 2p^2 + ... + 2p^{\frac{l_1-3}{2}} + 2p^{\frac{l_1-1}{2}} \\
&> 2 + 2p + 2p^2 + ... + 2p^{\frac{l_1-3}{2}} + 2p^{\frac{l_1-1}{2}} \\
&> 2 + 2p + 2p^2 + ... + 2p^{\frac{l-3}{2}} \\
&= \pi_{j \in W_{l_2}}(s_j^* = j+1)
\end{aligned}$$

Hence, $k + l_1 + l_2$ deviates to link to some $j \in W_{l_1}$.

Lemma 4 *For any t, $\nexists W_{l_1}, W_{l_2} \in g_t$ with $l_1 = l_2$.*

Proof: Suppose not. Say $\exists 2$ wheels of length y and no wheel of length of length $l > y$. Suppose, wlog, given a g_{t-1} agents $t+1, ...t+2y$ formed the two wheels and agents $j \geq t+2y+1$ chooses some agent $j' < j$. By Lemma 3, we know agent $j = t+2y+1$ is indifferent between agents $t+1, ...t+2y$. Therefore the payoff of $t \in W_y$ is

$$\pi_{t \in W_y}(.) = 2 + 2p + ... + 2p^{\frac{y-3}{2}} + \frac{N-t-2y}{2y}[1 + 2p + ... + 2p^{\frac{y-1}{2}}]$$

Now, given g_{t-1}, if the two wheels merge and forms W_{2y}, then $j = t + 2y + 1$ chooses $t \in W_{2y}$. In this case the payoff of agent t is

$$\begin{aligned} \pi_{t \in W_{2y}}(.) &= 2 + 2p + ... + 2p^{\frac{2y-3}{2}} + \frac{N-t-2y}{2y}[1 + 2p + ... + 2p^{\frac{2y-1}{2}}] \\ &> \pi_{t \in W_y} \end{aligned}$$

Hence in equilibrium, all agents belonging to the two wheels of size y are better off by offering to the next agent and accepting offers to form a wheel of size $2y$.

3.1 Special Cases

We first consider extreme values of the decay factor p and characterize the equilibrium networks in those cases.

Proposition 1 *There exists a* $p^* < 1$ *such that for* $p \in [p^*, 1]$,*the unique subgame-perfect equilibrium structure is a complete star.*

Proof. See Appendix A. ■

Proposition 1 characterises the subgame perfect equilibrium network structure for p high enough. When p is very high, the loss in benefit from an indirect source is not too much. In this case, the payoff from being a direct neighbour of the central agent is not too different from the central agent himself. Hence, the gain in payoff and hence the incentive to become the central agent (i.e. centre) is not strong. It is still true, though, that as long as $p < 1$ an agent would prefer to be the centre. But in order to do so he has to compete with the agent before him and this might be risky. For example, consider agent 3 at a history where 1 and 2 are linked. Agent 3 can link to 1 and lead to the formation of a star with 3 being a peripheral node. Else, 3 could try to compete and link to 4 making sure that some future agent, say 5, will be indifferent between 1,2,3, and 4. Now, in this case, 3 becomes the centre with probability 1/4 and with probability 1/4.he is a peripheral node in a star of $N-2$ agents. Also, with probability 1/2, agent 5 links to 1 or 2 and agent 3 is isolated from the star and hence gets a much lower payoff. When $p \simeq 1$ the loss in case 3 is isolated from the star is too high and hence 3 would prefer linking to 1 or 2 even though he will be a peripheral node.

We use backward induction to show first, that given any graph at time t an isolated agent would either abstain or choose some agent $j < t$ who had already moved. This implies that no agent starting from time 1 would choose to link to an agent who has not moved yet. Hence agent 2 either abstains or links to 1. The same holds for agent 3,4,5,....n. Also, an agent will abstain provided all agents before him have abstained and the cost c is higher than their expected gain from not linking. This is so because the payoff from being the centre of a star is higher than any other position. The expected gain from abstaining will be positive only when there is positive probability of being the centre of a star which obtains only when agent t is symmetric with all agent $k < t$ with

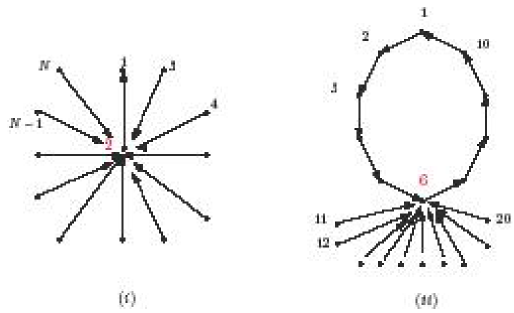

Figure 2: Equilibrium Networks

respect to his links. If agent $k < t$ have links, then an isolated agent t has zero probability of being the centre of a star if he abstains. Hence, he would not abstain and will choose some $j < t$.

The next proposition characterizes the equilibrium network structure for p low enough.

Proposition 2 *There exists a p^{**} such that for $p \leq p^{**}$, the unique equilibrium network structure is a wheel of length $L < N$ with a local star. Also, $L = N/2$ or $(N+1)/2$ if N is even or odd respectively.*

Proof. See Appendix A ■

First note that when $p \approx 0$, the difference in payoffs from being the centre and the peripheral node is high. For a low p, each agent has the incentive to compete with already connected agents and hence become the centre with positive probability by forming a wheel. This incentive decreases for agents moving later since the payoff from becoming the centre decreases. For p close to 0,however, even agent $N-1$ has such an incentive, provided the play reaches such a subgame.

The proof of proposition 2 identifies the condition under which even agent $N-1$ would have the incentive to belong to a wheel. This happens for a p close to 0. It goes on to use Lemma 4 and 5 to argue that in equilibrium agent $N/2$ would link to 1 thereby forming a wheel of length $N/2$.

Note that for p close to 1, we get a periphery sponsored or a mixed star with equal probability. Even in the mixed star, only one link is sponsored by the centre. This is consistent with Hojman and Szeidl (2004) who highlight the periphery sponsorship as a robust feature in models with decay. Also, since $p \approx 1$, the inequality in the payoffs of the central and the peripheral agents is small. For $p \approx 0$, however, the periperal (or spoke) agents get $(1+\varepsilon)$ much less than the central agent(close to $\frac{N}{2}+2$) with the other members of the wheel somewhere in between $(2+\varepsilon)$. This architecture is also interesting since it is similar to the hub-and-spoke architecture which is observed in many real-life networks like the airline industry, biotech firms etc.

3.2 General p

For general levels of decay, all the afore mentioned incentives will be at play and to various degrees. Note that as p increases, the agents at the end of the play will have less incentive to form a wheel since expected gain from being the centre decreases as p increases. This in turn will increase the potential future payoff from being the centre for the earlier players. This increase in incentives might result in some link rejection at the beginning of the order since the rejector expects to form a smaller wheel (hence probability of being centre higher) with larger number of agents as spokes. This increase in such incentives has to be high enough to compensate for the loss in myopic payoff from a smaller wheel. When p is high enough, however, the number of agents at the end of play who do not have incentive to form a wheel increases and in the limit as $p \to 1$, even agent 2 has no such incentive and the star obtains as the equilibrium structure.

Note that the magnitude of the two opposing incentives of maximising current payoff by connecting to the most connected agent and maximising future payoff for a single agent depends on the value of p. It also depends on the history and the number and incentives of agents.yet to move. Either one of the incentives might dominate for an arbitrary value of p. Till now, we do not have an equilibrium characterisation for an arbitrary $p \in (0, 1)$. But we do have some general properties of the equilibrium network and some examples of possible equilibria.

3.2.1 Properties of Equilibria

Let there be $m \geq 1$ components $(C_1, C_2, ..., C_m)$ in the equilibrium structure, formed in that specific order. Let the number of agents in each component be $n_1, n_2, ..., n_m$ respectively with $\sum\limits_{y=1}^{m} n_y = N$.

P1 : C_m is a wheel with a local star i.e. $C_m = W_m + S_m$ with $\#W_m \leq \#S_m$.

This property says that the component formed by the last set of agents (or the last component formed) must be a wheel with a local star with the number of agents in the wheel smaller than the number of spoke agents. Note that the wheel size may be 1, which is a star. This obtains when p$\simeq$ 1, in which case we get $m = 1$ with only 1 agent in the wheel. For $p \simeq 0$, $m = 1$ with $\#W_m = N/2 = \#S_m$.

P2 : $n_m > n_{m-1} > ... > n_2 > n_1$.

The next property says that the number of agents in each subsequent component must be increasing.

P3 : If $W_j \subset C_j$ and $W_{j'} \subset C_{j'}$, then $j < j'$ implies $\#W_j < \#W_{j'}$.

