

Supply signals, complementarities, and multiplicity in asset prices and information acquisition*

Jayant V Ganguli[†] and Liyan Yang[‡]

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Abstract

Allowing speculation based on private information on a stock's supply generates (i) complementarity in information acquisition and (ii) multiple equilibria in *the financial market and the information market* independently in a standard CARA-normal model. Information can be a complement irrespective of the financial market equilibrium coordinated upon and generates multiplicity in the information market. Multiplicity in the financial market exists irrespective of the information market equilibrium and is generated as traders seek to profit from information about the payoffs and the supply of the asset. The multiplicity and complementarity provides insights and explanations for market frenzies and other phenomenon. This extension also allows for general equilibrium analysis of the financial market suggesting that multiplicity is inherent to the CARA-normal framework.

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[†]Department of Economics, Cornell University, Ithaca, NY 14850, <jvg6@cornell.edu>

[‡]Department of Economics, Cornell University, Ithaca, NY 14850, <ly52@cornell.edu>

1 Introduction

Asset prices are affected by factors that are unrelated to the payoff(s) from the risky asset(s) or stock(s) and traders in a financial market often seek or possess information about aspects of the stock(s) in the market other than the payoff(s). One such aspect is the (net) supply of stock(s) in the market in any trading period.¹ The (net) supply of the stock(s) is determined by the trading decisions of a variety of traders including those who possess information about the payoff(s) of the stock(s) and those who trade for reasons, such as liquidity, which may be unrelated to the payoff(s) from the stocks. Trades that are unrelated to the fundamentals may occur, *inter alia*, in response to macroeconomic phenomena or to meet cash flow needs.

Traders who provide liquidity in the market [and may be informed of about the payoff(s)] will seek or possess information about the liquidity demanded, typically to avoid trades with anyone better informed than they are. Information about the supply of the risky asset(s) can also be thought of as the information obtained by dealers or market-makers who have access to the order book or as the information obtained by sentiment-oriented technical traders and frontrunners, or as information on the float maintained by investment banks. Finally, information about the aggregate supply can also be naturally obtained by a trader from her endowment of the risky asset(s), as in a general equilibrium model of trade in assets. Hence, it is of interest to assess the effects of traders who seek to profit from information about supply of the stock(s).

Markets with traders who have private information about the payoff have been widely analyzed, for example in the existing analyses of the CARA-normal REE (constant-absolute-risk-aversion-normal rational expectations equilibria)² models, while there are relatively few REE models that analyze markets where traders also have private information about the supply of the stock(s).³ We present possibly the simplest version of a CARA-normal REE model in which traders have private information

¹This is distinguished from the shares outstanding for a firm, which is fixed and publicly known.

²We use the abbreviation REE in the plural and the singular sense in what follows.

³We discuss these below.

about the aggregate supply of the stock and find that this dimension of information can have interesting implications. Our model can be extended to take into account more realistic assumptions about the supply information that can be held by traders in financial markets, as we discuss toward the end of this paper, and also allows for general equilibrium analysis of the market where assets are traded.

Do investors seek information that other investors have? Does the stock price always reveal more information when more investors with private information trade? Is this price uniquely predicted? Noisy REE models are commonly used to analyze trading in stocks by investors based on differences in information and the CARA-normal framework is possibly the most widely used framework for noisy REE modeling since it permits tractable analysis and closed-form solutions. The answers to the above questions in most existing analyses of the CARA-normal REE models have been no, yes, and yes respectively.

We analyze a natural extension of a standard CARA-normal model (Grossman, 1976; Grossman and Stiglitz, 1980; Diamond and Verrecchia, 1981; and Admati, 1985) and find that the answers to the first question can in fact be positive and the answers to the last two negative. In the CARA-normal framework, individuals (investors / traders) live for two periods, trading in risky and riskless assets in a financial market in the second period. In the first period, called the information market, the traders choose whether or not to acquire (costly) information, which may be diverse or identical across the traders, about the stock(s). In the previous models of the CARA-normal framework, only information about the payoff(s) per unit invested in the stock(s) can be purchased by traders.

We extend the analysis in this framework and allow each trader to independently obtain (costly) information in an additional dimension. The additional information, which is diverse across traders, is a noisy signal about the supply of the risky asset(s) traded in a financial market and allows for speculation based on private information about the stock's supply. We term this the *supply signal* and term the signal about

the payoff the *payoff signal*. We call the traders who purchase the signals *informed traders*.

Our extension of the model yields the following results. We find that strategic complementarity in information acquisition can exist within the normal distribution framework and this can lead to multiple equilibria in the information market. When information is a strategic complement then a trader will want to acquire this information even when other traders have it which can lead to the multiplicity. We also find the financial market has two (linear) partially revealing financial market REE price functions, which have opposing properties with regard to information content. An important aspect of our results, which we elaborate on below, is that the complementarity and the multiplicity in the information market do not generate the multiplicity in the financial market and vice versa.

Our results are in stark contrast to previous analyses of the CARA-normal model in the partial equilibrium framework by, inter alia, Grossman (1976), Grossman and Stiglitz (1980), and Admati (1985), and in the general equilibrium framework by Diamond and Verrecchia (1981) where information acquisition exhibits strategic substitutability and there is a unique (linear) partially revealing REE price in the financial market. If information is a strategic substitute then a trader has no incentive to acquire the information when other traders already possess it and as a consequence the information market also has a unique equilibrium. Also, in the financial market, the (unique) REE price always reveals more of the private information when more traders with private information are present.

These aspects of the existing CARA-normal models have led to studies that add additional imperfections to the CARA-normal environment (Genotte and Leland, 1991 and Yuan, 2005) or depart from the CARA-normal framework (Chamley, 2004 and 2005 and Barlvey and Veronesi 2000) to obtain strategic complementarities in information acquisition and / or multiple equilibria in the trading of assets.⁴ We

⁴A notable exception is a model by Veldkamp (2006a and 2006b) which does not depart from the CARA-normal framework and generates complementarity in information acquisition by endogenizing

show that retaining the CARA-normal framework, without adding additional imperfections, can lead to strategic complementarities and multiple equilibria.

The multiplicity of equilibria in the financial market in our model is driven by the presence of supply signals *and* payoff signals, which allow the informed traders to speculate based on their information about the supply of stock(s) or the payoff of stock(s). Speculation based on information about the payoff, i.e., direct speculation is already possible given the presence of payoff signals. Indeed, the assumptions in the previous analyses only allow for this type of speculation. As also pointed out by Grossman and Stiglitz (1980), a positive number of traders will acquire costly information on the payoff or the supply when the price in the financial market reveals the value of the payoff or the supply at most partially. The decisions of those informed traders who seek to profit from direct speculation leads to one of the financial market REE, while the presence of informed traders who seek to speculate based on supply information generates the other REE. The absence of either payoff signals or supply signals will lead to a unique (linear) partially revealing REE price.

In fact, in the Grossman and Stiglitz (1980) model, informed traders are also informed about the supply. In their model, once the costly (identical) payoff signal is purchased, the informed traders can infer the supply perfectly from the price of the risky asset, making a separate supply signal redundant. So, our model can be seen as a direct generalization of their's with diverse payoff signals, which we believe are a better approximation of reality. This generalization remarkably can yield results that are almost the opposite of those in Grossman and Stiglitz (1980).

The financial market equilibria in our model are closely related to the financial market equilibrium in the models by Grossman (1976), Grossman and Stiglitz (1980), and Diamond and Verrecchia (1981). In one of the equilibria, the price of the stock reveals more information about the payoff as the number of informed traders in the financial market increases similarly to the unique equilibrium in the previous studies.

the price of information.

Due to this and other similarities that we elaborate on in section 4, we refer to this equilibrium as the GS-REE (Grossman-Stiglitz REE). We refer to the other financial market equilibrium as the NGS-REE (non-Grossman-Stiglitz REE).⁵ The information about the payoff revealed by the NGS-REE price decreases as the number of informed traders increases in contrast to the unique equilibrium price in the previous studies. In fact, the NGS-REE price reveals more information about the supply of the stock as the number of informed traders increases.

Since the unique equilibrium in the financial market in the previous analyses of the standard CARA-normal model has been widely used to analyze a variety of phenomena, it is of interest to assess whether information can be a strategic complement if the traders coordinate on the GS-REE in our model. So, in this paper we focus primarily on the case where the traders coordinate on the GS-REE, while also providing results for the case where the traders coordinate on the NGS-REE. An additional reason for our focus on the GS-REE is the result that in our model the informed and uninformed traders are ex-ante better off in the GS-REE than in the NGS-REE.

We indeed find that even when the traders coordinate on the GS-REE, information acquisition can still exhibit strategic complementarities. Whether information is a strategic complement or not depends on the quality of the signals about the payoff and the supply of the stock. We show that if the supply signal is ‘relatively of better quality’ than the payoff signal, then information is a strategic complement. A precise statement of this is given in theorem 1. The strategic complementarity in information acquisition can lead to multiple equilibria in the information market, unlike the result in Grossman and Stiglitz (1980).

As we noted above, the NGS-REE price reveals less information about the payoff as the number of informed traders increases and unsurprisingly, this means that if the traders coordinate on the NGS equilibrium, then information acquisition may be a strategic complement. However, the conditions under which information is a strategic

⁵While we use the NGS label in very specific sense, we realize that it may seem vague and uninformative and welcome any suggestions for a better label.

complement in this scenario are the opposite of those when the traders coordinate on the GS-REE. This is a direct consequence of the fact that the two REE prices have opposite properties regarding information revelation as the number of informed traders increases.

In particular, there are two forces affecting information acquisition in our model and these can generate complementarity or substitutability depending on the financial equilibrium the traders coordinate upon. The first, the information about supply, tends to generate complementarities when the traders coordinate on the GS-REE and substitutability when the traders coordinate on the NGS-REE. However, the second force, which is the information about the payoff, generates substitutability if the traders coordinate on the GS-REE while it generates complementarities if the traders coordinate on the NGS-REE. Better information about the supply increases the utility of a trader via the improved informativeness of the stock price about the payoff, which depends positively on the number of informed traders in the GS-REE but varies inversely with the number of informed traders in the NGS-REE.

In general, traders seek information, i.e., it is a complement, when the price does not reveal more of the private information that is used by the informed traders for speculation. Since our model allows for two sources of speculation, private information about the payoff and about the supply, traders will seek to become informed when they believe that equilibrium price in the market reveals less information about the particular source of speculation.

Hence, the existence of multiple equilibria in the financial market in our model plays no role in the possibility of the existence of strategic complementarities in information acquisition. These exist no matter which financial market equilibrium the traders coordinate on. Also, multiple equilibria exist in the financial market even when the information is a substitute or the information market has a unique equilibrium. Indeed, the general equilibrium analysis that we can conduct with our model suggests that multiple equilibria are intrinsic to financial market models using

the CARA-normal framework.

