Asymmetric Information and Bank Runs

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(Preliminary Version)

Abstract

This paper extends Peck and Shell's (2003) bank run model to the enviornment in which people are endowed with different information technologies to observe the sunspots. We discuss bank runs in the traditional Diamond-Dybvig framework and in the more recent Peck-Shell framework. In an economy with small probability of run and small deviation in beliefs, an optimal contract might tolerate full bank runs and partial bank runs.

Acknowledgement 1 *I thank Prof. Karl Shell for guiding me through this topic.*

1 Introduction

In Diamond and Dybvig's (1983) seminal paper, bank runs are one of the Nash equilibria in the economy even in the absence of the weakness in the fundamentals. Banks arise as they help smooth consumption under the consumption shock. Bank runs occur due to panics. Knowing the possibility of having a run equilibrium in the post deposit game, we need to explain why people still want to deposit in the bank ex ante, and what the optimal contract will be given there is a run equilibrium ex post.

To formally model bank runs as an equilibrium phenomenon, Peck and Shell (2003) incorporate the pre-deposit decision. Knowing there is a run equilibrium in the post deposit game, consumers make decision whether to deposit in the bank or to stay in autarky in the pre-deposit game. The post deposit game which has a run equilibrium is coordinated by a sunspot variable. The sunspot variable, by definition, does not affect the fundamentals such as endowments, preference, and technology. All consumers choose to withdraw money if certain value of the sunspot variable is observed and not to run otherwise. The sunspot equilibrium is self-fulfilled and the probability of runs is determined by consumers' beliefs.

If the probability of runs is small enough, consumers still want to deposit in the bank as the expected return is higher. On the other side, the bank chooses an optimal contract based on the probability of runs. In some economy, the optimal contract tolerates runs, and a run proof contract will be provided otherwise.

So far, the literature models the panic runs as a sunspots phenomenon with perfect information on the sunspots. If the banking contract allows a run equilibrium, people share the same belief about the probability of bank runs ex ante, observe the exogenous signal perfectly, and make decisions knowing what other people will do. When some value of the sunspots variable is realized, all consumers withdraw their deposits as they expect everyone else will do so. Though the probability of bank runs is determined by an exogenous variable initially, this probability becomes bank's intrinsic instability in the sunspot equilibrium. The run equilibrium and the probability of bank runs are thus self-fulfilled. We are interested in the more realistic case in which people do not have perfect information about what the others will do. The information structure, that is, the ability to observe the sunspots, is asymmetric among the consumers. Consumers observe different partitions of the sunspot variable values and infer the partitions other consumers observe. In some economy, a banking contract allows consumers to make withdrawal decisions according to their own information sets in equilibrium. We first discuss given a contract, whether people make decisions according to their information structures. Then we design the optimal contract which allows consumers' heterogeneous decisions, and compare it with the contacts which do not allow heterogenous decisions in equilibrium to see whether it is the best one indeed.

We first analyze the simplest case in which there is no aggregate uncertainty in the economy and a demand deposit contract is offered. Since we do not aim to eliminate the run equilibrium, we assume there is no suspension of convertibility. We then extend the model to the economy with aggregate uncertainty where the optimal mechanism allows partial suspension of convertibility. Wallace (1990) suggests that in an economy with aggregate uncertainty, allowing partial suspension might be the best. Peck and Shell's optimal banking mechanism is within a broad class which includes partial suspension of convertibility, and we will follow their example. To simplify the notation, we assume a continuum of consumers in the traditional Diamond-Dybvig framework, while we assume a finite number of consumers in the Peck-Shell framework.

The paper is organized as follows. In next section, we will introduce the model set-up based on demand deposit contract. We then discuss the equilibrium concept, and introduce the optimal contract. Section 3 extend the model to allow aggregate uncertainty and partial suspension. We have a small discussion following section 3, and the last part gives the conclusions.

2 Benchmark Model - Demand Deposit Contract without Aggregate Uncertainty

2.1 Model Set Up

There are three periods ($t = 0, 1, 2$. period 0, 1 and 2 respectively) and a mass of 1 consumers in the economy. Each consumer is endowed with 1 unit of consumption good in period 0.