P3 follows directly from Lemma 3 and 4. At any stage if there exists a component C_j containing a wheel, then in the corresponding subgame, if agents are forming another component $C_{j'}$, then it must contain a bigger wheel so that a positive number of subsequent agents link to $C_{j'}$ and not C_j.

P4 : If $W_j \subset C_j$ for some $j < m$, then $n_j < \#W_m$.

This property compares the last component formed with any preceding component. By P1, the last components is a wheel with a star. Now, in order for this to form, the first spoke agent in C_m, say k_{m1}, must find it profitable to connect to an agent $k' \in W_m$ and not to the centre of C_j, say k''. Now if $W_j \subset C_j$, then there are agents in C_j connected as spoke agents, which implies that distance to these spoke agents through k'' is the smallest possible i.e. 1. If k_{m1} connects to $k' \in W_m$, then the number of agents in W_m has to be large enough so that it gives a higher payoff than the payoff from k''. A necessary condition for that is $n_j < \#W_m$. Note that lower the p, the greater the required difference between $\#W_m$ and n_j. In the extreme case of $p \simeq 0$, the difference is so high that two components are not possible.

Example 1 *Possibility of two components in equilibrium.*

Consider the equilibrium as depicted in the following figure. In equilibrium, 3 is rejecting 2's link and forming a W_4 with 4,5,6 with agents 7-N choosing one agent in the wheel. Now for this to be equilibrium p has to be such that the following conditions are satisfied:

(i) Given g_7, 7-10 cannot form a W_4 and 7-11 cannot form a W_5.

(ii) If 3-5 form W_3, 6-9 will form W_4.

(iii) if 2-5 form W_4, 6-10 can form W_5.

(iv) 3 prefers W_4 with 4,5,6 to W_5 with 2,4,5,6.

Condition (iii) actually implies (ii), since if 6-10 can form a W_5, 6-9 can form a W_4 because the incentives for forming a wheel are stronger for earlier agents. Cond (i) ensures that agent 7 onwards are spoke agents and do not form another wheel. If Cond (iii) does not hold, i.e. if 6-10 cannot form a wheel, then 3 could accept 2's link and form W_4 earlier with agents 6 onwards becoming spoke agents. This gives a higher payoff to 3 and hence, the proposed structure is not an equilibrium. Hence it must be the case that if 3 accept's 2's link he has to form a W_5 to have a positive probability of becoming the centre. Condition (iv) gives the conditions for 3 to prefer rejecting (and forming W_4) to accepting 2's link (and having to form W_5).

The conditions imply bounds on p. The bounds on p are given by the following conditions:

(ia) $\frac{N-11}{5}(1 + 2p + 2p^2) < p + 2p^2 + p^3 + (N - 11)p$

(ib) $46p - p^2 - 9 > 0$

(ii) $2p + p^3 - p^4 < 1$

(iii) $\frac{N-10}{5}(1 + 2p + 2p^2) > p + 2p^2 + p^3 + (N - 10)p$

(iv) $(N - 6)(1 + 2p - 3p^2) > 20p$

These conditions are satisfied for a $p \in N_\varepsilon(p = 0.3)$ *for* $N = 20$.

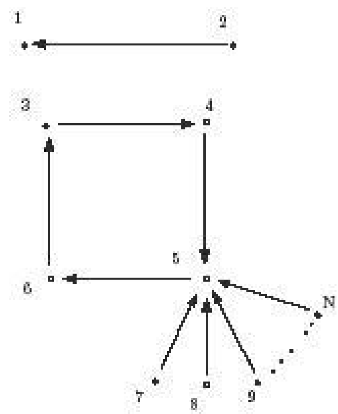

Figure 3: Two Components

The conditions in Example 1, however, need not be satisfied for a N higher or lower. For example, with p fixed at 0.3, the conditions do not hold for $N = 25$ or $N = 8$.

4 Welfare and Efficiency

In this section we focus on the efficient network architecture for different values of the decay factor. An *efficient* network architecture is defined as one that maximises aggregate payoffs of all agents. Note that when there is decay i.e.$p < 1$, increasing distances between agent reduce their payoffs. Hence, the efficient structure should minimize the distances between agents.

Proposition 3 *The efficient network architecture is (i) a star if* $p > \frac{2-c}{2}$ *and (ii) a wheel of length 3 with a local star if* $p < \frac{2-c}{2}$.

Proof. (See Appendix A). ■

It is fairly simple to see that when $c < 1$, all agents must be connected (in a single component) in an efficient network. Now, N agents have to be connected with at least $N-1$ links, Also, given the capacity, the maximum number of total links in a network with N agents is N where all agents initiate a link. First we show that the aggregate payoff in a WS_k architecture is higher than that in a WS_{k+1} architecture for $k > 3$ which implies that among networks with N links, the efficient one is WS_3. This involves a cost of Nc. Among networks with $N-1$ links, the star network minimizes distances between agents though the centre has the most links. Any redistribution of links from the center to a peripheral nodes would only increase the distances between all the agents in the network and hence lower aggregate payoff. Note that the star involves a total cost of

$(N-1)c$ since 1 abstains and is the efficient one among networks with $N-1$ links. Now, note that the condition $p < \frac{2-c}{2}$ can be rewritten as $c < 2-2p$. This implies that an additional link between any two peripheral nodes increases the total payoff by $2-2p$ at a cost of c. If $p < \frac{2-c}{2}$, it is an improvement and hence WS_3 obtains as the efficient structure since it does not increase the distances between agents (as compared to a star) but gives a positive net benefit to 2 agents connected by this extra link.

This also shows the tension between efficiency and the equilibrium. The conflict disappears for $p \simeq 1$, particularly $p > Max\{\frac{2-c}{2}, p^*\}$, when the equilibrium is a star which is also efficient. But for lower values of p, the equilibrium is inefficient as seen from the equilibrium for $p \simeq 0$. There might also be another range of p where the equilibrium is efficient. This happens if there is a $p < \frac{2-c}{2}$, for which the equilibrium structure is WS_3. In general, the relationship between equilibrium efficiency and p is non-monotonic.

5 Heterogeneous Agents (not complete)

In most networks agents are not likely to be symmetric and identical. Individuals differ in their ability to provide favours as well as in the time they spend on social relationships. In this section we consider the first type of heterogeneity by making the benefits agents provide to the network, individual-specific. In particular, we consider two types of agents according to whether they provide a high benefit θ_h or a low one θ_l. The high (low) type could represent those agents in an ethnic network who are (not so) well-placed in the labour market. Examples include a manager of a local bank who can help in credit and a labourer who can lend a hand in farm-work for a friend. Introducing heterogeneity, therefore, brings forth the issue of whether to connect to an isolated high type agent or a well-connected low type agent. This of course, would depend on the difference $\theta_h - \theta_l$. Also, the proportion of high types is likely to matter since low types may not be succeed to become well-connected in presence of a high proportion of $h-$agents. In this section, we first analyse the same game when there is a single high type in the population.

For this purpose, let $\theta_l = 1 < \theta_h$.

Proposition 4 *With one $h-type$, θ_h in the population and $p \simeq 1$, the equilibrium network structure is a complete star or a WS_3 with the θ_h as the centre if $\theta_h > 1 + p$.*

Proof. See Appendix B. ■

With a high type agent in the population and $p \simeq 1$, a θ_l agent fails to compete with a connected θ_h with regard to centrality. The only way to compete with a θ_h is to keep him isolated. But when $p \simeq 1$, the incentive to directly connect to θ_h dominates and a θ_h*centred star* obtains.

6 Robustness

6.1 Random Proposers

One restrictive feature of our model is the specific order of moves of players. In particular the fact that period t's responder becomes next period's proposer does have some implications on the equilibrium architecture. This order reduces the possible types of histories that agent t faces at any stage t and simplifies the analysis for $p \simeq 1$ and $p \simeq 0$. It is also responsible for possible rejection by a responder in a subgame (leading to 2 components in equilibrium) since the order gives some power to the responder by making him the proposer of the in the next stage. If the order of proposers is made completely random at each stage, then for $p \simeq 1$however, our result will not change and a complete star network would form with either the 1st or 2nd mover as the centre since it is still the case that the incentive to become the centre is very small for each agent.

Proposition 5 *With the order of proposers completely random in each period, the equilibrium network is a complete star for $p \simeq 1$.*

Proof. The Proof is similar to the proof of Proposition 1. The difference is that the payoff functions in this case are expected payoffs, the expectation taken with respect to the random order. The payoffs in Prop 1 are valid for a unique realisation of the order which chooses the responder of a period as the next proposer. For all other realisations of draws for proposers each period, the payoff functions are modified but the inequalities still hold. We will just point out the difference in the payoffs and show that the inequalities hold in each case.