The multiplicity of financial market equilibria and complementarity in information acquisition provide insights into and explanations for market frenzies, the cost of capital, price crashes and excess volatility, and the twin shares phenomenon. These are discussed in sections 4 and 5.6. In particular, we find that in contrast to the existing analysis by Easley and O'Hara (2004) the cost of capital to a firm can in fact increase if the private information about the payoff is more precise or the number of informed traders increases. The multiple financial market equilibria in our model also suggest that two independent stocks that have the same fundamentals and are traded in the same market could have completely different prices, which could help explain the twin stocks phenomenon.

As noted above, when information is a complement, there can be multiple equilibria in the information market also, independently of the equilibria in the financial market. This means that there can be a large number of overall equilibria in the model. In particular, prices can crash as traders switch from one information market equilibrium to another without switching between financial market equilibria or as traders switch between financial market equilibria while staying at the same information market equilibrium. The switches between equilibria can also lead to price volatility that is excessive compared to those of the payoff- or supply-fundamentals.

The multiplicity of equilibria in our model also provides an insight into market frenzies, or 'price surges' that can be thought of as occurring due to changes in information about the stock as in Veldkamp (2006a). A new result is that a price surge may occur with a rise in information about the supply of the stock, suggesting that supply-based speculation has a role in explaining large market fluctuations.

The rest of the paper is organized as follows. We discuss some of the related literature on complementarities and on the financial market in section 2, while section 3 describes the model. We discuss the equilibria in the financial market with its applications in section 4 and we discuss the general equilibrium analysis of the

CARA-normal framework in sub-section 4.1. Section 5 describes the equilibria in the information market and its applications. The two main results are presented in proposition 1 and theorem 1. All the results are proved in the appendix and section 6 concludes the paper.

2 Related literature

We are not the first to explore the consequences of a signal about the supply of the risky asset(s). This dimension of information is also considered by Gennotte and Leland (1991), who then use the equilibrium of their model to study price crashes. The signal in their model is identical across traders, who may be thought of as market-makers, while we consider signals that are diverse across agents as we believe this is a better approximation of reality. Palomino (2001) also considers markets in which traders speculate based on private information about the supply of the risky asset and finds that supply signals improve the informational efficiency of imperfectly competitive markets, within the CARA-normal setup.

2.1 Financial market equilibrium

The financial market equilibrium in the CARA-normal framework has been widely applied. As we noted above, Gennotte and Leland (1991) introduce hedging trades, imperfectly observed by supply informed traders, in the supply of the stock. The equilibrium price function in their model is discontinuous in the fundamentals, which means that the price can crash, i.e., jump down discontinuously with a small change in the fundamentals. Yuan (2005) introduces borrowing constraints for the informed traders in a CARA-normal REE model with only payoff informed traders and generates multiple equilibria in the financial market and a non-linear equilibrium price function, both of which can cause price crashes. Using borrowing constraints on informed traders to link markets for different assets, she is also able to generate con-

tagion across financial markets. Kodres and Pritsker (2002) use a standard multi-asset CARA-normal REE model to study contagion across markets through a cross-market portfolio-rebalancing effect. The noise in the identical payoff signal is correlated across the markets, which generates the cross-market rebalancing effect.

Angeletos and Werning (forthcoming) use the financial market equilibrium as an endogenous public signal of the fundamental in a Morris and Shin (1998) coordination game modeling currency crises and generate multiple equilibria in the game. They also introduce feedback effects from the coordination game to the financial market, which then generates multiple equilibria in the market. The multiplicity in their model is useful for understanding excess volatility in financial markets. Our results are complementary to their analysis, with the understanding that using the financial market price(s) from CARA-normal REE models may generate multiplicity in the coordination game simply because there are multiple prices, especially given that closing the CARA-normal REE model, i.e., considering its general equilibrium analysis generates multiplicity.

2.2 Complementarity and substitutability

In Grossman and Stiglitz (1980) traders who choose to become informed receive an identical noisy payoff signal. As a result information acquisition exhibits strategic substitutability, so that as the number of informed traders increases, the (REE) price of the stock reveals more information about the payoff to the uninformed traders. This makes it easier for the uninformed traders to free-ride on the information acquired by others and leads to a unique equilibrium in the information market. Verrechia's (1982) more general (CARA-normal) model about traders with differing risk aversion, who can purchase diverse payoff signals and improve the signal-precision at increasing cost can be simplified to one with identical risk aversion across traders and a single level of precision available at a cost. Here too the REE price is more informative about the payoff as the number of informed traders increases and as a result, information

acquisition will be a strategic substitute.⁶

Barlevy and Veronesi (2000) leave the CARA-normal framework and consider risk-neutral traders, who trade risk-free and risky assets with a binomial distribution of the returns of the risky asset and an exponential distribution of the aggregate supply of the risky asset. These assumptions generate the possibility that learning (or information acquisition) is a strategic complement in the information market. However, their claim (on p86) that “(o)ne needs to move away from the normal distribution framework and towards distributions which place greater mass on the extreme in order to uncover our result” is not necessary as we show.

Chamley (2005) also departs from the CARA-normal framework to develop a dynamic asset-trading model where information is a complement and generates multiple equilibria in asset trading with sudden changes in the volume of trade, in the information generated by the market, and in the volatility and evolution of the asset price. Hellwig and Veldkamp (2006) study games where a large number of agents choose information and then interact strategically. They conclude that when the game played has strategic complementarity (substitutability), then the information is complementary (substitutable) as well. They explain the substitutability of information in the Grossman-Stiglitz model (1980) by stating that “[i]nvestment is a strategic substitute: investors prefer purchasing assets that others don’t want, because these assets have low prices” (p 4). Although they note that allowing for freely observable aggregate variables, like prices, may or may not change their result, they present the Grossman-Stiglitz model as an example where it does not. Our results show that the payoff signal being identical is crucial to this result, since in our model, although investment is still a strategic substitute in the financial market, the information choice of the traders can exhibit complementarity.

We let the traders purchase (costly) payoff and supply signals, which are diverse

⁶Although the stock’s supply is the aggregate of individual endowments, these are i.i.d. and Verrecchia (1982) only considers the limit as the number of traders approaches infinity, so that individual endowment provides no information about aggregate supply.

and not identical across traders, in the information market. This aspect generates complementary information acquisition. As noted above, the payoff signal is diverse across traders because an identical signal makes the model exactly the same as Grossman and Stiglitz (1980), in which information can only be a substitute. Also, as noted above, just introducing a diverse payoff signal, without a supply signal (as in the simplification of Verrecchia 1982) is not enough to generate complementarity, a supply signal must be present.

We are not the first to be able to generate complementarities in a CARA-normal framework. Veldkamp (2006a and 2006b) generalizes the Grossman-Stiglitz (1980) model by introducing an information production sector that supplies the payoff information at an endogenous price in a dynamic setting. She then uses the complementarities to explain the occurrence of media ‘frenzies’ in information markets and the formation of media herds in multiple investment markets (2006a) and to explain the excess covariance of asset prices relative to the covariance of their fundamentals (2006b). In her model, the endogenous price for information generates the complementarity in information acquisition by making the net benefit from being informed non-monotonic in the number of informed traders. In our model, on the other hand, the net benefit curve is always monotonic in the number of informed traders if they commit to one of the two REE, while it can be U-, inverted U-, or S-shaped if they randomize over the REE, and complementarities arise from the net benefit provided by the supply signal. Veldkamp’s (2006a) analysis of data on emerging markets suggests that complementarities in information acquisition are useful in understanding how asset price volatility generates demand for news and how news increases asset price and price dispersion across markets. We are able to generate similar results to Veldkamp’s (2006a) dynamic model in a static setting.

Complementarities occur in a large number of economic interactions, such as bank runs; regime switches; and currency, debt, and financial crises resulting in, inter alia, synchronization of actions, multiplicity of equilibria, and amplification of volatility.

Our results suggest that the analytically tractable CARA-normal REE models can be used to analyze situations where the information is complementary across agents and multiplicity is potentially useful to understand the movement of economic data and possibly puzzling economic phenomena. We discuss these in sub-sections 4 and 5.6 and now turn to the formal model.

3 The model

There is a continuum of traders, indexed by $[0, 1]$, who are identical ex ante and live for 2 periods. There are 2 assets - one riskless (money) and one risky (a stock) - traded in a financial market that opens in period 2. The payoff to each unit invested in money is normalized to one and each trader is endowed with \bar{N} units of money and zero units of the stock at the beginning of period 1.⁷

The payoff to each unit invested in the stock is denoted by v , and v is normally distributed with mean \bar{v} and precision $\rho_v > 0$, i.e.,

$$v \sim N(\bar{v}, 1/\rho_v) \text{ with } \rho_v > 0.$$

We assume that the aggregate supply (x) of the stock is normally distributed, i.e.,

$$x \sim N(\bar{x}, 1/\rho_x) \text{ with } \rho_x > 0.$$

We call the pair (v, x) the fundamentals in this financial market.

At the beginning of period 1, the information market opens. In this market, at a cost $\kappa \in (0, \bar{N})$, trader i can independently purchase a two-dimensional private information signal about the fundamentals in the financial market,⁸ $S_i = (y_i, x_i)$,

⁷The assumption about zero endowment of the stock is not important for our results. We obtain the same results, if for example, we assume that the endowment of the stock is random across the traders and is independent of the aggregate supply of the stock. See also section 4.1

⁸Alternatively, we can assume that the traders pay $\kappa_x \geq 0$ to buy a supply signal, $\kappa_y > 0$ to buy a payoff signal, and $\kappa = \kappa_x + \kappa_y \in (0, \bar{N})$ to buy both. Then there will be four groups of traders in the

where y_i and x_i are mutually independent and

$$\begin{aligned} y_i &= v + \varepsilon_i, \text{ with } \varepsilon_i \sim N(0, 1/\rho_\varepsilon), \rho_\varepsilon > 0, \text{ and} \\ x_i &= x + \eta_i, \text{ with } \eta_i \sim N(0, 1/\rho_\eta), \rho_\eta > 0. \end{aligned}$$

The noise $(\varepsilon_i)_{i \in [0,1]}$ and $(\eta_i)_{i \in [0,1]}$ in the signals are i.i.d. across the traders.⁹ Trader i is called *informed* if she chooses to purchase the signal S_i and *uninformed* otherwise. Once the traders have made their information acquisition decision, period 1 ends and financial market opens in period 2. We relabel the traders so that the set of traders who are informed is $[0, \lambda]$, i.e., the fraction of informed traders is λ .

Note that unlike the model of Grossman and Stiglitz (1980), the private information is diverse. While, this assumption is not uncommon in the literature, see for example Grossman (1976), Hellwig (1981), Diamond and Verrecchia (1981), Admati (1985), and Verrecchia (2001), the results that we derive are new. Simply introducing diverse supply signals in a CARA-normal REE model with diverse payoff signals is what drives these new findings.

