Run Equilibria in a Model of Financial Intermediation

Huberto Ennis

Federal Reserve Bank of Richmond

Todd Keister

Federal Reserve Bank of New York

September 22-23, 2007

CU-PSU Macroeconomics Workshop

Introduction

- There is a substantial literature on “self-fulfilling” bank runs
  - coordination failure in an equilibrium model
  - classic reference: Diamond & Dybvig (1983)

Q: What features of the environment allow such runs to occur?

  - some partial answers, but much remains unknown

- Green & Lin (2003): study a finite-agent version of the Diamond-Dybvig model
  - show that self-fulfilling runs cannot occur in equilibrium
  - “What’s missing?”

What We Do

- Ask: What is needed to generate a run equilibrium in a fully-specified model of financial intermediation?
- Study 3 variations on the Green-Lin model
  - each differs from original model in only one respect
  - building on work of Peck & Shell (2003)
- Show that in each case a run equilibrium can exist
  - construct examples
- General theme: information frictions are the key
  - need to be "strong enough" for runs to be possible

Approach

- Intermediary (planner) tries to implement efficient allocation
    - subject to a sequential service constraint
    - has full commitment power
- Traders play a *withdrawal game*
    - each reports type to the intermediary
    - receive payments according to efficient allocation
- Ask if this game has a run equilibrium
    - if so, a run can occur with positive probability in the "overall" game where intermediary chooses contract (as in Peck-Shell)

Why Do We Care?

- Perceived “fragility” of banks is the justification for important (and costly) policy interventions
  - “Federal deposit insurance ... should not be considered necessary for banking system stability.” Nosal (2006)
- Recent events highlight importance of topic (Northern Rock)
  - policy makers want to understand what makes an institution susceptible to a run
- Issue may also be relevant for non-bank financial institutions
  - BNP Paribas funds, other “funds” of various types

Environment

- 2 time periods, $t = 0, 1$
- Finite number I of traders
  - utility:

$$v\left(a_i^0, a_i^1; \omega_i\right) = \frac{\left(a_i^0 + \omega_i a_i^1\right)^{1-\gamma}}{1-\gamma} \qquad \gamma > 1$$

$$\text{where } \omega_i = \left\{ \begin{matrix} 0 \\ 1 \end{matrix} \right\} \text{ if trader } i \text{ is } \left\{ \begin{matrix} \text{impatient} \\ \text{patient} \end{matrix} \right\}$$

- Type ω_i is private information
- π = probability of $(\omega_i = 0)$ [i.e., being impatient]
  - probability of type profile ω is $P(\omega)$

Technologies

- Traders are isolated; markets cannot meet (Wallace, 1988)
- Each trader has an opportunity to contact the intermediary in each period
  - opportunity arrives sequentially, ordered by i (Green & Lin)
- Intermediary has I units of good in period 0
  - return on investment is $R > 1$
- Utility cost ϕ_i of i^{th} trip to the intermediary (new)
  - set $\phi_1 = 0$ (normalization)
  - study two benchmark cases: $\phi_2 = 0$ and $\phi_2 = \infty$

Efficient Allocation

- An allocation maps states into consumption profiles

$$\mathbf{a} : \omega \mapsto \left\{a_i^0, a_i^1\right\}_{i=1}^{\mathrm{I}} \quad \text{with} \quad \sum_i \left(a_i^0 + \frac{a_i^1}{R}\right) \leq \mathrm{I}$$

- Sequential service: a_i^0 can only depend on the information received prior to i
  - if all traders visit intermediary in period 0, must have

$$a_i^0\left(\omega\right) = a_i^0\left(\widehat{\omega}\right) \text{ for all } \omega, \widehat{\omega} \text{ with } \omega^i = \widehat{\omega}^i$$

- Efficient allocation $\mathbf{a}^*$ maximizes sum of expected utilities
  - subject to feasibility, sequential service

Implementation

- Intermediary wants to implement the efficient allocation $\mathbf{a}^*$
- Withdrawal game: each trader reports his/her type to intermediary
  - if $\phi_2 = 0$, all visit intermediary and report type in period 0
  - if $\phi_2 = \infty$, only reports of 'impatient' are received in period 0
  - direct revelation, subject to sequential service
  - intermediary makes payments assuming actions are truthful
- Strategies:

$$\mu_i : \omega_i \mapsto \{0, 1\}$$

- Resulting allocation in state ω is $\mathbf{a}^* (\mu (\omega))$
- Equilibrium: μ^* such that:

$$U_i \left(\mathbf{a}^* \circ \left(\mu^*_{-i}, \mu^*_i\right)\right) \geq U_i \left(\mathbf{a}^* \circ \left(\mu^*_{-i}, \mu_i\right)\right) \quad \forall \ \mu_i \ \forall i$$

- If $\mathbf{a}^*$ is incentive compatible, $\mu^* = \omega$ is an equilibrium

Q: Does game have an equilibrium where $\mu^*_i \neq \omega_i$ for some i?