Step 1: Any agent $t > 1$, if isolated chooses v_t^{iso}.

This statement is true for the last agent moving, $t = N$. Now consider k with the statement true for all $\tau > k$.

Consider the payoff for strategy $s_k = k + 1$.

For Case I, the payoff is $E\pi_k(k+1) = \pi_k(k+1|order\ r).\Pr ob(order\ r)$.

Now, $\pi_k(k+1)$ for each order r is of the form

$$1 + p + p^2 + ... + p^{j_0-k-y-1} + p^{j_0-k-y}(1 + [a] + p * y) + +(N - j_o)p^{d'}$$

where the order r is such that y agents between k and j_0 were isolated when chosen as proposers and chose v_k^{iso} since statement if true for all $\tau > k$. It is easy to see that

$$\begin{aligned}\pi_k(k+1|order\ r) &= 1 + p + p^2 + ... + p^{j_0-k-y-1} + p^{j_0-k-y}(1 + [a] + p * y) + +(N - j_o)p^{d'} \\ &< 1 + (j_0 - k - y)p + [a] + py + (N - j_0)p \\ &= 1 + [a] + (N - k)p = \pi_k(v_k^{iso})\end{aligned}$$

Hence, $E\pi_k(k+1) < \pi_k(v_k^{iso})$. Case II is similar.

For Case III, due to the random proposers the wheel length that can form following $s_k = k+1$, is random and hence $E\pi_k(k+1)$ includes expected length El instead of some $l > 1$. Note again that $El \leq l$ since with positive probability a positive number y of isolated agent is chosen as the proposer and he chooses v^{iso} which implies that he cannot belong to the wheel component. (It could also be the case that no wheel forms). Hence

$$E\pi_k(k+1) = [2 + 2p + ..\frac{El}{2} terms] + E[\frac{N-k-l-y}{l+1}(1 + p + ..l + 1 terms)]$$

As before, $\pi_k(v_k^{iso}) = 1 + [a] + (N-k)p$

For $p = 1, E\pi_k(k+1) = N-k-y \leq N-k < 1 + [a]|_{p=1} + (N-k) = \pi_k(v_k^{iso})|_{p=1}$

By similar logic, for $p \simeq 1, E\pi_k(k+1) < \pi_k(v_k^{iso})$ and hence $s_k^* = v_k^{iso}$.

The rest of the proof is exactly same as that of proposition 1. ■

The difficulty with random proposers arises in analysing the case for lower p. For $p \simeq 0$, the order considered in the model facilitates formation of the wheel since there is no randomness in the future play at any stage. With random proposers, the responder of stage t might not be the proposer at $t+1$. Consider the following example.

Example

Let $N = 8$. Consider the subgame at the beginning of period 4 where agents $1, 2, 3, 4'$ are connected in a chain. With the particular order we have in our model, $4'$ is the next proposer for sure and and hence proposes to 1 to complete the wheel. But suppose the proposer chosen at $t = 4$ is some other (isolated) agent j', then the wheel cannot be completed. In this case, j' can connect to 1, can connect to 4 or connect to some central agent (2 or 3 in this case). For $p \simeq 0$, j' s optimal choice would be agent 1. To see this consider the following subgame G. $G = \{12\} + \{23\} + \{34'\} + \{41\} + \{54\} + \{65\}$. Hence there are two periods remaining and 2 agents, $4'$ and $8'$ left to propose. With probability 1/2 agent $4'$ is chosen as the proposer. He has 3 options.

(i) $s = 6$

(ii) $s = v^{con}$

(iii) $s = 8'$.

If agent $4'$ who is now renumbered as 7 chooses 6, a wheel of length 7 is formed and the last agent $8' \equiv 8$ link sto one of them with equal probability. Hence, $\pi(s = 6) = 2 + 2p + 2p^2 + \frac{1}{7}(1 + 2p + ... + 2p^3)$.

If $s = j = v^{con}$, then the last agent chooses j and $\pi(s = v^{con}) = 1 + r_1p + r_2p^2 + .. + r_qp^q$

If $s = 8'$, then agent $8'$ in the last period chooses v^{con} and $\pi(s = 8') = 2 + r'_1p + ... + r'_qp^q$

For $p \simeq 0$, strategy $s = 6$ dominates the others and $4'$ will choose 6.

After period 6 however, with probability 1/2 agent $8'$ is chosen as the proposer at $t = 7$. In that case, 7 knows that the last agent $4'$ will choose v^{con} and hence agent 7 cannot be the centre (since he is one the extreme end agents). Hence 7 chooses v^{iso}.

Now, at $t = 6$, when an isolated agent is chosen and faces the graph $G - \{65\}$, his payoff from linking to 5 is

$$\pi_6(s = 5, G) = \frac{1}{2}[2 + 2p + 2p^2 + \frac{1}{7}(1 + 2p + ... + 2p^3)] + \frac{1}{2}[1 + r_1 p + ... r_q p^q]$$

His payoff from $s = v^{iso}$ is

$$\pi_6(v^{iso}, G) = 1 + r'_1 p + ... + r'_q p^q$$

For $p \simeq 0$, $\pi_6(s = 5, G) > \pi_6(v^{iso}, G)$ and agent 6 links to agent 5.

The same incentive works for any isolated agent chosen as the proposer. But the exact network formed depends on the realisations of the orders of prposers. If at some stage, responder at stage t is not chosen as the proposer at any $\tau, t < \tau < N$, then the structure contains no wheel. If responder of stage t is chosen as the proposer as any time before the last period, then a wheel forms and the remaining agents link to one of the agents in the wheel. The wheel size in this case is weakly bigger since the responder at time 4, could be chosen to propose only at time 6, say, resulting in a wheel of size 6.

Hence with random proposers, the outcome is not deterministic. With positive probability the architecture could be any of $WS_{N/2}$ and $WS_{k>N/2}$. The architecture could also not have a wheel if an agent connected in the chain gets to propose only in the last period. For a general p, the order of random proposers would take away power from the responder since he might be chosen to propose only at the end of play. In this case, we might not see rejections in any subgame and a single equilibrium component unlike example 1.

7 Discussion

This paper focusses on the network architectures that arise in equilibrium when agents are farsighted and capacity constrained and there are decreasing returns (represented by the decay factor p). In our model, the constraint is albeit an extreme one, in that each agent is allowed to intiate only a single link. This restriction, though not realistic, does allow us to get clear patterns and architectures for some decay factors. The extreme level of constraint is particularly important to reinforce/highlight the strategic concerns when agents are farsighted/non-myopic. With an increase in capacity agents still face the same tradeoff but the conflict between the current and the future reduces. For example, if we remove the constraint altogether i.e. agents can initiate any number of links, then for $c < 1$ the complete network obtains in equilibrium with earlier agents having the advantage of some level of free-riding.

We also assumed that $c < 1$. Our results also hold for any $c < 1 + p$. For higher values of cost, the result will be similar to Derorian (2006) when $p \simeq 1$ since agents might choose to abstain rather than incur the high cost. This will give rise to more than one component in the equilibrium configuration since some of the agents who abstain will remain isolated.

Our first result of a mixed or periphery-sponsored star for p close to 1 is consistent with the results of Bala and Goyal (2000), Galeotti et al. (2006), Kannan et al (2007) and Hojman and Szeidl (2004) who show the emergence of mixed or periphery sponsored stars in models with small amount of decay. Most of the work in the literature does not have a characterisation for a general p. The result of a wheel with local star for p close to 0 has some similarity to the flower networks in the one-way flow model with decay in BG (2000). Galeotti (2006) also finds a wheel with a (multiple) centre-sponsored star(s) as the equilibrium network for some parameters in a one-way flow model with heterogeneous values and costs and without decay. The higher value agents belong to the wheel and become the centres of the stars. The payoff inequality in this case is even more extreme since benefits flow one way only. The formation of wheels is similar to Watts (2001b) who proposes a dynamic model of network formation and shows that circle networks may be formed and supported by a strategy similar to trigger strategy. For a general level of p we show that there might be two components in equilibrium. The reason for this is that an agent wants a positive probability of being the centre and also share this probability will least number of agents possible. This incentive to share with less agents might cause a rejection of a link in some (off-equilibrium) subgame. Watts (2001a) also points out in a discussion that one might expect rejections in some off-equilibrium path because of the forward looking behaviour of agents. This rejection decreases the probability of the formation of a star when agents are not myopic in her model since the payoff to being a central agent is lower than being non-central and hence no agent wants to be the centre. There is, however no result for forward-looking agents..Another point to note is that, in our model, it might be the case that middle-ranked agents have an advantage relative to intial or end agents, as seen in Example 1 where intial agents were stranded and end agents were spoke agents with the middle rank being the centre. In fact, initial agents are worst-off in the specific example.