Each trader only cares about wealth W at the end of period 2 and has the (von-Neumann-Morgenstern) utility function u with CARA parameter $\gamma > 0$,

$$u(W) = -e^{-\gamma W}.$$

We normalize the price of money to 1 and denote by P the price of the stock. The initial wealth of trader i is $W_1^i = \bar{N}$. We denote trader i 's demand for the stock by $D^i(P)$ and for the bond by $B^i(P)$. Then trader i 's wealth at the end of period 2 is

financial market: supply-informed, payoff-informed, supply-payoff-informed, and uninformed. This generalization is straightforward and the main results of this paper still hold under this extension. For the sake of simplicity, we adopt the current version.

⁹Here and in what follows, when we are dealing with a continuum of i.i.d. random variables $(Y_i)_{i \in [0,1]}$ with mean $\bar{Y} < \infty$ and variance $(\rho_Y)^{-1} < \infty$, we adopt the convention that $\int_0^1 Y_i di = \bar{Y}$ with probability one. The interested reader is referred to Admati (1985) for this convention and to Judd (1985) and Bewley (1980) for details on this issue.

$$W_2^i = B^i(P) + vD^i(P).$$

Since we use the concept of an REE, the price, P , of the stock is a function, $P(v, x)$, of the fundamentals (v, x) of the economy.¹⁰ Denoting by $D_I^i(P)$ (respectively, $D_U(P)$) the demand function of an informed (respectively, uninformed) trader i in the financial market, an *overall equilibrium* in the model is a tuple

$$\left(\lambda^*, P, (D_I^i)_{i \in [0, \lambda^*]}, (D_U^i)_{i \in (\lambda^*, 1]} \right) \quad (1)$$

such that the tuple $\left(P, (D_I^i(P))_{i \in [0, \lambda^*]}, (D_U^i(P))_{i \in (\lambda^*, 1]} \right)$ constitutes an REE in the financial market and λ^* is the equilibrium fraction of informed traders. This is formalized in definitions 1 and 2 where $E[V_I(\lambda)]$ denotes the expected (indirect) utility of an informed trader and $E[V_U(\lambda)]$ denotes the expected (indirect) utility of an uninformed trader, for any $\lambda \in [0, 1]$, and $R(\lambda) = E[V_I(\lambda)] / E[V_U(\lambda)]$. Note that the expected (indirect) utility of being informed (uninformed) is identical across traders, since they are identical ex-ante.

Definition 1 (*Financial market equilibrium*) Given a fraction (λ) of informed traders in the market, a price function $P(x, v)$ and demand functions $(D_I^i(P))_{i \in [0, \lambda]}$ and $(D_U^i(P))_{i \in (\lambda, 1]}$ constitute an REE if (i) $D_I^i(P)$ (respectively $D_U^i(P)$) maximizes the expected utility of informed (respectively, uninformed) trader i conditional on her information, including that provided by the prices, given the price P and (ii) the markets for the stock and the bond clear for each realization of (v, x) .

Noting that we have defined utility to be negative, an equilibrium $\lambda^* \in [0, 1]$ in the information market is given by the following.

Definition 2 (*Information market equilibrium*)

$$\lambda^* = 0 \text{ if } R(0) > 1, \lambda^* = 1 \text{ if } R(1) < 1, \text{ or } \lambda^* \in [0, 1] \text{ if } R(\lambda^*) = 1. \quad (2)$$

¹⁰To be precise, in an REE, the price P is a function of the information of all agents in the economy, i.e., $P \equiv P((y_i, x_i)_{i \in [0, \lambda]})$. However, in equilibrium the price will aggregate all the diverse information and hence will be a function of only the aggregates v and x , i.e., $P = P(v, x)$.

Hence, if a trader does not benefit from becoming informed when no other trader is informed, i.e., $R(0) > 1$, then it is an equilibrium in the information market for no one to buy the information, i.e., $\lambda^* = 0$. On the other hand, if a trader is strictly better off from being informed when all other traders are also informed, i.e., $R(1) < 1$, then in equilibrium all traders in the market will be informed, i.e., $\lambda^* = 1$. In general, for a given fraction of informed traders (λ) if a trader is indifferent between becoming informed and staying uninformed, then that fraction λ is an information market equilibrium.

4 Equilibria in the financial market

We now establish the existence of two REE in the financial market. We are interested in REE that have price $P(v, x)$ as a linear function of the fundamentals. So, suppose the traders conjecture the price function as

$$P = a + bv - cx, \text{ with } b > 0 \text{ and } c > 0. \quad (3)$$

Then the information contained in the price can be expressed by the public signal

$$s = \frac{P - a + c\bar{x}}{b} = v - \frac{c}{b}(x - \bar{x}).$$

Let $\phi = -\frac{c}{b}(x - \bar{x})$, then ϕ is normally distributed with mean 0 and precision given by

$$\rho_\phi = \left(\frac{b}{c}\right)^2 \rho_x. \quad (4)$$

So, informed trader i , uses the sufficient private signal^{11,12}

$$z_i = v - \frac{c}{b} (x - E[x|x_i]) = v - \frac{c}{b} \left[\frac{\rho_x}{\rho_x + \rho_\eta} x - \frac{\rho_\eta}{\rho_x + \rho_\eta} \eta_i \right]. \quad (5)$$

Let $\theta_i = -\frac{c}{b} (x - E[x|x_i])$. So, conditional on x_i , θ_i is normally distributed with mean 0 and precision given by

$$\rho_\theta = \left(\frac{b}{c} \right)^2 (\rho_x + \rho_\eta). \quad (6)$$

Equation (6) captures the benefit to informed trader i from the supply signal x_i in the form of the additional precision $(b/c)^2 \rho_\eta$ in the information she tries to extract from the price. This additional precision depends on (b/c) , which in turn will be determined by the value of λ in the (overall) equilibrium. This provides the channel through which the benefit from acquisition of supply information depends on the fraction of informed traders, which underlies the complementarity or substitutability of information as formalized in section 5. Note that (b/c) can be used as a measure of the *informativeness* of the price about the payoff. When (b/c) is larger, price changes relatively more in response to changes in v than changes in x . Similarly, (c/b) is a measure of the informativeness of price about x . This is consistent with the measures used by Grossman and Stiglitz (1980) and Verrecchia (1982).

Informed traders An informed trader i has information $\{x_i, y_i, P\}$ and uses $\{x_i, y_i, z_i\}$ to update her beliefs. $v | \{x_i, y_i, z_i\}$ is normally distributed with mean μ_I^i and precision ρ_I given by

$$\mu_I^i = \frac{\rho_v \bar{v} + \rho_\varepsilon y_i + \rho_\theta z_i}{\rho_I}, \rho_I = \rho_v + \rho_\varepsilon + \rho_\theta. \quad (7)$$

Given her posterior beliefs about the stock (7), the demand function of informed

¹¹In the sense of Blackwell, see Grossman, Kihlstrom, and Mirman (1977) and the references therein.

¹²Throughout the paper, we use $A | \{B_1, B_2, \dots, B_n\}$ to mean the random variable A conditioned on the random variables $\{B_1, B_2, \dots, B_n\}$.

trader i is

$$D_I^i(P) = \frac{\mu_I^i - P}{\gamma \rho_I^{-1}}. \quad (8)$$

Note that each informed trader has two sources of information to possibly profit from – the information about payoff v , which constitutes direct speculation, and the information about supply x .

Uninformed traders Each uninformed trader only has information $\{P\}$, or equivalently $\{s\}$ and uses it to update their beliefs. $v|s$ is normally distributed with mean μ_U and precision ρ_U given by

$$\mu_U = \frac{\rho_v \bar{v} + \rho_\phi s}{\rho_U}, \rho_U = \rho_v + \rho_\phi. \quad (9)$$

Given her posterior beliefs about the stock (9), each uninformed trader's demand function is

$$D_U(P) = \frac{\mu_U - P}{\gamma \rho_U^{-1}}. \quad (10)$$

In equilibrium, the stock market clears, i.e.,

$$\int_0^\lambda D_I^i(P) di + (1 - \lambda) D_U(P) = x. \quad (11)$$

We find the REE by solving equation (11) for P and then verifying that P is of the form conjectured in (3). Proposition 1 characterizes the REE. As we elaborate below, one of the financial market equilibria in our model exhibits the same properties as the financial market equilibrium in the model by Grossman and Stiglitz (1980). So, we refer to this equilibrium as the GS-REE (Grossman-Stiglitz REE) and label the corresponding values of the variables by GS. We refer to the other financial market equilibrium as the NGS-REE (non-Grossman-Stiglitz REE) and label the corresponding values of the variables by NGS.

Proposition 1 *If $\gamma^2 > 4\lambda^2 \rho_\varepsilon \rho_\eta > 0$, for any given λ , and $\rho_\varepsilon > 0, \rho_\eta > 0$, there exist*

two partially revealing rational expectations equilibria in which,¹³

$$P = a + bv - cx,$$

where

$$a = \frac{\rho_v \bar{v} + \beta \rho_x \bar{x}}{K}, b = \frac{\lambda(\rho_\varepsilon + \rho_\theta) + (1 - \lambda)\rho_\phi}{K}, c = \frac{\beta \rho_x + \gamma}{K},$$

and $\beta = \frac{b}{c}$ takes one of two values,

$$\beta^{GS} = \frac{\gamma - \sqrt{\gamma^2 - 4\lambda^2 \rho_\varepsilon \rho_\eta}}{2\lambda \rho_\eta}, \beta^{NGS} = \frac{\gamma + \sqrt{\gamma^2 - 4\lambda^2 \rho_\varepsilon \rho_\eta}}{2\lambda \rho_\eta}.$$

with $K = \lambda \rho_I + (1 - \lambda) \rho_U$, $\rho_I = \rho_v + \rho_\varepsilon + \rho_\theta$, $\rho_\theta = \beta^2 (\rho_x + \rho_\eta)$, $\rho_U = \rho_v + \rho_\phi$, $\rho_\phi = \beta^2 \rho_x$.