- Green & Lin: When $\phi_2 = 0$ and types are independent, answer is 'no'
  - surprising result; information frictions not "strong enough"
  - see Andolfatto *et al.* (2007) in importance of independence

The Three Cases

We study:

(1) Costly reporting: $\phi_2 = \infty$ (or finite but large)

- limits information available to the intermediary when choosing payments

(2) Correlated types

- exacerbates private information problem

(3) Early decisions: traders do not know index i (Peck-Shell)

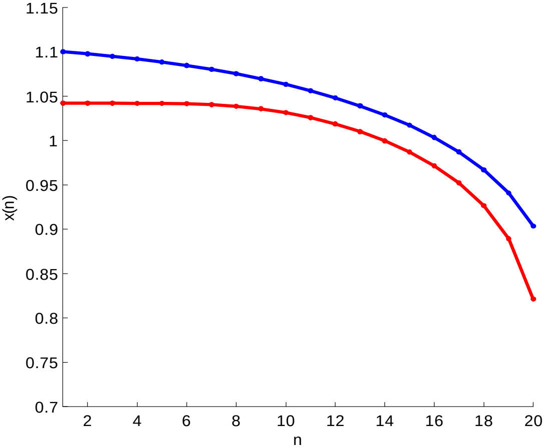
- limit information available to traders when making decisions

- No other changes (to utility function, etc.)

Case 1: Costly Reporting

- Now intermediary only observes some reports in period 0
  - first impatient trader could be $i = 1$ or $i = 2$ or ...
  - period-0 payments are chosen based on *less* information
- Sequential service implies a payment vector $\{x(n)\}_{n=1}^{\mathrm{I}}$
  - $x(n) =$ payment to n^{th} trader to arrive in period 0
  - used by Peck-Shell, but not critical for their results
- Note: the set of allocations satisfying sequential service is strictly smaller with $\phi_2 = \infty$
  - the efficient allocation under $\phi_2 = 0$ lies outside this set

- Efficient allocation: $\mathrm{I} = 20$, $R = 1.1$, $\pi = \frac{1}{2}$, $\gamma = 10$



- Notice: resembles a demand-deposit contract

Run Equilibrium?

- If patient, depositor I will always wait (as in Green & Lin)
  - any run must be *partial*
- Conjecture:

$$\mu_i^* = \left\{ \begin{array}{c} 0 \\ \omega_i \end{array} \right\} \text{ for } \left\{ \begin{array}{c} i \leq i^* \\ i > i^* \end{array} \right\} \text{ for some } i^* < \mathrm{I}$$

- Let $z(i)$ = expected utility of waiting if:
  (a) all traders before i run, and
  (b) all traders after i act truthfully
- Note: z is a strictly decreasing function

- Repeat: $\mu_i^* = \left\{ \begin{array}{c} 0 \\ \omega_i \end{array} \right\}$ for $\left\{ \begin{array}{c} i \leq i^* \\ i > i^* \end{array} \right\}$

- This strategy profile is an equilibrium if

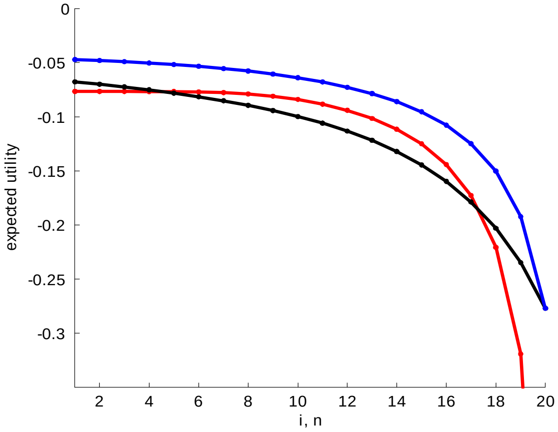
  (i) $u(x(i^*)) > z(i^*)$, and

  (ii) $u(x(i^*+1)) < z(i^*+1)$

- Condition (i) : trader i^* prefers to run
  - monotonicity of $x \Rightarrow$ traders before i^* choose to run