8 Conclusion

This paper studies the network formation process when agents are capacity constrained and forward looking. The types of networks that form under different environments is crucial since the exact structure of these networks significantly affect economic outcomes. We find that the shapes of equilibrium networks depend crucially on the rate of decreasing returns in the payoff function. With a low decay a complete star network forms while with high levels of decay the equilibrium architecture involves a wheel with a local star. This incidentally resembles the hub-and-spoke architecture observed in studies of R&D firms, the airline industry and social groups. For intermediate levels of decay, we have an example where some agents might be isolated from a bigger component, which again has the wheel-star structure.

A promising area of further work is to analyse the effect of a changing marginal cost of link formation when agents have a capacity of r links, $1 < r < N-1$ One could also analyse a form of heterogeneity encompassing both one's value in the network and one's capacity. For example, suppose an agent is some particular ability θ which is also the value of benefits he provides in the network. One situation could be that a high θ agent has a higher wage in the labour market and hence a higher opportunity cost of investing in social links. This implies that the optimal level of social links that a high type intiates would be less than a low type. Hence the heterogeneity in innate ability determines both one's value in network and his capacity which are, in this case, negatively related. The other possibility is that a high type agent is overall more efficient in the market and hence has more time available for his social network which implies a positive correlation between one's level of human capital and his social investment. A potential extension would be to explore the equilibrium networks with both these types of heterogeneity.

References

Acemoglu, D.; G. Egorov and K. Sonin (2007), "Coalition Formation in Political Games", mimeo,

Aumann, R and R. Myerson (1988) "Endogenous Formation of Links Between Players and of Coalitions: an Application of the Shapley Value" in: Roth, A.E. ed., The Shapley Value, Cambridge University Press, 175-191.

Bala, V. and S. Goyal (2000a) "A Noncooperative Model of Network Formation", *Econometrica*, 68, 1181-1229

Bala, V. and S. Goyal (2000b) "A Strategic Analysis of Network Reliability," Review of Economic Design, 5, 205-228.

Banerjee, A. and K. Munshi (2003), "How Efficiently is Capital Allocated? Evidence from the Knitted Garment Industry in Tirupur" BREAD Working Paper no 4.

Billand, P., C. Bravard and S. Sarangi (2006) "Heterogeneity in Nash Networks", Working paper, Louisiana State University.

Burt, R. (1992) "Structural Holes:the Social Structure of Competition", Harvard University Press.

Chatterjee, K.; B. Dutta; D. Ray; K.Sengupta (1993) "A Noncooperative Theory of Coalitional Bargaining", *Review of Economic Studies*, 60, 463-477.

Coleman, J. (1988), "Social Capital in the Creation of Human Capital", *American Journal of Sociology*, 94, 95-120.

Conley, T and C. Udry (2001), "Social Learning Through Networks: The Adoption of New Agricultural Technologies in Ghana" *American Journal of Agricultural Economics*, 83(3), 668-673.

Delapierre, M. and L. Mytelka (1998) " Blurring Boundaries: new inter-firm relationship and the emergence of networked, knowledge-based oligopolies. In: Colombo, M.G. (Ed.) The Changing Boundaries of the Firm. Routledge Press, London.

Dutta, B and S. Mutuswami (1997) "Stable Networks", *Journal of Economic Theory*, 76, 322-344.

Dutta, B., A. van den Nouweland and S. Tijs (1998) "Link Formation in Cooperative Structures", *International Journal of Game Theory*, 27, 245-256.

Ellison, G. and D. Fudenberg (1995), "Word-of-Mouth and Social Learning", *Quarterly J. of Economics*, 110, 93-126.

Fafchamps, M and S. Lund (2001) "Risk-Sharing Networks in Rural Philippines", mimeo, Oxford University.

Feick L.F. and L.L. Price (1987) "The Market Maven: A Diffuser of Marketplace Information" Journal of Marketing, 51-1, 83-97

Galeotti, A. (2006) "One-way Flow Networks: the Role of Heterogeneity", *Economic Theory*, 29, 163-179.

Galeotti, A., S.Goyal and J. Kamphorst (2006) "Network Formation with Heterogeneous Players", *Games and Economic Behaviour*, 54, 353-372.

Goyal, S and S. Joshi (2003) "Networks of Collaboration in Oligopoly", *Games and Economic Behaviour,* 43, 57-85.

Goyal, S. and J.L. Moraga-Gonzalez (2001) "R&D Networks", *RAND Journal of Economics*, 32, 686-707.

Granovetter, M (1974) *Getting a Job:a study of contacts and careers*, University of Chicago Press, Chicago.

Haller, H and S. Sarangi (2001) " Nash Networks with Heterogeneous Agents", mimeo, Virginia Polytechnic Institute.

Hojman, D and A.Szeidl (2004) "Core and Periphery in Endogenous Networks", mimeo,

Holzer, H (1987) " Hiring Procedures in the Firm: Their Economic Determinants and Outcomes", NBER Working Paper No. 2185.

Jackson, M.O. (2003) "A Survey of Models of Network Formation: Stability and Efficiency", mimeo, Caltech.

Jackson, M.O. and A. Wolinsky (1996) "A Strategic Model of Social and Economic Networks", *Journal of Economic Theory*, 71, 44-74.

Johnson, C. and R.P. Gilles (2000) "Spatial Social Networks", Review of Economic Design, 5, 273-299.

Lazerson, M. (1993) "Factory or Putting-Out? Knitting Networks in Modena" in Grabher, G, ed., The Embedded Firm: On the Socioeconomics of Industrial Networks, 203-226, Routledge, New York.

McMillan, J. and C. Woodruff (1999) "Interfirm Relationships and Informal Credit in Vietnam", Quarterly Journal Of Economics, 114, 1285-1320.

Montgomery, J. (1991) "Social Networks and Labour Market Outcomes: Towards an Economic Analysis", American Economic Review, 81, 1408-1418.

Myerson, R. (1977) "Graphs and Cooperation in Games", Mathematics of Operations Research, 2, 225-229.

Nishiguchi, T. (1994) Strategic Industrial Sourcing, Oxford University Press, New York.

Okada, A. (1996) "A Noncooperative Coalitional Bargaining Game with Random Proposers", Games and Economic Behaviour, 16, 97-108.

Powell, W.W. ; K W. Koput, L. Smith-Doerr;(1996) "Interorganizational Collaboration and the Locus of Innovation: Networks of Learning in Biotechnology", *Administrative Science Quarterly*, 41,

Sarangi, S., R. Kannan and L. Ray (2003) "The Structure of Information Networks", mimeo, Louisiana State University.

Selten, R. (1981) "A Noncooperative model of Characteristic Function Bargaining", in Bohm,V. and Nachtkamp, H (eds), Essays in Game Theory and Mathematical Economics in Honour of Oskar Morgenstern.

Shaked, A. (1986), "Three-Person Bargaining", mimeo, University of Bonn.

Wasserman. S. and K. Faust (1994) "Social Network Analysis: Methods and Applications", Cambridge University Press.

Watts, A. (2001a) "A Dynamic Model of Network Formation" *Games and Economic Behaviour*, 34, 331-341.

Watts, A. (2001b) "Formation of Circle Networks", *Economic Letters*, 74, 272-282

Appendix A

Proposition 1 *There exists a p^* such that for $p \in [p^*, 1]$,the unique subgame-perfect equilibrium structure is a (complete or incomplete) star.*

Proof:

Step 1: We claim that any agent $t > 1$, if isolated chooses v_t^{iso}.

This statement is obviously true for $t = N$.

Suppose that the statement is true for agent $\tau \in \{k+1, k+2, ..., N\}$. We will prove it true for agent $\tau = k$.

Agent k has two options:

i) $s_k = k + 1$

ii) $s_k = j < k$

Note that if $s_k = j < k$, then $s_k = v_k^{iso}$ since in that case the myopic payoff π_k^{myo} is maximised and $v_k^{iso} = v_\tau^{iso}, \tau > k$ and hence all future agent choose v_k^{iso}. This implies that the future payoff is $p(N-k)$ which is the maximum possible future payoff given that k is isolated. (i.e. if k chooses some $j < k$, then $k \neq v_{k+1}^{iso}$ and $\pi^f \leq (N-k)p$). We can write the payoff from (ii) explicitly as

$$\pi_k(v_k^{iso}) = V_{k-1} + p(N-k)$$

where $V_{k-1} = 1+[a]$ denotes the maximum total payoff from agents $1, 2, .., k-1$ (i.e. through v_k^{iso}).

Now the payoff from (i) depends on the equilibrium strategy of the players starting this subgame. Denote the subgame by G_k.