We now comment further on the GS and NGS labels we have adopted for the two REE. In the model of Grossman and Stiglitz (1980), β increases as the fraction of informed traders increases, i.e., $\frac{\partial \beta}{\partial \lambda} > 0$. This is true of the equilibrium with GS variables in our model,

$$\frac{\partial \beta^{GS}}{\partial \lambda} = \frac{\gamma \beta^{GS}}{\lambda \sqrt{\gamma^2 - 4\lambda^2 \rho_\varepsilon \rho_\eta}} > 0.$$

In contrast, in the equilibrium with NGS variables, β decreases as λ increases,

$$\frac{\partial \beta^{NGS}}{\partial \lambda} = -\frac{\gamma \beta^{NGS}}{\lambda \sqrt{\gamma^2 - 4\lambda^2 \rho_\varepsilon \rho_\eta}} < 0.$$

However, note that for any given value of λ , the NGS-REE price is always more informative than the GS-REE price since $\beta^{NGS} \geq \beta^{GS}$. The GS and NGS labels are used to indicate that the equilibrium price with GS label has similar properties to the

¹³Although all the variables here depend on λ , we do not express the dependence explicitly for simplicity of notation. This convention is followed in the next section also.

equilibrium price in Grossman and Stiglitz (1980), while the equilibrium price with the NGS label does not. The GS-REE is thus a generalization of the equilibrium in Grossman (1976), Grossman and Stiglitz (1980), and Diamond and Verrecchia (1981) in our setup, while the NGS-REE is a new (partially revealing) equilibrium that does not exist in the previous studies.

The change in the informativeness of price about the payoff can be clearly understood in the limiting case when there are no informed traders in the market, i.e., $\lambda = 0$. When $\lambda = 0$, there is no information about v or x among the traders and there is no linear partially revealing REE price in the financial market. There are only two fully revealing REE prices, which exist due to self-fulfilling nature of an REE, one of which fully reveals the value of v while the other fully reveals the value of x . As λ converges to zero, the GS-REE approaches the REE which fully reveals x and provides no information on v , while the NGS-REE approaches the REE that fully reveals v and provides no information on x .

As we noted above, β is a measure of the informativeness of the price about the payoff v , so that $(1/\beta)$ measures the informativeness of the price about x . As λ increases above zero, there is more information about v and x present among the traders. This introduces more information about v into the GS-REE and more information about x into the NGS-REE, making these partially revealing. As a consequence, the GS-REE price is more informative about v as λ increases and the NGS-REE price less informative about v (and more informative about x) as λ increases.

Proposition 1 suggests that for a linear (partially revealing) equilibrium price function to exist in the financial market, the signals can not be very sharp, that is, we need low ρ_ε or ρ_η .¹⁴ As we noted above, each informed trader can possibly profit from trade by using her information about v and by using her information about x . The benefit to each trader from being informed can be decomposed into that from

¹⁴Fully revealing REE prices exist irrespective of the sign of $(\gamma^2 - 4\lambda^2\rho_\varepsilon\rho_\eta)$. However, we focus on the more interesting partially revealing REE prices. Also, we have nothing to say about the existence of a partially revealing REE price when $\gamma^2 < 4\lambda^2\rho_\varepsilon\rho_\eta$ beyond noting that such a linear price function does not exist.

the supply signal and that from the payoff signal.

A trader will purchase costly information about v or x only when she expects the price *not* to fully reveal that information so that she can profit from it. This is exactly the paradox considered by Grossman and Stiglitz (1980) in their seminal study. There will be a positive fraction of informed traders in an (overall) equilibrium only if the financial market REE price reveals their information only partially (or not at all).

In our model, when $\rho_\eta = 0$, the supply signal provides no information and the informed trader can only use her payoff signal to make profitable trades. In the limit as ρ_η converges to zero, the NGS-REE price approaches an REE price that fully reveals v . The GS-REE on the other hand approaches a partially revealing REE price. Now, traders seeking to profit from information about v , i.e., from direct speculation will purchase costly information when the REE price only partially reveals v and not if it does so fully. Hence, the financial market will be in equilibrium with a positive fraction of informed traders only when the traders' decisions lead to the GS-REE. This is just the familiar result that the GS-REE is driven by the direct speculation motives of traders informed about v .

On the other hand, when $\rho_\varepsilon = 0$, there is no information contained in the payoff signal and any profit from private information can only be made using the information about supply x . As ρ_ε converges to zero, the GS-REE approaches an REE price that fully reveals x , while the NGS-REE approaches a partially revealing REE price. Symmetrically to the previous case, the financial market will be in equilibrium with a positive value of λ if the traders' decisions lead to the NGS-REE. We thus have the symmetric result that the NGS-REE exists due to the desire to profit from trades that are based on information about the stock's supply x . The nature of the informativeness of the GS- and NGS-REE in each of the limiting cases is summarized in

Table 1.

Table 1 Information characteristics of the financial market equilibria

	β^{GS}	β^{NGS}
$(\lambda, \rho_\varepsilon) \gg 0, \rho_\eta = 0$	$\frac{\lambda \rho_\varepsilon}{\gamma}$ [Partially revealing]	∞ [Fully reveals v]
$(\lambda, \rho_\eta) \gg 0, \rho_\varepsilon = 0$	0 [Fully reveals x]	$\frac{\gamma}{\lambda \rho_\eta}$ [Partially revealing]
$(\rho_\varepsilon, \rho_\eta) \gg 0, \lambda = 0$	0 [Fully reveals x]	∞ [Fully reveals v]

Similarly to Grossman and Stiglitz (1980) our analysis suggests that REE that only reveal information about v or x are interesting and we consequently restrict attention to these. Allowing traders to possibly profit from information about supply of the stock, i.e., allowing $\rho_\eta > 0$ and $\rho_\varepsilon > 0$, provides the result that a new partially revealing REE exists.

In the previous studies such as by Diamond and Verrecchia (1981), $\rho_\eta = 0$, which permits the existence of a unique (linear) partially revealing REE price. Our results show that this result is not robust to small perturbations in the value of ρ_η and that multiple partially revealing REE prices exist for small positive values of ρ_η . Figure 1 illustrates the jump in the number of partially revealing REE prices in a CARA-normal setup as ρ_η varies.

In order to ensure that the multiplicity is not an artifact of the continuum of traders, we also considered an economy identical to the one here but with a finite number of traders. Indeed, when $\rho_\eta > 0$ and there is a large, but finite number of traders in the economy there exist three partially revealing REE in the economy. As the number of traders converges to infinity, the three REE of the finite economy converge to the GS-REE, the NGS-REE, and a REE which fully reveals v . So, the multiplicity of equilibria is not an artifact of the continuum economy, it is generated by the presence of supply signals.

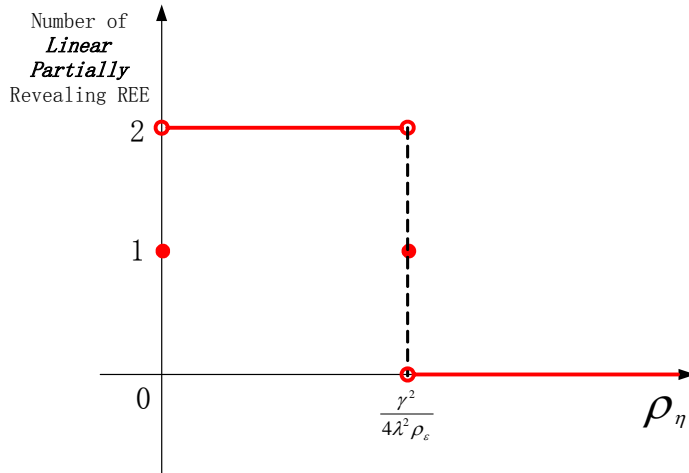


Figure 1 The number of linear partially revealing REE prices and the precision of the supply signal

In Grossman and Stiglitz (1980), the payoff signal is identical across traders and the price only communicates (noisily) the information of the informed traders (the payoff signal) to the uninformed traders, it has no role as an aggregator of information. The identical payoff signal in Grossman and Stiglitz (1980) also means that the informed traders are perfectly informed about the aggregate supply of the stock once they are informed of the price, which is publicly observable. As noted before, our model generalizes that of Grossman and Stiglitz (1980) to a case with diverse noisy information. In Diamond and Verrecchia (1981), the price does aggregate the diverse payoff information across traders, but the absence of a supply signal allows the existence of a unique equilibrium. We now turn to applications of the financial market equilibria in the following section.

Applications The (unique) financial market equilibrium in the previous studies of the CARA-normal framework has been extensively used in the finance literature. These studies have used the properties of the unique equilibrium to examine the policy and theoretical implications for issues such as the cost of capital (Easley and

O'Hara 2004). The multiplicity in the financial market means that the policy and theoretical implications can be very different depending on which equilibrium the traders coordinate upon.

In Easley and O'Hara (2004) the partially revealing REE is unique and exhibits the same properties as the GS-REE in our model. On the other hand, if the traders coordinate on the NGS-REE in our model, then the policy and theoretical implications are opposite those in Easley and O'Hara (2004). First, on average the NGS-REE has a higher price and consequently a lower cost of capital than the GS-REE. Second, when the traders coordinate on the NGS-REE, the larger the fraction (λ) of the informed traders, the larger is the cost of capital.¹⁵ In the NGS-REE, when the number of informed traders increases, the information revealed by the price falls, this makes the stock riskier for the uninformed traders, who demand a higher risk premium raising the cost of capital. Finally, in the NGS-REE, when the payoff signal becomes more precise, i.e., ρ_ε increases, then the cost of capital increases, unlike in Easley and O'Hara (2004).

Also, the multiplicity of market-clearing prices in our model means that price crashes can occur in our model and the stock prices can exhibit volatility that is excessive to that of the fundamentals. Multiplicity also means that our model can explain the twin stocks phenomenon (Barberis and Thaler, 2003).

4.1 General equilibrium analysis: removing noise traders

The previous analyses of the CARA-normal framework are in a partial equilibrium framework, since the aggregate supply of the stock is driven by noise (liquidity) traders who are not modeled. The behavior of the noise traders or the reasons thereof are not analyzed and the behavior is simply summarized in the normally distributed random variable x representing the trading decisions of these noise traders. Introducing

¹⁵Here we assume that the traders commit to one financial market equilibrium for all possible values of the parameters.

a supply signal in the CARA-normal framework allows us to remove (unmodeled) noise traders from the set up and hence permits general equilibrium analysis of the financial market where the aggregate supply of the stock is in fact an aggregate of the endowments of the (rational) traders whose behavior is explicitly analyzed.

Diamond and Verrecchia (1981) introduced the idea of doing away with noise traders and considering the aggregate supply of the stock to be the aggregate of the rational traders endowments of the stock. Although Diamond and Verrecchia (1981) carry out general equilibrium analysis of the CARA-normal framework, they do so only for the case of a finite number of traders. They follow this route since they assume that the endowments are i.i.d. across the traders and consequently, assuming an infinite number of traders would make the aggregate supply of the stock a fixed number, by the law of large numbers, instead of a random variable.