- Condition (ii) : trader i^*+1 prefers not to run
  - monotonicity of $x \Rightarrow$ traders after i^*+1 choose not to run

- Example: $\mathrm{I} = 20$, $R = 1.1$, $\pi = \frac{1}{2}$, $\gamma = 10 \Rightarrow i^* = 17$



- Note: only one equilibrium of this form exists (i^* is unique)

Intuition

- Traders run if they fear the intermediary will give away "too much" in period 0
  - but trader i^* knows all subsequent traders will act truthfully
  - why does she run?
- Critical issue: compare beliefs about number of additional early withdrawals
  - depends on number of traders who have yet to act (and their types, but types are i.i.d.)
- Trader i^* knows $\mathrm{I} - i^*$ have yet to act
- Intermediary does not (it thinks i^* is likely the last)

- Result: $x(i^*)$ is "large" given trader $i^{*\prime}s$ (correct) beliefs
  - gives higher utility than waiting until period 2
- Note: with $\phi_2 = 0$, this cannot happen
  - intermediary knows $i \Rightarrow$ has same belief as each trader
- Then efficient payment schedule gives i^* an incentive to wait
  - regardless of actions taken by prior traders
  - Green & Lin (2003), Lemma 5
- Costly reporting is an additional information friction
  - can open the door to differing beliefs, runs

Case 2: Correlated Types

- Let $\phi_2 = 0$, but suppose ω_i are not i.i.d.
- Intermediary always knows i, but ...
  - traders have private info about *others'* types
- Efficient allocation is more complicated
  - a_0^1 depends on entire partial history ω^i, but ...
- Example: $\mathrm{I} = 4$, $R = 2$, $\gamma = 6$

	number of impatient traders				
	0	1	2	3	4
probability	0.01	0.01	0.96	0.01	0.01

- Example is "close" to a model with no aggregate uncertainty
    - useful for gaining intuition
- Efficient allocation:
    - large payments for first two early withdrawals ($\sim c_1^*$)
    - much lower payments if > 2 early withdrawals
- Run equilibrium strategies:

$$\mu_i^* = \left\{ \begin{array}{c} 0 \\ \omega_i \end{array} \right\} \text{ for } i = \left\{ \begin{array}{c} 1,2 \\ 3,4 \end{array} \right\}$$

    - again, a *partial* run

- Why does trader 2 run?
  - knows she is patient
  - knows trader 1's report was uninformative
  - if trader 1 was patient, 3 & 4 are likely to withdraw early
- Emphasize: similar theme of the two examples
  - in both, traders have better information than intermediary about additional early withdrawals
  - information frictions keep this info from the intermediary
  - result: intermediary is too optimistic, sets $x(i^*)$ too high

Case 3: Early Decisions

- Let $\phi_2 = 0$ and let ω_i be independent
  - want to implement $\mathbf{a}^*$ from Green & Lin
- Traders must choose an action before knowing their index i
  - after strategies are set, places in order assigned randomly
  - follows Peck & Shell (2003)
- Strategy: $\mu_j : \omega_j \mapsto \{0, 1\}$
  - all traders face same decision problem
- Difference from Peck-Shell: patient and impatient traders have same utility function

- Example: $\mathrm{I} = 6$, $R = 1.1$, $\gamma = 6$, $\pi = 0.1$
- Run equilibrium strategy:

$$\mu_j = 0 \quad \text{for all } j$$

- Why do depositors run?
    - would wait if they knew $i = 6$ (or $i = 5$)
    - given that others are running, prefer to run if $i = 1, \ldots, 4$
    - take expectation over possible places in line
- In this case traders have less information than intermediary about additional early withdrawals

Concluding Remarks

- We ask: What features of the environment allow self-fulfilling runs to occur?
- Show: information frictions must be important enough
  - must sustain different beliefs for traders and intermediary
  - this leads intermediary to offer "large" early payments
- Studied frictions in:
  - how quickly the intermediary observes traders' decisions
  - what traders know when they make their decisions
- Showed how correlated types exacerbates information frictions