Case I: On the equilibrium path, let j_0 be defined as the first player such that $s_{j_o} = j < k; j_0 \in \{k+1, k+2, ..., N\}$[9]

Hence, $j_0 + 1$ is isolated and chooses $v_{j_0+1}^{iso} \neq k$. In this case

$$\begin{aligned}
\pi_k(k+1) &= 1 + p + p^2 + ... + p^{j_0-k-1} + p^{j_0-k}V_{k-1} + (N-j_0)p^{d'} \\
&= 1 + p + p^2 + ... + p^{j_0-k-1} + p^{j_0-k}(1+[a]) + (N-j_0)p^{d'} \\
&= 1 + (p + p^2 + ... + p^{j_0-k-1} + p^{j_0-k}) + p^{j_0-k}[a] + (N-j_0)p^{d'} \\
&< 1 + (j_0-k)p + [a] + (N-j_0)p = \pi_k(v_k^{iso})
\end{aligned}$$

where $d' = d(v_{j_0+1}^{iso}, k, g)$.

QED

Case II: As before, define j_1 as the first player who chooses some agent j', $k < j' < j_1 - 1$ on the equilibrium path. Since, $j' \neq k$, $k \neq v_{j_1+1}^{iso}$. The maximum payoff for k is when all agents $\{j_1+1, ..., N\}$ connect to some agent j'' such that $d(k, j'') = d' = 1$. In this case,

$$\pi_k(k+1) = 1 + r_1p + r_2p^2 + ... + r_qp^q$$

where $1 + r_1 + r_2 + ... + r_q = N - k$

[9] j_0 exists since agent N would link to some player $j < N$.

Now, it is immediate that

$$\begin{aligned} \pi_k(k+1) &= 1 + r_1 p + r_2 p^2 + ... + r_q p^q \\ &\leq 1 + (N-k-1)p \\ &< 1 + (N-k)p \\ &\leq 1 + [a] + (N-k)p \\ &= \pi_k(v_k^{iso}) \end{aligned}$$

Case III : Suppose agent $k+l$ chooses k, hence forming a wheel. The best situation for k is when the probability of being chosen by agents $\tau > k+l$ is positive. Let us focus on that situation. In equilibrium the wheel contains (l +1) agents and $N-k-l$ agents choose $j \in W_l$ randomly.

Then

$$\pi_k(k+1) = [2+2p+.....\frac{l}{2} \text{ terms}] + \frac{N-k-l}{l+1}(1+p+p+p^2+p^2+.....l+1 \text{ terms})$$

Recall that

$$\pi_k(v_k^{iso}) = 1 + [a] + p(N-k)$$

Now, both payoffs are monotone increasing in p.

For $p = 0, \pi_k(k+1) > \pi_k(v_k^{iso})$

For $p = 1, \pi_k(k+1) = N-k < 1 + [a]|_{p=1} + N - k = \pi_k(v_k^{iso})$

Hence there exists a $p_k^* < 1$, such that for $p > p_k^*$, $\pi_k(v_k^{iso}) > \pi_k(k+1)$ and hence $s_k^* = v_k^{iso}$.

Since this is for a general k, we can conclude that for $p > p^* = Max\{p_N^*, p_{N-1}^*, ..., p_2^*\}$, any isolated agent $t > 1$ would connect to v_t^{iso}.

Step 2: We argue that agent 1 would abstain so that 2 is isolated and 2 chooses 1 by Step 1.

Given a p high enough, 1 knows that if 1 abstains then 2 is isolated and chooses 1. In that subgame, 3 is isolated and chooses one of $\{1, 2\}$. Hence the expected payoff of 1 from the strategy ϕ is

$$E\pi_1(\phi) = \frac{1}{2}[N-1+1+(N-1)p]$$

If 1 chooses 2, then following that subgame, any equilibrium network involves one of the following

(i) 2 abstains.

(ii) 2 chooses 3.

In case (i), 2 and 1 are symmetric and hence equal probability of being chosen as the centre (decided by the choice of 3). The payoff of 1 in this case if

$$\frac{1}{2}[N-1+1+(N-1)p] - c = E\pi_1(\phi) - c$$

In case (ii), 3 could abstain, choose 1 or choose 4. If 3 abstains then 2 becomes the most central agent and 1 the peripheral node for sure. Agent 1's payoff is therefore $1 + (N-1)p - c$ which is less than $E\pi_1(\phi)$. If 3 chooses 1, then 4 is indifferent between 1, 2 and 3. In this case 1's probability of being centre is lower at 1/3 and thus gets a lower payoff, that too at a cost. The same argument can be applied when 3 links to 4.

Hence, 1's payoff by the strategy $s_1(2) < s_1(\phi)$. So, 1 abstains and 2 links to 1 followed by agent 3 who chooses one of 1 or 2 randomly.

Proposition 2: *There exists a p^{**} such that for $p \leq p^{**}$, the unique equilibrium network structure is a wheel of length $L < N$ with a local star. Also, $L = N/2$ or $(N+1)/2$ if N is even or odd respectively.*

Proof: Consider a subgame where agent $N-1$ is connected and hence by Lemma 1 belong to a chain of length l denoted by Ch_l. Suppose there is another component C'' with $C'' \cap Ch_l = \phi$. Let the maximum value of C'' to any agent connecting to C'' be denoted by $V(C'')$. This will depend on the number of agents in C'' and its architecture.In general it is of the form $r_1p + r_2p^2 + ... + r_qp^q$ where $r_1 + r_2 + ... + r_q = \#C''$.

We know by Lemma 4 and 5 that if $W_{l'} \in g_{N-l-1}, l' > l$, then $N-1$ will not choose to form a wheel since probability of being chosen by N is 0. Now say $l' \leq l$. Hence we focus on subgames such that, if $s_{N-1} = N-l$,i.e. $N-1$ forms a W_l, then $s^*_N = j \in W_l$. Hence, $N-1$ might form the wheel for the precise reason that N will link to $N-1$ with positive probability.

Now, among this type of subgames $N-1$ has the following options:

i) $s = N - l(wheel)$

ii) $s = j \notin \{N, N-l\}$

iii) $s = N$

Step 1: We rule out strategy (iii) for p low enough.

Suppose, $s_{N-1} = N$. In the subgame following $s_{N-1} = N$, agent N has 2 options

a) $s_N = j \in C_l$

b) $s_N = j \in C''$

Note that if N chooses $j \in C_l$, he will choose j=argmax$_{j\in C_l}$ $\pi_N^{myo} = j^*$. Similarly, $b)$ implies $j = j^{**} \in C''$.

Now, if (a) $\succ$ (b)$\Rightarrow \pi_N(a) > \pi_N(b)$ i.e.

$$\begin{aligned} \pi_N(a) &= \pi_N(j^*) = 2 + 2p + ... + 2p^{\frac{l+1}{3}} + [p + p^2 + ... + p^{\frac{l+1}{3}}] \qquad (@) \\ &> \pi_N(b) = [1 + p + p^2 + ... + p^{l-1}] + V(C'') \end{aligned}$$

In this case,

$$\pi_{N-1}(iii) = 2 + 2p + ... + 2p^{\frac{l+1}{3}} + [p^2 + p^3 + ... + p^{\frac{l+4}{3}}]$$

Suppose $N-1$ deviates to $s_{N-1} = j^*$. Now in this (deviation) subgame, say, g^d, N is isolated and can choose between j^* and j^{**}.

It is simple to verify that $\pi_N(j^*, g^d) > \pi_N(j^{**}, g^d)$ given @.
$[\pi_N(j^*, g^d) = 1 + [p + p^2 + ... + p^{\frac{l+1}{3}}] + [2p + 2p^2 + ... + 2p^{\frac{l+1}{3}-1} + p^{\frac{l+1}{3}}]$
$= 2 + 2p + ... + 2p^{\frac{l+1}{3}} + [p + p^2 + ... + p^{\frac{l+1}{3}}] - 1 - p^{\frac{l+1}{3}}$
$> [1 + p + p^2 + ... + p^{l-1}] + V(C'') - 1 - p^{\frac{l+1}{3}}$
$= V(C'') + p - p^{\frac{l+1}{3}} + [p^2 + ... + p^{l-1}]$
$> V(C'') = \pi_N(j^{**}, g^d)]$
Therefore

$$\begin{aligned}
\pi_{N-1}(j^*) &= [2 + 2p + ... + 2p^{\frac{l+1}{3}-1} + p^{\frac{l+1}{3}}] + [p + p^2 + ... + p^{\frac{l+1}{3}}] + \pi^f \\
&= [2 + 2p + ... + 2p^{\frac{l+1}{3}}] - p^{\frac{l+1}{3}} + [p + p^2 + p^3 + ... + p^{\frac{l+1}{3}}] + p \\
&> [2 + 2p + ... + 2p^{\frac{l+1}{3}}] + [p - p^{\frac{l+1}{3}}] + [p^2 + p^3 + ... + p^{\frac{l+4}{3}}] \\
&> 2 + 2p + ... + 2p^{\frac{l+1}{3}} + [p^2 + p^3 + ... + p^{\frac{l+4}{3}}] \\
&= \pi_{N-1}(iii)
\end{aligned}$$

Hence, $N - 1$ will deviate and $s_{N-1} \neq N$

If (b) $\succ$(a), then

$$\pi_{N-1}(iii) = [1 + p + p^2 + ... + p^{l-2}] + 1 + pV(C'') \qquad \text{(P(iii))}$$

If $N - 1$ chooses $s = N - l, i.e.(i)$

$$\pi_{N-1}(i) = 2 + 2p + ... + 2p^{\frac{l-2}{2}} + \frac{1}{l}[1 + 2p + ... + 2p^{\frac{l}{2}}] \qquad \text{(P(i))}$$

Now compare P(i) and P(iii).Both are monotone in p. At $p = 1, P(iii) > P(i)$. At $p = 0, P(iii) < P(i)$.