As pointed out by Hellwig (1980) and Laffont (1985), there is a conceptual problem in using the REE concept with a finite number of traders. The behavior of traders seems odd in an REE when only a finite number of them are present. In particular, while discussing Grossman (1976), Hellwig (1980) states that “[b]ut then Grossman’s agents are slightly schizophrenic... one should expect that agents ... will also notice the effect they have on the price. Yet, Grossman’s agents are price takers. They do not attempt to manipulate the price and the information content of price.” Laffont (1985) also raises this issue, stating “this equilibrium notion makes sense only when each agent is negligible. Indeed if an agent is able to know the mapping from private signals to prices, he must be able to know the inverse demand functions; if he is not negligible, he will use this knowledge to behave strategically. In a finite economy, one must take into account strategic behavior of agents both with respect to prices and with respect to the disclosure of their own information, since the equilibrium concept used provides the agent with the information necessary to behave strategically.”

Grundy and McNichols (1989) seek to analyze an infinite number of traders and avoid degenerateness of the aggregate supply by assuming that the variance of the

(random) aggregate supply is proportional to the number of traders so that with an infinite number of traders the variance of the aggregate supply is infinite. On the other hand, we avoid this potentially problematic assumption, and simply assume that there is a common component across the endowments of the traders. This assumption then generates the new result that multiple (partially revealing) REE (prices) exist in the CARA-normal setup.

The general equilibrium analysis of the financial market is possible using a special case of our model, in which the supply signal is available freely to all traders while the payoff signal is available only to a fraction of the traders. The free supply signal for each trader is in fact her (random) endowment of the risky asset, the realization of which is only observed by her. The aggregate supply of the risky asset then is just the aggregate of the endowments of the traders. There are no noise or liquidity traders. The traders in the financial market do not know the aggregate supply in the market since the aggregate of traders' endowments is still random. In this case, information about payoff is available as a private noisy signal to only $\lambda > 0$ of the traders who are called *informed*. The private payoff signal is diverse across the traders. Note that the information structure of the model is considered exogenous, since we are interested in general equilibrium analysis of only the financial market.

Trader $i \in [0, 1]$ is endowed with \bar{n} units of the bond and x_i units of the stock, where

$$x_i = x + \eta_i \text{ and } x \sim N(\bar{x}, 1/\rho_x), \eta_i \sim N(0, 1/\rho_\eta), \rho_x > 0, \text{ and } \rho_\eta > 0.$$

As before, $(\eta_i)_{i \in [0, 1]}$ are i.i.d across the traders. The realization of x_i is observed only by trader i . The aggregate supply of risky asset is then

$$\int_0^1 x_i di = x.$$

We can interpret x as a common shock due to the fluctuation of the whole economy

and η_i as an idiosyncratic shock due to variation in individual ability.¹⁶ For example, if the risky asset is corn, then x can be thought of as the effect of weather on the output of corn. In the studies by Grossman and Stiglitz (1980), Diamond and Verrecchia (1981), Grundy and McNichols (1989), Kim and Verrecchia (1991) that have analyzed CARA-normal models where traders are endowed with random amounts of the risky asset, the endowments do not have a common component across traders.

The analysis can now be done similarly to that for the derivation of proposition 1 and will yield a similar result. Hence, simply closing the model by considering its general equilibrium version yields multiple equilibria in the financial market without resorting to any extra constraints or feedback effects. This suggests that multiple equilibria may in fact be an inherent feature of financial markets based on the CARA-normal framework.

5 Equilibrium in the information market

We now consider the existence and properties of equilibria in the information market. In particular, we show that information acquisition can exhibit strategic complementarity in the CARA-normal framework, which leads to multiple equilibria in this market. The properties of the NGS-REE price with regard to information content clearly indicate that if traders coordinate on this equilibrium then information acquisition can be a strategic complement. We establish this result in proposition 4. However, the GS-REE in our model is a generalization of the unique financial market equilibrium of the previous studies, which has been applied to study a variety of issues as noted above. Hence, it is of interest to assess whether information can be a strategic complement when the traders coordinate on the GS-REE in the financial market. We indeed find that information can be a strategic complement when the

¹⁶The existence of multiple equilibria in the financial market does not require a common shock to the endowments of *all* agents. A common shock to a positive fraction of the agents is enough to generate multiple equilibria. A proof of this assertion is available from the authors on request.

traders coordinate on the GS-REE. This is an important result since in the previous studies of the CARA-normal framework and its applications, with the exception of Veldkamp (2006a and 2006b), information is a strategic substitute given the (unique) financial market equilibrium.

Also, as we show in proposition 3 the (rational) informed and (rational) uninformed traders have higher utility ex-ante when they coordinate on the GS-REE than when they coordinate on the NGS-REE for a reasonable range of values for the parameters, which further suggests that coordination on the GS-REE may be more likely. In what follows, we assume $\bar{v} = \bar{x} = 0$.¹⁷

The next result shows that ex-ante the expected indirect utility of becoming informed is proportional to that of staying uninformed for any trader i , irrespective of the REE coordinated upon, and the respective expected utilities are identical across traders, since the traders are identical ex-ante.

Proposition 2 *Suppose traders coordinate on the GS-REE (or the NGS-REE), then for any given λ , the expected indirect utility of the uninformed traders ($E[V_U]$) and that of the informed traders ($E[V_I]$) is given by*

$$\begin{aligned} E[V_U] &= -\frac{1}{\sqrt{1+A}} e^{-\gamma\bar{N}}, \\ E[V_I] &= e^{\gamma\kappa} \sqrt{\frac{\rho_U}{\rho_I}} E[V_U], \end{aligned}$$

where $A = \rho_U \left(\frac{1}{b} \frac{\rho_\phi}{\rho_U} - 1 \right)^2 \left(\frac{b^2}{\rho_v} + \frac{c^2}{\rho_x} \right)$ and values of the parameters $b, c, \rho_\phi, \rho_U, \rho_I$ are as given in proposition 1 corresponding to the GS-REE (or the NGS-REE, respectively).

¹⁷Assuming non-zero means does not affect the qualitative results regarding complementarity in this paper. In several examples, we use non-zero means.

5.1 Ex-ante welfare analysis

As noted before, as ρ_η converges to zero, we get that the GS-REE price remains partially revealing, with

$$\lim_{\rho_\eta \rightarrow 0} \beta^{GS} = \frac{\lambda \rho_\varepsilon}{\gamma}.$$

On the other hand, the NGS-REE price converges to a fully revealing price, with

$$\lim_{\rho_\eta \rightarrow 0} \beta^{NGS} = \infty \Rightarrow c \rightarrow 0 \Rightarrow P \rightarrow v.$$

The properties in the limit as ρ_η converges to zero of the two equilibria can be used to understand the implications for the welfare of the traders in the two equilibria.

When $\rho_\eta \rightarrow 0$, if the traders coordinate on the NGS-REE, in the limit the stock becomes risk-free since its price perfectly reveals its value. The market is reduced to one with only risk-free assets (money and the risk-free stock). As a consequence, after trade, holding the equilibrium portfolio is utility-equivalent to holding the initial endowment of money, which is always affordable in a partially-revealing equilibrium, which in the limit is the GS-REE. So, the informed and uninformed traders are better off when coordinating on the GS-REE than on the NGS-REE in this case and the missing risky asset accounts for the welfare loss. In other words, more precise information makes the price adjust such that the risk premium is low enough to more than offset the benefit of lower risk due to the sharper information. This result is formalized in proposition 3. Analyses of similar phenomena can be found in Hirshleifer (1971) and Citanna and Villanacci (2000).

Proposition 3 *In a neighborhood of $\rho_\eta = 0$ or $\rho_\varepsilon = 0$, the (rational) informed and (rational) uninformed traders have higher utility ex-ante when coordinating on the GS-REE instead of the NGS-REE, for any $\lambda \in [0, 1]$.*

This result is not very restrictive in light of proposition 1, which shows that low values of ρ_ε or ρ_η are needed for the linear REE prices to exist in the financial market.

Although we cannot formally prove that informed and uninformed traders are better off in the GS-REE than in the NGS-REE for all reasonable parameters, we do many numerical experiments, which show this result is very robust. For example, figure 2 shows the case $\gamma = 2$, $\rho_\varepsilon = \rho_\eta = 1$, $\rho_v = 2$, $\rho_x = 4$, $\kappa = \bar{N} = 0$.

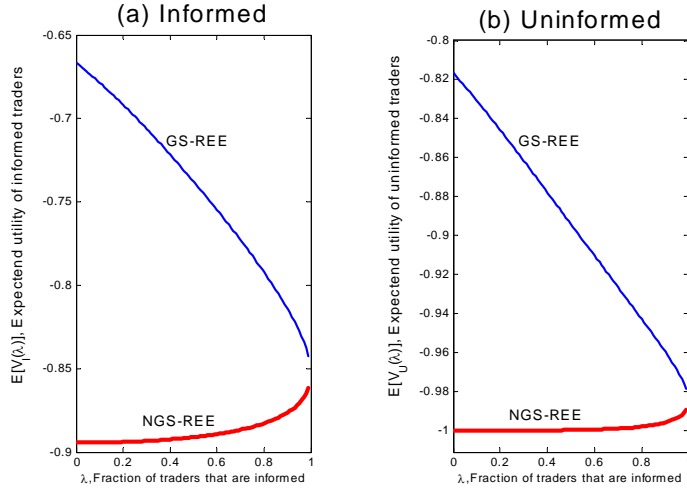


Figure 2 Expected indirect utility in the GS-REE and the NGS-REE

We now make precise the sense in which information acquisition is a strategic complement (substitute), using the result of proposition 2, which provides that $R(\lambda) = e^{\gamma\kappa} \sqrt{\frac{\rho_U}{\rho_I}}$.

Definition 3 (*Strategic complement / substitute*) *If $R'(\lambda) < 0$, then learning is a strategic complement, and if $R'(\lambda) > 0$, then learning is a strategic substitute.*

In other words, strategic complementarity (substitutability) will give the traders more (less) incentive to get informed as the fraction of informed traders is getting larger. This definition corresponds to those in Grossman and Stiglitz (1980) and Barlevy and Veronesi (2000).

We now state our main results, which say that whether information acquisition is a strategic substitute or a strategic complement, depends on the comparison of the

relative precision of the payoff and supply signals.

5.2 Complementarity with the GS-REE

As we noted above, the GS-REE is an equilibrium that exists as informed traders seek to profit from their costly information about v , i.e., from direct speculation and also GS-REE price reveals more information about v as λ increases. We now let the traders coordinate on the GS-REE in the financial market, which corresponds to the unique financial market equilibrium of previous studies, and unless otherwise stated, all the variables in period 2 will refer to the GS-REE.