So, there exists a $\widetilde{p}_{N-1} > 0, s.t. for\ p < \widetilde{p}_{N-1}$, $s_{N-1} = N$ is dominated by $s_{N-1} = N - l(s_w)$.

Step 2: To rule out strategy (ii).
Consider the strategies $s = s_w$ and $s = j < N - l$.
The payoffs to $N - 1$ from the two strategies are:
$\pi_{N-1}(W_l) = 2 + 2p + ... + 2p^{\frac{l-3}{2}} + \frac{1}{l}(1 + 2p + 2p^2 + ... + 2p^{\frac{l-1}{2}})$
$\pi_{N-1}(j < N - l) = 1 + p + p^2 + ... + p^{l-2} + 1 + k_1p + k_2p^2 + ... + k_rp^r$
$\pi_{N-1}(W_l) > \pi_{N-1}(j < N - l)$
$\Leftrightarrow 2 + 2p + ... + 2p^{\frac{l-3}{2}} + \frac{1}{l}(1 + 2p + 2p^2 + ... + 2p^{\frac{l-1}{2}}) > 1 + p + p^2 + ... + p^{l-2} + 1 + k_1p + k_2p^2 + ... + k_rp^r$
$\Leftrightarrow 2 + 2p + ... + 2p^{\frac{l-3}{2}} + \frac{1}{l}(1 + 2p + 2p^2 + ... + 2p^{\frac{l-1}{2}}) > 2 + p + p^2 + ... + p^{l-2} + k_1p + k_2p^2 + ... + k_rp^r$
Both LHS and RHS are monotone in p.
Also for $p = 0$, LHS > RHS
and for $p = 1$, RHS $> l = LHS$
Hence $\exists$a $\widehat{p}_{N-1} \in (0, 1)$, such that $LHS > RHS$ for $p < \widehat{p}_{N-1}$ and agent $N - 1$ prefers to form the wheel.

Now consider $s = s_w$ and $s = j_0 \in intC_l$.

If $s_{N-1} = j \in C_l$ and N chooses $j \in V(C'')$ in this subgame, the maximum $N-1$ can get is

$$\pi_{N-1}(j_0) = 2 + 2p + ... + 2p^{\frac{l}{3}-1} + [p + p^2 + ... + p^{\frac{l}{3}}]$$

which is $< \pi(s_w)$ for all p.

If N does not choose $j \in V(C'')$, then he chooses j_0 and

$$\pi_{N-1}(j_0) = 2 + 2p + ... + 2p^{\frac{l}{3}-1} + [p + p^2 + ... + p^{\frac{l}{3}}] + p$$

which is $< \pi(s_w)$ for any $p < 1$.

So, $s_{N-1} = s_w$ strictly dominates $s = s' \neq s_w$ for $p < p_{N-1} = Min\{\widehat{p}_{N-1}, \widetilde{p}_{N-1}\}$.[10]

For agent $N-2$ similarly we can get the threshold value of p such that for any $p < \widehat{p}_{N-2}$, agent $N-2$ would prefer to form the wheel. It is fairly simple to show that $\widehat{p}_{N-1} \leq \widehat{p}_{N-2} \leq .. \leq \widehat{p}_{N-l}$.

Hence for $p < \widehat{p}_{N-1}$, an agent at time t, given graph g_{t-1} would prefer to form a wheel if the probability of being chosen by any future agent is positive. This implies that if agent t completes a wheel of length l and the number of remaining agents $N-t \geq l+2$, then agents $\{t+1, t+2, ..., t+l+1\}$ would form a wheel of length $l+1$.Hence in equilibrium no agent t would complete W_l if $N-t \geq l+2$. Let the number of remaining agents be x_t for agent t. Hence if t forms a W_l, the upper bound for x_t for getting any future payoff is l. Hence

$$\begin{aligned}
\pi_t(W_l) &= 2 + 2p + ... + 2p^{\frac{l-3}{2}} + \frac{x_t}{l}(1 + 2p + ... + 2p^{\frac{l-1}{2}}) \\
&= \frac{x_t}{l} + [2 + 2p + ... + 2p^{\frac{l-3}{2}}]\frac{px_t}{l} \\
&\leq 1 + [2 + 2p + ... + 2p^{\frac{l-3}{2}}]p
\end{aligned}$$

Note that the payoff $\pi_t(W_l)$ of any agent belonging to W_l is increasing in x but may increase of decrease with l. It is in fact increasing in the ratio $\frac{x}{l}$. The agents want to maximise $\pi_t(W_l)$ with $\frac{x}{l} \leq 1$. At the optimum, $\frac{x}{l}$ should be the maximum i.e. 1. Fixing $\frac{x}{l} = 1$, the payoff is increasing in l. This implies that agents would want to form the largest wheel possible with $x = l$. This is possible only when $N/2$ agents form a wheel with $N/2$ agents remaining who link to an agent in W randomly to form a local star.

(it is easy to see that no agent will reject any link to create 2 components. E.g. say N=20. Consider the subgame where2 proposes to 3. 3 could accept and go on to form a wheel of 10 with last 10 agents as spokes. or 3 could reject in order to form a smaller wheel. But for p low enough, if 3-11 forms a wheel of 9, agents 12-20 become spokes but the max value of $\frac{x}{l} = 1$ which is same as accepting 2 and forming W_{10}. But rejecting 2, implies loss of payoff from 2 (and possibly 1) for any $p > 0$. Hence agents will form the largest wheel possible

[10] If the graph g_{N-1} is such that there are two distinct components C'', C''' with $C'' \cap Ch_l = \phi, C''' \cap Ch_l = \phi$, then the more valuable component would matter and the same proof applies.

(i.e. with $x \leq l$) in equilibrium. It can also be shown that W_{10} entails a higher payoff than W_{11}, since in that case $\frac{x}{l} < 1$ and the loss due to the fall in $\frac{x}{l}$ is higher than the gain due to the extra agent in the wheel for any $p > 0$).

Proposition 3: *The efficient network architecture is (i) a star if $p > \frac{2-c}{2}$ and (ii) a wheel of length 3 with a local star if $p < \frac{2-c}{2}$.*

Proof: First note that due to the capacity constraints of agents, the maximum number of links in any architecture is N. Also, for $c < 1$, each agent would prefer forming a link unless it is redundant (e.g. linking to a person who has already made the link with the agent).

Step 1: The first step of the proof is a direct result from Jackson and Wolinsky (1996)((Proposition 1 section 3.1.1) which shows that for any c, p the star is the efficient structure among all networks with $N-1$ links. Step 2 of the proof shows that WS_3 is the efficient structure among all networks with N links.

Step 2: We show that for any p, the aggregate payoff in a connected network with a wheel of length $k, k > 3$ is less than that in a wheel of length 3 with a local star. For this purpose, we compare the aggregate payoffs for a WS_k and WS_{k+1} for any $k > 3$. Let us assume that k is odd.

Note that the payoff to an agent belonging to the wheel from other agents in the wheel is $(2 + 2p + 2p^2 + ... + 2p^{\frac{k-3}{2}}) = z_w(say)$.

Also, the payoff to any spoke agent from the agents in the wheel is $(1 + 2p + 2p^2 + ... + 2p^{\frac{k-1}{2}}) = z_s(say)$.