Following definition 5 and proposition 2, if $\sqrt{\rho_I/\rho_U} > e^{\gamma\kappa}$, then a trader would decide to become informed. So, we could measure the benefit from being informed by ρ_I/ρ_U , while $e^{\gamma\kappa}$ is directly related to the cost of information acquisition. In particular, the relative benefit to an informed trader can be decomposed into the benefit from the payoff signal and that from the supply signal, i.e., we have

$$\frac{\rho_I}{\rho_U} = \frac{\rho_\varepsilon}{\rho_U} + \frac{\beta^2 \rho_\eta}{\rho_U} + 1$$

Here, the first term (ρ_ε/ρ_U) is contributed by the payoff signal, while the second term ($\beta^2 \rho_\eta/\rho_U$) is contributed by the supply signal. When the payoff signal is relatively sharper than the supply signal, an informed trader has more precise information about the payoff and so, more to gain from direct speculation in the financial market than from supply-signal-based speculation. Now, as more traders seek private information about v to engage in direct speculation, i.e., λ increases, the GS-REE price reveals more information about v , i.e., β rises making it easier for the uninformed traders to free-ride on the information acquired by the informed, i.e., ρ_U becomes larger. Thus, the benefit from the payoff signal decreases making information a substitute.

On the other hand, when the supply signal is more precise than the payoff signal,

an informed trader has more to gain from speculation based on the information about the stock's supply x . As more traders seek private information about x , λ increases and the GS-REE price reveals less information about x , i.e., $(1/\beta)$ decreases. This makes it harder for the uninformed traders to infer the informed traders private information from the price, and the relative benefit of being uninformed decreases, i.e., when β becomes larger,

$$\frac{\beta^2 \rho_\eta}{\rho_U} = \frac{\rho_\eta}{(\rho_v/\beta^2) + \rho_x}$$

increases, so that information will be a complement.

Comparing the precision-ratios of the two signals thus indicates which effect is dominant and provides a means of identifying whether information is a substitute or complement as formalized below.

Theorem 1 *Let the traders coordinate on the GS-REE in the financial market. If $\frac{\rho_\varepsilon}{\rho_v} > \frac{\rho_\eta}{\rho_x}$, then information acquisition is a strategic substitute while if $\frac{\rho_\varepsilon}{\rho_v} < \frac{\rho_\eta}{\rho_x}$, then information acquisition is a strategic complement.*

Hence, the presence of diverse payoff signals and diverse supply signals generates complementarities in information acquisition. Before illustrating the content in theorem 1 with some examples, we make the following observations regarding the strategic substitutability and complementarity results in the literature.

Remark 1 *In the standard case (Diamond and Verrecchia, 1981, Verrecchia, 1982), $\rho_\eta = 0$, so, learning is always a strategic substitute.*

Remark 2 *In a GS-REE, $\frac{\partial \beta}{\partial \lambda} > 0$, i.e., learning always makes it easier for traders to infer information from prices. In Barlevy and Veronesi (2000, p88) “the crucial component generating learning complementarities is that learning makes identification more complicated for uninformed agents.” In our model of the CARA-normal*

framework, the possibility of speculation based on private information about supply is the relevant property which makes information complementary or substitutable.

By theorem 1, if $\frac{\rho_\varepsilon}{\rho_v} > \frac{\rho_\eta}{\rho_x}$, learning is a strategic substitute, like Grossman and Stiglitz (1980), and there is a unique equilibrium in the information market. However, if $\frac{\rho_\varepsilon}{\rho_v} < \frac{\rho_\eta}{\rho_x}$, i.e., when learning is a strategic complement, multiple equilibria are possible. See figure 3 for an illustration of both cases.

In figure 3(a), learning is a strategic substitute, with $\gamma = 2$, $\rho_\varepsilon = \rho_\eta = 1$, $\rho_v = 2$, $\rho_x = 4$, and $\kappa = 1/10$ and there is a unique equilibrium: $\lambda^* = 0.26$. In figure 3(b), learning is a strategic complement, with $\gamma = 2$, $\rho_\varepsilon = \rho_\eta = 1$, $\rho_v = 4$, $\rho_x = 2$, and $\kappa = 1/15$, and there are three equilibria: $\lambda^* \in \{0, 1, 0.96\}$. At $\lambda^* = 0$, the expected indirect utility of a trader from being informed is strictly less than that from being uninformed given that all the other traders are uninformed, i.e., $R(0) > 1$. On the other hand, at $\lambda^* = 1$, the expected indirect utility of a trader from being informed is strictly greater than that from being uninformed given that all the other traders are informed, i.e., $R(1) < 1$.¹⁸

While the zero-information equilibrium ($\lambda^* = 0$) may seem strange,¹⁹ our model can be extended to allow the cost of acquiring the same information to differ across traders, an extreme example being where a very small fraction get the information for free. This will lead to equilibria where λ is always positive in equilibrium.

When $\frac{\rho_\varepsilon}{\rho_v} = \frac{\rho_\eta}{\rho_x}$, then $R(\lambda)$ is independent of λ . In this situation if $R(\lambda) \equiv 1$, then any $\lambda \in [0, 1]$ is an equilibrium.

In Veldkamp (2006a), information is produced by a fixed-cost technology in a competitive information production sector and sold at an endogenous price. Once produced information can be distributed (but not resold) at zero marginal cost and as a result, the equilibrium price of information declines as the number of informed traders increases. So, an increase in λ makes information cheaper and so more de-

¹⁸Recall that the utilities as defined are negative.

¹⁹Veldkamp (2006a) also has examples of cases where in equilibrium λ is zero.

sirable to uninformed traders. On the other hand, an increase in λ also makes the financial market price more informative about the payoff, decreasing the value of information. When λ is low, this price-complementarity effect more than offsets the decreased desirability of information causing the net benefit from being informed to increase with λ . A consequence of Veldkamp's (2006a) model is that λ is positive in equilibrium only for high values of v . In contrast, since the complementarity in our model arises from the supply signal, λ can be positive in equilibrium even for low values of v .

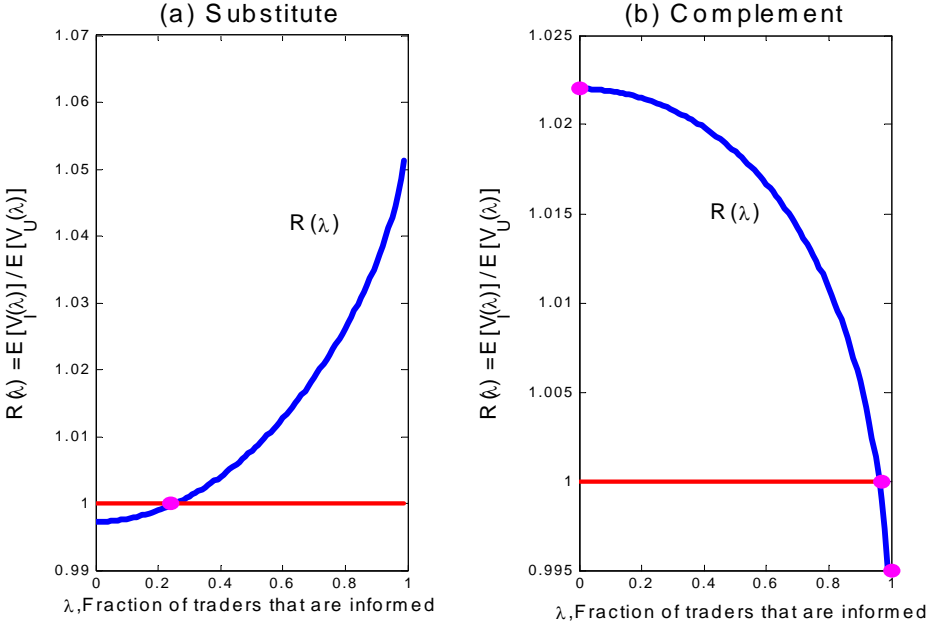


Figure 3 Information market equilibria [GS-REE]

5.3 Complementarity with the NGS-REE

The NGS-REE exists as traders seek to profit from speculation based on private information about supply. When the traders coordinate on the NGS-REE, arguments analogous to those in the case of coordination on the GS-REE suggest when informa-

tion is a complement and when it is a substitute.

There is a trade-off between speculation based on private information about v and x , respectively. However, the NGS-REE price always reveals less information about v and more about x as the fraction of informed traders, λ , increases. This means that it is easier for uninformed traders to free-ride on the information revealed by prices when the informed traders seek to speculate about x , making information a substitute. On the other hand, more traders seeking to speculate using private information about v makes price reveal less informative about v , so that information is a complement. This is formalized as follows.

Proposition 4 *Let the traders coordinate on the NGS-REE. If $\frac{\rho_\varepsilon}{\rho_v} < \frac{\rho_\eta}{\rho_x}$, then information acquisition is a strategic substitute while if $\frac{\rho_\varepsilon}{\rho_v} > \frac{\rho_\eta}{\rho_x}$, then information acquisition is a strategic complement.*

If the traders coordinate on the NGS-REE, then by proposition 2, $R(\lambda)$ would depend on λ only through β^{NGS} . Now, β^{NGS} affects $R(\cdot)$ in exactly the same way as β^{GS} , i.e., $\frac{\partial R(\cdot)}{\partial \beta^{NGS}} = \frac{\partial R(\cdot)}{\partial \beta^{GS}}$ when $\beta^{NGS} = \beta^{GS}$.²⁰ However, $\frac{\partial \beta^{NGS}}{\partial \lambda}$ has exactly the opposite sign to $\frac{\partial \beta^{GS}}{\partial \lambda}$. So, the complementarity and substitutability result would be exactly the opposite: if the payoff signal is relatively sharper $\left(\frac{\rho_\varepsilon}{\rho_v} > \frac{\rho_\eta}{\rho_x}\right)$, then information acquisition is a complement.

This result is illustrated in figure 4(a), where $\gamma = 2$, $\rho_\varepsilon = \rho_\eta = 1$, $\rho_v = 2$, $\rho_x = 4$, and $\kappa = 1/16$, and information is a strategic complement with $\lambda^* \in \{0, 0.85, 1\}$. In figure 4(b), information is a strategic substitute with $\gamma = 2$, $\rho_\varepsilon = \rho_\eta = 1$, $\rho_v = 4$, $\rho_x = 2$, and $\kappa = 1/12$, so that $\lambda^* = 0.88$.

²⁰Obviously $\beta^{NGS} = \beta^{GS}$ is true only for two different values of λ .