The other part of the payoffs are those from the spoke agents. The payoff of each spoke agent from other spoke agents is $(N-k-1)p$. The payoff of a wheel agent from a spoke agent varies according to his position in the wheel. The aggregate payoff of the wheel agents (except the sentre) from the spoke agents can be calculated as

$$2[(N-k)p + (N-k)p^2 + ... + (N-k)p^{\frac{k-1}{2}}] = 2z'$$

while the payoff of the centre from the spoke agents is simply $(N-k)$. Hence the aggregate payoff $P(WS_k)$ is

$$\begin{aligned} P(WS_k) &= k(2 + 2p + 2p^2 + ... + 2p^{\frac{k-3}{2}}) \\ &\quad +(N-k)(1 + 2p + ... + 2p^{\frac{k-1}{2}}) \\ &\quad +(N-k)(N-k-1)p \\ &\quad +2[(N-k)p + (N-k)p^2 + ... + (N-k)p^{\frac{k-1}{2}}] \\ &\quad +(N-k) \\ &= kz_w + (N-k)z_s + 2z' + (N-k)(N-k-1)p + (N-k) \end{aligned}$$

Similarly, we can write out the aggregate payoff from WS_{k+1} as

$$\begin{aligned} P(WS_{k+1}) &= (k+1)(2+2p+...+2p^{\frac{k-3}{2}}+p^{\frac{k-1}{2}}) \\ &+(N-k-1)(1+2p+...+2p^{\frac{k-1}{2}}+p^{\frac{k+1}{2}}) \\ &+(N-k-1)(N-k-2)p \\ &+2(N-k-1)[p+p^2+...p^{\frac{k-1}{2}}]+(N-k-1)p^{\frac{k+1}{2}} \\ &+(N-k-1) \end{aligned}$$

We can show that $P(WS_k) - P(WS_{k+1}) > 0$ for any p, N and $k > 3$.
A similar exercise can be done for k even.
Step 3: Comparing the payoffs from the star and the WS_3, we see that Star $\succ_{eff}$ WS_3 iff

$$\begin{aligned} P(star) &> P(WS_3) \\ 2(N-1)+(N-1)(N-2)p-(N-1)c &> (N-1)+2(2+(N-3)p) \\ &+(N-3)(1+(N-2)p)-Nc \\ (N-1)+(N-1)(N-2)p+c &> 4+2Np-6p+N-3 \\ &+(N-3)(N-2)p \\ c &> 2+2Np-6p-(N-2)2p \\ c &> 2-2p \\ p &> \frac{2-c}{2} \end{aligned}$$

Appendix B

Proposition 6 *With one $h-type$ in the population and $p \simeq 1$, the equilibrium network structure is a complete star or a WS_3 with the θ_h as the centre if $\theta_h > 1+p$.*

Proof. The proof obtains from a series of lemmas. ■

Lemma B.1: Consider a history h_t such that θ_h has moved and $g_t \neq g^e$. Then an (low type) isolated agent at t would choose v_t^{iso} for $p \simeq 1$.
Proof. The proof proceeds as that of Proposition 1 with the term V_{k-1} weakly larger due to the presence of θ_h. Hence, $\pi_k(v_k^{iso})$ is larger for any p and any isolated agent at $t > 1$ will choose v_t^{iso} to maximise the myopic payoff for $p \simeq 1$. This implies that the threshold value of p for agent k is weakly lower. i.e.$p > \widetilde{p}_k, \pi_k(v_k^{iso}) > \pi_k(k+1)$ and $\widetilde{p}_k < p_k^*$ for any k. This in turn implies that $\widetilde{p} = Max\{\widetilde{p}_N, \widetilde{p}_{N-1}, ..., \widetilde{p}_2\} < p^* = Max\{p_N^*, p_{N-1}^*, ..., p_2^*\}$. ■

Lemma B.2: Consider a subgame where an isolated low-type agent at $t-1, t > 2$, links to the high type agent. Then at time t, θ_h will link to some agent $j < t-1$.

Proof. Let the component with agents $1, 2, ..., t-2$ be denoted by $\widetilde{C}$. Let the payoff from linking to $j \in \widetilde{C}$ be denoted by $V(\widetilde{C}, j) = 1 + [a(j)]$.Define $j^* = \arg\max_{j \in \widetilde{C}} V(\widetilde{C}, j)$. We could be in one of the following 3 cases, given this subgraph $g_t = \widetilde{C} + \{\theta_{l,t-1}\theta_{ht}\}$.

Case I: $s_{ht} = j^*$ and $v_{t+1}^{iso}(g_t + \{\theta_{ht}j^*\} = \theta_h$

In this case, θ_h is maximising myopic payoff $V(\widetilde{C}, j)$ and future payoff $\pi^f = N - t$. No other strategy s_{ht} would increase his payoffs. Hence, if g_t is such that $v_{t+1}^{iso}(g_t + \{\theta_{ht}j^*\} = \theta_h$, $s_{ht}^* = j^*$.

Case IIa: $s_{ht} = j^*$ and $v_{t+1}^{iso}(g_t + \{\theta_{ht}j^*\} = j^*$

In this case, $\pi_h(s_{ht} = j^*) = 1 + V(\widetilde{C}, j^*) + (N-t)p = 2 + [a(j^*)] + (N-t)p$.

Now, if θ_h deviates to link to some agent $j > t$ then in the subgame that follows there are additional agents towards $\widetilde{C}$ with respect to θ_h. If j^* is more central than θ_h at the subgame with graph $g_t + \{\theta_{ht}j^*$, then for the graph $g = g_t + \{\theta_{ht}\theta_{l,t+1}\} + \{\theta_{l,t+1}, \theta_{l,t+2}\} ... + \{\theta_{l,t+l}, j^*\}$, θ_h cannot be the central agent at any time $\tau > t+1$. Hence the maximum payoff for $s_h = j \not< t$ is

$$\begin{aligned}
\pi_h(s_{ht} &= t+1) = 1 + 1 + pV(\widetilde{C}, j^*) + (N-t-1)p \\
&= 2 + p + p[a(j^*)] - p + (N-t)p \\
&< \pi_h(s_{ht} = j^*)
\end{aligned}$$

Hence, θ_{ht} will not link to $j \not< t-1$.

(Note that Case IIa is impossible if $\widetilde{C}$ contains no link since $\theta_h > 1$).

Case IIb: Suppose it is the case that if $s_{ht} = j^*$, then $v_{t+1}^{iso}(g_t + \{\theta_{ht}j^*\} = j^*$. Also suppose, $\exists j^0 \neq j^*, j^0 \in \widetilde{C}$ such that if $s_{ht} = j^0$, then $v_{t+1}^{iso}(g_t + \{\theta_{ht}j^0\} = \theta_h$.and θ_{ht} prefers j^0 to j^*, *i.e.*

$$1 + 1 + [a(j^0)] + (N-t) > 1 + 1 + [a(j^*)] + (N-t)p$$

Hence,

$$\pi_h(s_{ht} = j^0) = 1 + 1 + [a(j^0)] + (N-t)$$

If θ_h links to some agent $j \not< t-1$, then in the subgame that follows, either he is still the centre but only one link further away from the component $\widetilde{C}$ or he is not the centre anymore and also further away from $\widetilde{C}$ which implies a strictly lower payoff for θ_{ht}.

Hence θ_{ht} will link to some agent $j \in \{1, 2, ..., t-1\}$. ■

Lemma B.3: Consider a subgame where θ_h has not moved. Then at time t an isolated low type agent θ_{lt} will link to $j \in L = \{1, 2, ...t-1, \theta_h\}$.

Proof. Suppose at time t, an isolated low-type agent is selected to move. Let the subgraph at time t be $\widetilde{C}$ and the value of $\widetilde{C}$ through agent j is $V(\widetilde{C}, j) = 1 + [a(j)]$. As before, define $j^* = \arg\max_{j \in \widetilde{C}} V(\widetilde{C}, j)$. Also, note that if θ_{lt} links to θ_h, we have the 3 situations as in Lemma B.2. In each case, θ_h links to some agent $j < t$ and θ_{lt} cannot be the centre.

In case I, $\pi_l(s_{lt} = \theta_h) = \theta_h + p(1 + [a(j^*)]) + (N - t)p$. Alternatively, linking to an agent outside set L, will not make θ_{lt} the centre and increases his distance from other players. The associated payoff is

$$\pi_l(s_{lt} = j \notin L) = 1 + p + p^2 + ..p^{l-2} + p^{l-1}(1 + [a(j^*)]) + p^{l'}(N - t - l) + p^d\theta_h$$

where l is the chain length that follows the subgame,$l \geq 1; d$ i sthe distance to $\theta_h, d > 1; l'$ is the distance to the central agent to which isolated end agents connect to,$l' \geq 1$. It can easily be shown that for any $\theta_h > 1$, this payoff is strictly less than $\pi_l(s_{lt} = \theta_h)$.

However, if $V(\widetilde{C}, j) = 1$, then θ_{lt} could link to $j < t$ and have some positive probability of being the centre, depending on the value of θ_h. Let the payoff in that case be denoted by $\pi_l(s_{lt} = j < t)$. If $\pi_l(s_{lt} = j < t) < \pi_l(s_{lt} = \theta_h)$, then $s_{lt} = \theta_h$ dominates strategies $s'_{lt} \in \{j < t, j \notin L\}$. If $\pi_l(s_{lt} = j < t) > \pi_l(s_{lt} = \theta_h)$, then the previous analysis implies that $\pi_l(s_{lt} = j < t) > \pi_l(s_{lt} = j \notin L)$. Hence, $s_{lt} = j \notin L$ is always dominated by $s_{lt} = j \in L$.

Similarly, we can show that for cases IIa and IIb too, $\pi_l(s_{lt} = j \notin L) < \pi_l(s_{lt} = \theta_h)$. ■

Lemma B.4: Consider a subgame where θ_h has not moved and g_t is such that $V(\widetilde{C}, j^*) \geq 1 + p$.Then at time t an isolated low type agent θ_{lt} will link to θ_h if $\theta_h > V(\widetilde{C}, j^*)$.

Lemma 5 ***Proof.*** *For the first part we need to show that if $\theta_h > V(\widetilde{C}, j^*)$, then $v^{iso}_{t+2}(g) = \theta_h$ for $g = g_t + \{\theta_{lt}\theta_h\} + \{\theta_h j^*\}$ and in that case, the optimal choice for θ_{lt} is $s^*_{lt} = \theta_h$.*

Suppose $\theta_h > V(\widetilde{C}, j^*)$. Also suppose, $g_{t+1} = g_t + \{\theta_{lt}\theta_h\} + \{\theta_h j^*\}$. Now, consider an isolated agent $t+1$ who is chosen to propose a link. For $t+1$,

$$\pi^{myo}_{t+1}(\theta_h) = \theta_h + p + p(1 + [a(j^*)])$$

and

$$\pi^{myo}_{t+1}(j^*) = (1 + [a(j^*)]) + p\theta_h + p^2$$

Hence $\pi^{myo}_{t+1}(\theta_h) > \pi^{myo}_{t+1}(j^*)$ according as

$$\theta_h + p + p(1 + [a(j^*)]) > (1 + [a(j^*)]) + p\theta_h + p^2$$

$$\Leftrightarrow (1 - p)\theta_h > (1 - p)(1 + [a(j^*)]) - (1 - p)p$$

$$\Leftrightarrow \theta_h > V(\widetilde{C}, j^*) - p$$

which holds. Hence, $t+1$ chooses θ_h, for $p \simeq 1$.

In this case, payoff to θ_{lt} is

$$\pi_{lt}(\theta_h) = \theta_h + pV(\widetilde{C}, j^*) + (N - t - 1)p$$

If θ_{lt} deviates to j^*, the maximum possible payoff is when all future players link to j^* and is

$$\begin{aligned} \pi_{lt}(j^*) &= p\theta_h + V(\widetilde{C}, j^*) + (N-t-1)p \\ &< \pi_{lt}(\theta_h) \text{ for } \theta_h > V(\widetilde{C}, j^*). \end{aligned}$$

Hence, θ_{lt} will link to θ_h if $\theta_h > V(\widetilde{C}, j^*)$. ■

Proof of Proposition

Consider agent 1. If agent 1 is the high type then given Lemma B.1, the optimal strategy is for θ_h to abstain and agent 2 connects to θ_h, agent 3 connects to θ_h and so on.

Suppose agent 1 is a θ_l. He has 3 choices:

$i)s_1 = \phi$

$ii)s_1 = \theta_l$

$iii)s_1 = \theta_h$

If $s_1 = \theta_h$, then given Lemma B.1, the optimal strategy for θ_h is to abstain since then agent 3 will link to θ_h and the equilibrium structure is a complete star with θ_h as the center. Hence, $\pi_1(s_1 = \theta_h) = \theta_h + (N-2)p - c$.

If $s_1 = \phi$, then with probability $\frac{1}{N-1}$, θ_h is chosen as the 2nd proposer and we are in a case similar to Case I of Lemma B.2. Hence it is optimal for him to link to 1, since from period 3 agents would link to θ_h and $\pi_1 = \theta_h + (N-2)p$.With probability $\frac{N-2}{N-1}$ however, some θ_l is chosen as the proposer. Now, $\theta_{l,2}$ can link to θ_h who by Case I, Lemma B.2, will choose $\theta_{l,1}$ and the network will be a $\theta_h - star$. In this case,

$$\pi_{l,2} = \theta_h + (N-2)p - c$$

$\theta_{l,2}$ can also link to 1. In this case, with probability $\frac{N-3}{N-2}$, the 3rd agent is θ_l. By Lemma B.4, for $\theta_h > 1 + p$, θ_{l3} links to θ_h. By Lemma B.2, θ_h links to 1 or 2 (chooses randomly) and from period 5 onwards agents link to θ_h. The payoff to 2, therefore, is

$$\pi_2|_{agent3=\theta_l} = 1 - c + \frac{1+p}{2}(\theta_h + (N-3)p)$$

With $\frac{1}{N-2}$ probability the 3rd agent is θ_h. Since $\theta_h > 1 + p$, the high type knows that even if $s_{h3} = 1/2$, $v_4^{iso} = \theta_h$. Hence, θ_{h3} links to 1 or 2 and agent 4 onwards links to $v_4^{iso} = \theta_h$. In this case, $\pi_2|_{agent3=\theta_h} = \pi_2|_{agent3=\theta_l} = 1 - c + \frac{1+p}{2}(\theta_h + (N-3)p)$.

Hence the payoff to $\theta_{l,2}$ from $s_2 = 1$ is

$$\pi_{l,2}(s_2 = 1) = 1 - c + \frac{1+p}{2}(\theta_h + (N-3)p)$$

Hence if $s_1 = \phi$, then $\theta_h \succ_2 1$ iff

$$\theta_h + (N-2)p - c > 1 - c + \frac{1+p}{2}(\theta_h + (N-3)p)$$

$$or, 2\theta_h + 2(N-2)p > 2 + (1+p)(\theta_h + (N-3)p)$$
$$or, (1-p)\theta_h > 2 + p(N-3-2N+4) + (N-3)p^2$$

$$\begin{aligned} or, (1-p)\theta_h &> 2 - p(N-1) + (N-3)p^2 \\ &= 2 - 2p^2 + (N-1)(p^2 - p) \\ &= 2(1-p)(1+p) - (1-p)p(N-1) \\ &= (1-p)(2 - p(N-3)) \end{aligned}$$

$$or, \theta_h > 2 - p(N-3)$$

which holds for $p \simeq 1$. Hence if $s_1 = \phi$, 2 chooses θ_h who in turn chooses agent 1. Hence,

$$\pi_1(s_1 = \phi) = \theta_h + (N-2)p$$

If $s_1 = \theta_l$, then in this subgame, 2 can choose ϕ or to link to some other agent. If $s_2 = \phi$, then the next agent, if θ_l chooses θ_h and if θ_h, he chooses one of 1 or 2. In either case, θ_h becomes the central agent for all future agents and $\pi_1(\theta_l) = 1 - c + \frac{1+p}{2}(\theta_h + (N-3)p) < \pi_1(\phi)$.

It could be the case that $s_2 = \theta_h$, In this case θ_h links to 1 and becomes the centre and hence, $\pi_1(\theta_l) = 1 - c + \theta_h + (N-3)p$. For $p > 1-c$, $\pi_1(\phi) > \pi_1(\theta_l)$ and vice versa. If $s_2 = \theta_l$, then on the path we could have either of the 2 cases. One where some agent $j > 2$ links to θ_h who then would link to $k > 1$ and $\pi_1(\theta_l) = 1 + r_1 p + r_2 p^2 + ... + r_q p^q + p^d \theta_h + (N-l-1)p^{d+1} < \theta_h + (N-2)p = \pi_1(\phi)$. In the other case, suppose l low types form a wheel so that probability of centre is positive. Hence, $\pi_1(\theta_l) = (2 + 2p + ...(l-1) terms) + \frac{1}{l}(1 + 2p + ...l terms)(\theta_h + N - l - 1)$. Its is easily shown that for $p = 1$, $\pi_1(\theta_l) = \theta_h + (N-2) - c < \pi_1(\phi)|_{p=1}$ and the inequality holds for $p \simeq 1$.

Hence, for $p \simeq 1$, agent 1 would either abstain and hence $\theta_h - star$ obtains for any $\theta_h > 1 + p$ or agent 1 links to 2 who then links to θ_h, hence forming a WS_3 with the centre being the high type agent.