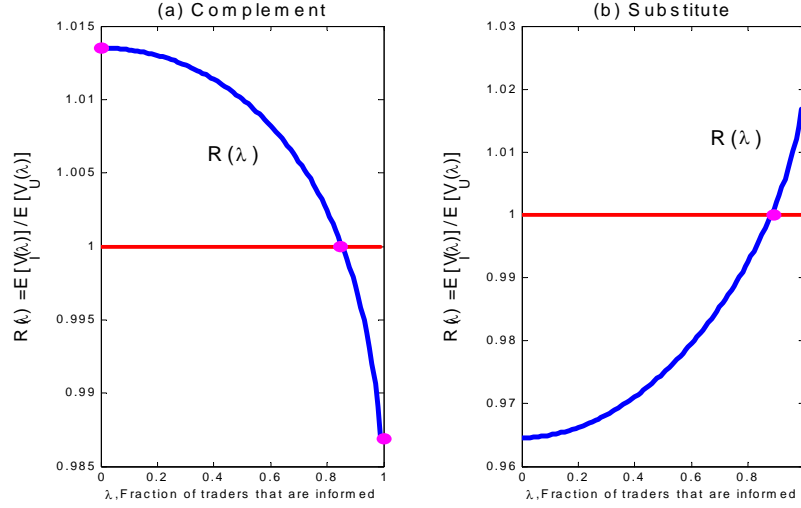


Figure 4 Information market equilibria [NGS-REE]

5.4 Complementarity with sunspots

Since there are two REE in the financial market, the traders could use sunspots to coordinate on these. If the traders do so, we could have a U-shaped, an inverted U-shaped, or a slanted S-shaped R-curve, since the GS- and NGS-REE contribute in exactly opposite ways. Figure 5 provides illustrations of these possibilities with π denoting the probability of the sunspot variable realization that leads to coordination on the GS-REE.

In figure 5(a), we have $\pi = 3/4$, $\gamma = 2$, $\rho_\varepsilon = \rho_\eta = 1$, $\rho_v = 4$, $\rho_x = 2$, $\kappa = 0.068$ and there are three equilibria with $\lambda^* \in \{0.48, 0.77, 1\}$. In figure 5(b), $\pi = 1/2$, $\gamma = 2$, $\rho_\varepsilon = \rho_\eta = 1$, $\rho_v = 2$, $\rho_x = 4$, $\kappa = 0.076$ and there are again three equilibria with $\lambda^* \in \{0, 0.05, 0.31\}$. Figure 5(c) presents an interesting case with a slanted S-shaped curve, where $\pi = 2/5$, $\gamma = 2$, $\rho_\varepsilon = \rho_\eta = 1$, $\rho_v = 4$, $\rho_x = 2$, $\kappa = 0.071$ and there are five equilibria with $\lambda^* \in \{0, 0.03, 0.61, 0.99, 1\}$.

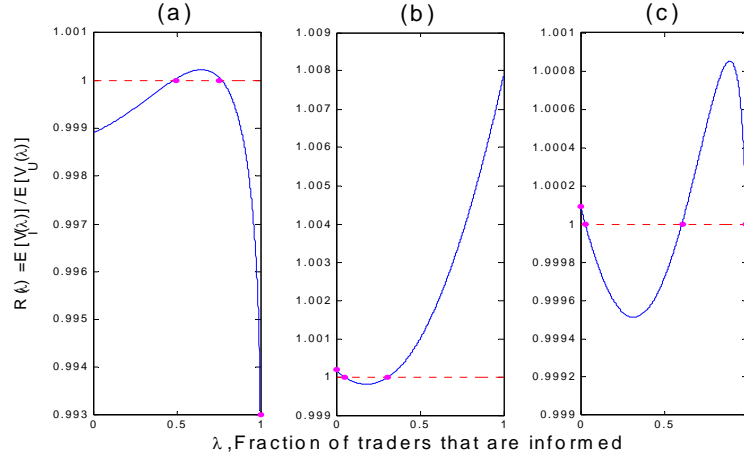


Figure 5 Information market equilibria [sunspots]

5.5 Multiplicity of equilibria in the model

As a consequence of the fact that multiplicity in the financial market occurs independently of multiplicity in the information market and vice versa, there can be a large number of overall equilibria, $(\lambda^*, P, (D_T^i)_{i \in [0, \lambda^*]}, (D_U^i)_{i \in (\lambda^*, 1]})$ in our model. For example, while there exist two REE for each information market equilibrium and there can exist three information market equilibria for each REE, there are also overall equilibria where each information market equilibrium corresponds to a different REE. Hence, a change in the coordination decision of the traders in financial market could lead to a change in the information market equilibrium. On the other hand, the price of the stock could switch without a change in the fundamentals as the information market equilibrium changes.

Applications Our results provide two sources of excess price volatility - multiplicity in the financial markets and multiplicity in the information market, so that the volatility in stock prices can be excessive relative to that of the fundamentals. A change in the information market equilibrium when the traders coordinate on one of

the two REE in the financial market can cause the volatility of the stock price to increase without a change in the volatility of the fundamentals. In the example of figure 5, the price volatility, measured by $Var(P^{GS}|\lambda^*)$ jumps from 0.125 to 0.18 as λ^* changes from 0 to 0.96 even though the volatility of the payoff ($1/\rho_v$) and that of the supply ($1/\rho_x$) do not change from 0.5 and 0.25, respectively. A jump from the GS-REE to the NGS-REE can also cause the volatility of price to increase. For example, the volatility of price increases from $Var(P^{GS}|\lambda) = 0.15$ to $Var(P^{NGS}|\lambda) = 0.23$ with a shift from the GS-REE to the NGS-REE for $\lambda = 1/2$, $\gamma = 2$, $\rho_\varepsilon = \rho_\eta = 1$, $\rho_v = 4$, and $\rho_x = 2$.

In multi-asset markets, an implication of the multiplicity of information market equilibria in our model is that the prices of two stocks that have very similar (and independent) fundamentals can differ even if the prices exhibit similar properties. In other words, even if traders coordinate on the same REE for both stocks, for example the GS-REE, the information market could be in different equilibria for the two stocks, for example in one there could be no informed traders and in the other a positive fraction of them, causing the price to be different on average. The information market could be in a different equilibrium for each stock due to historical reasons or extrinsic factors that affect the information market equilibria the traders coordinate on for each stock.

5.6 Application: information and trade frenzies

‘Frenzies’ in financial markets are occasional surges in asset prices that can be thought of as occurring when “a shift in information suddenly raises an asset’s price above what a model without information would predict” (p 577, Veldkamp, 2006a). As noted before, Veldkamp (2006a) generates complementarities in information acquisition in a dynamic model of the CARA-normal framework by endogenizing the price of information. In her model, complementarities can make the change in equilibrium information provision large and abrupt across periods as the fundamentals move beyond

a cut-off value. As the unique equilibrium stock price is increasing in the fraction of informed traders, the jump from a no-information equilibrium to a positive-information equilibrium can explain the occurrence of frenzies in financial markets. The equilibrium stock price in her model is similar to that in the equilibrium of Grossman and Stiglitz (1980) and consequently to the GS-REE in our model.

As noted in proposition 5 in the GS-REE the expected price of the stock increases as the number of informed traders increases, while in the NGS-REE the expected price of the stock decreases as the number of informed traders increases. This result suggests that while media frenzies can occur, the reverse phenomenon is also likely.

Proposition 5 *[Information and asset prices]* *If $\bar{x} > 0$, in the GS-REE, the expected stock price $E[P]$ is a strictly increasing function of λ , and in the NGS-REE, the expected stock price $E[P]$ is a strictly decreasing function of λ .*

Unlike Veldkamp (2006a, 2006b), our model is a static one and multiple equilibria (in the information market) are necessary to explain the occurrence of frenzies. A frenzy in our model occurs when the information market jumps from one equilibrium to another when the traders coordinate on the GS-REE. For example, a jump from $\lambda^* = 0$ to $\lambda^* = 0.96$ in figure 5, causes an abrupt surge in the (expected) price of the stock when the financial market is in the GS-REE. The change in $E[P]$ is 40% when the information market equilibrium changes from $\lambda^* = 0$ to $\lambda^* = 0.96$, specifically $E[P]$ changes from 0.5 to 0.7 as λ^* changes. Another measure of a frenzy is the ratio of the variances of price in the two information market equilibria, $\frac{Var(P^{GS}|\lambda^*=0.96)}{Var(P^{GS}|\lambda^*=0)} = 1.43$. These values are calculated using $\bar{v} = \bar{x} = 1$.

Hence, it is possible that media frenzies can also occur when information about the stock's supply is available and are caused by an increase in information (measured by λ) like the result of Veldkamp (2006a). However, a fall in the fraction of informed traders when the traders are coordinating on the NGS-REE can also cause a rise in the stock price, the exact opposite of the frenzy phenomenon. For example, a jump

from $\lambda^* = 0.85$ to $\lambda^* = 0$ causes a change in $E[P]$ of 17% while the ratio of price variances is $\frac{Var(P^{NGS}|\lambda^*=0)}{Var(P^{NGS}|\lambda^*=0.85)} = 1.21$.

6 Conclusion

In this paper, we consider a natural extension of the standard CARA-normal asset trading model in which the traders are allowed to purchase a signal about the supply of the risky asset. Introducing this feature into the CARA-normal framework generates complementarities in information acquisition among the traders and multiple equilibria in the financial market. The complementarity exists even for financial market equilibria that share the same properties with regard to information content as previous analyses of the framework. Although other studies have explored the consequences of introducing information about supply in the framework, information acquisition remains a strategic substitute in those analyses.

Our results mean that the analytically tractable CARA-normal framework can still be used to study financial markets when information complementarity seems a natural phenomenon. Our model also allows for many further extensions. The first extension is already mentioned in footnote 8, which could allow for information market equilibria that have differing fractions of supply- and payoff-informed traders. Our main results still hold in this extension.

Muti-asset markets can also be studied using an extension of our model, which could shed light on the comovement of stock prices. Endogenizing the price of information, as in Veldkamp (2006a and 2006b), would also be interesting in our model since that would add an additional source of complementarity in information acquisition.

Another straightforward extension is to consider a model with trading in many periods. Then the prices in consecutive periods are correlated by the (random) asset supply. This may help people infer more through the realization of a price process

and can have an effect on the information acquisition decisions. Further, we could allow for borrowing constraints (Yuan, 2005), which may help explain asymmetric and correlated behavior of prices.

Finally, we could allow for consumption in the first period in our model, i.e., have the traders care about wealth over two periods and not just the terminal wealth. We believe the intertemporal consumption choice and multiplicity would generate some interesting results as in the study by Muendler (forthcoming).

7 Appendix

Proof of proposition 1. From equations (5) to (11), we solve for P and get the following

$$\begin{aligned} & [\lambda\rho_I + (1 - \lambda)\rho_U]P \\ = & \left[\rho_v\bar{v} + \lambda\frac{c}{b}\rho_\theta\frac{\rho_x}{\rho_x + \rho_\eta}\bar{x} + (1 - \lambda)\frac{c}{b}\rho_\phi\bar{x} \right] \\ & + [\lambda(\rho_\varepsilon + \rho_\theta) + (1 - \lambda)\rho_\phi]v - \left[\lambda\frac{c}{b}\rho_\theta\frac{\rho_x}{\rho_x + \rho_\eta} + (1 - \lambda)\frac{c}{b}\rho_\phi + \gamma \right]x \end{aligned}$$

Comparing with equation (3), we have the polynomial

$$\lambda\rho_\eta\left(\frac{b}{c}\right)^2 - \gamma\left(\frac{b}{c}\right) + \lambda\rho_\varepsilon = 0.$$

Then defining $\beta = b/c$, the result follows directly.

Proof of proposition 2 The ex post indirect utility of informed trader i

$$\begin{aligned} E[V_I^i|x_i, y_i, P] &= E\left[-e^{-\gamma W_{2I}^i}|x_i, y_i, P\right] \\ &= -\exp\left\{\left[-\gamma(\bar{N}_i - \kappa) - \frac{(E[v|x_i, y_i, P] - P)^2}{2\rho_I^{-1}}\right]\right\} \end{aligned}$$

Of course, conditioning on $\{x_i, y_i, P\}$ is equivalent to conditioning on $\{x_i, y_i, s\}$.

Define $h = \text{Var} (E [v|x_i, y_i, P] |s)$. By the conditional variance identity formula, we have

$$h = \frac{1}{\rho_U} - \frac{1}{\rho_I}.$$

Define

$$Z_i = \frac{E [v|x_i, y_i, P] - P}{\sqrt{h}}.$$

So,

$$E [V_I^i |s] = e^{\gamma^\kappa u (W_1^i)} E \left[\exp \left(-\frac{h}{2\rho_I^{-1}} Z_i^2 \right) |P \right].$$

Conditional on P or s , $E [v|x_i, y_i, P]$ is normally distributed. Hence conditional on P or s , Z_i^2 has a non-central chi-squared distribution. Then, for $t > 0$, the MGF (moment generating function) for Z_i^2 can be written as

$$E \left(e^{-tZ_i^2} |s \right) = \frac{1}{\sqrt{1+2t}} \exp \left[\frac{-(E [Z_i |s])^2 t}{1+2t} \right],$$

where,

$$E [Z_i |s] = \frac{E [v|s] - P}{\sqrt{h}}.$$

So, set $t = \frac{h}{2\rho_I^{-1}}$,

$$E \left[\exp \left(-\frac{h}{2\rho_I^{-1}} Z_i^2 \right) |P \right] = \sqrt{\frac{\rho_U}{\rho_I}} \exp \left[\frac{-(E [v|s] - P)^2}{2\rho_U^{-1}} \right],$$

which implies

$$E [V_I^i |s] = e^{\gamma^\kappa \sqrt{\frac{\rho_U}{\rho_I}} u (W_1^i)} \exp \left[\frac{-(E [v|s] - P)^2}{2\rho_U^{-1}} \right]. \quad (12)$$

The uninformed traders has ex post indirect utility

$$E [V_U^i | s] = u(W_1^i) \exp \left[-\frac{(E[v|s] - P)^2}{2\rho_U^{-1}} \right]. \quad (13)$$

Then, by (12) and (13),

$$E [V_I^i | s] - E [V_U^i | s] = \left(e^{\gamma\kappa} \sqrt{\frac{\rho_U}{\rho_I}} - 1 \right) E [V_U^i | s].$$

Then taking expectation on both sides of the above equation gives us

$$E [V_I^i] = e^{\gamma\kappa} \sqrt{\frac{\rho_U}{\rho_I}} E [V_U^i],$$

which proves the second equality in the proposition. Now we turn to the first one.

By (13),

$$E [V_U^i | s] = -e^{-\gamma\bar{N}_i} \exp \left[-\frac{(E[v|s] - P)^2}{2\rho_U^{-1}} \right].$$

By (9),

$$E[v|s] - P = \left[\frac{1}{b} \frac{\rho_\phi}{\rho_U} - 1 \right] P,$$

which has a normal distribution. Then the MGF of Chi-squared distribution gives us the desired result.

Proof of proposition 3 By proposition 2, the expected utility of the uninformed traders is positively related to A . By proposition 1, after some algebra,

$$A = \left(\frac{\lambda (\rho_\varepsilon + \beta^2 \rho_\eta)}{\rho_v + \beta^2 \rho_x + \lambda (\rho_\varepsilon + \beta^2 \rho_\eta)} \right)^2 \frac{\rho_v}{\beta^2 \rho_x}. \quad (14)$$

By proposition 2, the expected utility of the informed traders is positively related to $I = (1 + A) \rho_I / \rho_U$. By proposition 1 and equation (14),

$$I = \left[1 + \left(\frac{\lambda (\rho_\varepsilon + \beta^2 \rho_\eta)}{\rho_v + \beta^2 \rho_x + \lambda (\rho_\varepsilon + \beta^2 \rho_\eta)} \right)^2 \frac{\rho_v}{\beta^2 \rho_x} \right] \frac{\rho_v + \rho_\varepsilon + \beta^2 (\rho_x + \rho_\eta)}{\rho_v + \beta^2 \rho_x}. \quad (15)$$

Case 1. $\rho_\eta = 0$. According to the remark after proposition 1, we have

$$\beta^{GS} = \frac{\lambda \rho_\varepsilon}{\gamma}, \beta^{NGS} = \infty.$$

So,

$$A^{GS} > 0 = A^{NGS}, I^{GS} > 1 = I^{NGS}.$$

Case 2. $\rho_\varepsilon = 0$. In this case,

$$\beta^{GS} = 0, \beta^{NGS} = \frac{\gamma}{\lambda \rho_\eta}.$$

So,

$$A^{GS} = \infty > A^{NGS}, I^{GS} = \infty > I^{NGS}.$$

The neighborhood in the two cases can now be obtained using continuity.

Proof of theorem 1 By proposition 1,

$$R(\lambda) = e^{\gamma \kappa} \sqrt{\frac{\rho_U}{\rho_I}} = e^{\gamma \kappa} \sqrt{\frac{\rho_v + \beta^2 \rho_x}{\rho_v + \rho_\varepsilon + \beta^2 (\rho_x + \rho_\eta)}}.$$

According to the remark after proposition 1, we know in the GS-REE, $\frac{\partial \beta}{\partial \lambda} > 0$. So, $R'(\lambda)$ has the same sign as $\frac{\partial(\rho_U/\rho_I)}{\partial(\beta^2)}$.

Since

$$\frac{\partial (\rho_U/\rho_I)}{\partial (\beta^2)} = \frac{\rho_x \rho_\varepsilon - \rho_\eta \rho_v}{[\rho_v + \rho_\varepsilon + \beta^2 (\rho_x + \rho_\eta)]^2},$$

then $R'(\lambda) > 0$ if and only if $\rho_x \rho_\varepsilon - \rho_\eta \rho_v > 0$, i.e.,

$$\frac{\rho_\varepsilon}{\rho_v} > \frac{\rho_\eta}{\rho_x}.$$

Proof of proposition 4 The proof is similar to that of theorem 1 and hence omitted.

Proof of proposition 5 By proposition 1,

$$E(P) = a + b\bar{v} - c\bar{x} = \bar{v} - \frac{\gamma}{K}\bar{x}$$

Thus, if $\bar{x} > 0$, then $\frac{\partial E(P)}{\partial \lambda}$ has the same sign as $\frac{\partial K}{\partial \lambda}$.

Note that

$$K = \rho_v + \lambda \rho_\varepsilon + \beta^2 \rho_x + \lambda \beta^2 \rho_\eta.$$

Thus, obviously $\frac{\partial K^{GS}}{\partial \lambda} > 0$ since $\frac{\partial \beta^{GS}}{\partial \lambda} > 0$. Now,

$$\begin{aligned} \frac{\partial K^{NGS}}{\partial \lambda} &= (\rho_\varepsilon + \beta^2 \rho_\eta) + 2(\rho_x + \lambda \rho_\eta) \beta \frac{\partial \beta^{NGS}}{\partial \lambda} \\ &= (\rho_\varepsilon + \rho_\eta \beta^2) - \frac{2(\rho_x + \lambda \rho_\eta) \gamma}{\lambda \sqrt{\gamma^2 - 4\lambda^2 \rho_\varepsilon \rho_\eta}} \beta^2 \\ &= \rho_\varepsilon - \left[\left(\frac{2\gamma}{\sqrt{\gamma^2 - 4\lambda^2 \rho_\varepsilon \rho_\eta}} - 1 \right) \rho_\eta + \frac{2\rho_x \gamma}{\lambda \sqrt{\gamma^2 - 4\lambda^2 \rho_\varepsilon \rho_\eta}} \right] \left[\frac{\gamma + \sqrt{\gamma^2 - 4\lambda^2 \rho_\varepsilon \rho_\eta}}{2\lambda \rho_\eta} \right]^2. \end{aligned}$$

Note that

$$\frac{\gamma + \sqrt{\gamma^2 - 4\lambda^2 \rho_\varepsilon \rho_\eta}}{2\lambda \rho_\eta} \geq \frac{\gamma}{2\lambda \rho_\eta} \geq \frac{\sqrt{4\lambda^2 \rho_\varepsilon \rho_\eta}}{2\lambda \rho_\eta},$$

where the last inequality follows because $\gamma^2 > 4\lambda^2 \rho_\varepsilon \rho_\eta$ in order for the REE to exist.

So,

$$\begin{aligned}
& \left[\left(\frac{2\gamma}{\sqrt{\gamma^2 - 4\lambda^2 \rho_\varepsilon \rho_\eta}} - 1 \right) \rho_\eta + \frac{2\rho_x \gamma}{\lambda \sqrt{\gamma^2 - 4\lambda^2 \rho_\varepsilon \rho_\eta}} \right] \left[\frac{\gamma + \sqrt{\gamma^2 - 4\lambda^2 \rho_\varepsilon \rho_\eta}}{2\lambda \rho_\eta} \right]^2 \\
& \geq \left(\frac{2\gamma}{\sqrt{\gamma^2 - 4\lambda^2 \rho_\varepsilon \rho_\eta}} - 1 \right) \rho_\eta \left[\frac{\sqrt{4\lambda^2 \rho_\varepsilon \rho_\eta}}{2\lambda \rho_\eta} \right]^2 \\
& = \left(\frac{2\gamma}{\sqrt{\gamma^2 - 4\lambda^2 \rho_\varepsilon \rho_\eta}} - 1 \right) \rho_\varepsilon \geq \rho_\varepsilon.
\end{aligned}$$

Thus,

$$\frac{\partial K^{NGS}}{\partial \lambda} = \rho_\varepsilon - \left[\left(\frac{2\gamma}{\sqrt{\gamma^2 - 4\lambda^2 \rho_\varepsilon \rho_\eta}} - 1 \right) \rho_\eta + \frac{2\rho_x \gamma}{\lambda \sqrt{\gamma^2 - 4\lambda^2 \rho_\varepsilon \rho_\eta}} \right] \left[\frac{\gamma + \sqrt{\gamma^2 - 4\lambda^2 \rho_\varepsilon \rho_\eta}}{2\lambda \rho_\eta} \right]^2 \leq 0.$$

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