There is a measure of α ($\alpha < 1$) impatient consumers, the rest are patient. Impatient consumers derive utility only from consumption in period 1. Their utility is described by $u(c^1)$, where c^1 is the consumption received at $t = 1$. Patient consumers consume in the last period. If a patient consumer receives consumption at $t = 1$, he can costlessly store it and consume it at $t = 2$. Thus, a patient consumer's utility is described by $u(c^1 + c^2)$, .where c^2 is the consumption received at $t = 2$. The coefficient of relative risk aversion of the utility function $u(x)$ is less than -1 for $x \geq 1$. Whether a consumer is patient or impatient is private information, and is revealed to the individual consumer at $t = 1$.

The investment technology is as follows. One unit of endowment invested in period 0 yields 1 unit of consumption good in period 1, and R ($R > 1$) units in period 2.

The bank behaves competitively, and offers a demand deposit contract which describes the amount of consumption good paid to the consumers who withdraw in period 1 (c^1) and 2 (c^2) respectively. The bank observes sequential service constraint. It pays c^1 to the consumers until it runs out of resource, and distributes the remaining resource plus the interest equally among the consumers who wait until the last period, therefore $c^2 = \max\{0, \frac{1-nc^1}{1-n}R\}$, where n ($0 < n < 1$) denotes the measure of consumers who withdraw the deposits in period 1.

A demand deposit banking mechanism $m = (c^1, c^2)$ satisfies

$$c^2 = \max\{0, \frac{1 - nc^1}{1 - n}R\}, \text{where } n \geq \alpha, c^1 \geq 0. \tag{1}$$

Consumers are isolated from each other. They do not know how many have already withdrawn the deposits before they come to the bank.

The least incentive compatibility constraint is defined as

$$u(\frac{1 - \alpha c^1}{1 - \alpha}R) \geq u(c^1), \tag{2}$$

which means that the patient consumers should get at least as much as the impatient consumers get in period 1 if all patient consumers wait. This is the minimum requirement for a banking mechanism so that the patient consumers are willing to wait.

Let M denote the set which includes all $m's$ which satisfy (1)-(2). This is the set which includes all feasible banking mechanisms in the traditional bank run literature.

The first-best allocation achieves highest welfare ex ante assuming the consumption types are observable.

$$\max \alpha u(c^1) + (1-\alpha)u(\frac{1-\alpha c^1}{1-\alpha}R)$$

$$s.t.(1)-(2).$$

As the coefficient of relative risk aversion of the utility function's is less than -1, the optimal (c^{1*}, c^{2*}) satisfies $1 < c^{1*} < c^{2*} < R$. The first-best allocation is Nash implementable even the types are not observable as there is a Nash equilibrium in the post deposit game in which all patient consumers choose to wait until the last period.

A $m = (c^1, c^2)$ has a run equilibrium if

$$Eu(\text{Not Run}|\text{All patient consumers run}) < Eu(\text{Run}|\text{All patient consumers run}). \tag{3}$$

That is, the expected utility if a patient consumer does not run given everyone else runs is strictly lower than that if he runs on the bank.

The optimal contract has a run equilibrium as $c^1 > 1$,

$$u(0) < \frac{1}{c^1}u(c^1).$$

A banking mechanism $m \in M$ is run proof if it satisfies (4)

$$Eu(\text{Not Run}|\text{All patient consumers run}) \geq Eu(\text{Run}|\text{All patient consumers run}). \tag{4}$$

Let M^{RP} denote the set which contains all run proof $m's$. It is a subset of M.

Consumers are endowed with different information technologies in period 1. They imperfectly observe a sunspot variable in period 1. Sunspots do not affect the fundamentals and preference. Given private consumption information, banking contract and the realization of the sunspots, patient consumers infer whether other patient consumers will withdraw deposits and decide whether to run on the bank in period 1.

Let sunspot variable ω be uniformly distributed on $[0,1]$. Patient consumers are divided into two groups according to their ability to observe the realization of ω. A measure of n_1 of the patient consumers observe partition $T_1 = \{[0,\sigma_1), [\sigma_1, 1]\} = \{A_1, B_1\}$, where $A_1 = \{[0,\sigma_1)\}$, and $B_1 = \{[\sigma_1, 1]\}$. A measure of n_2 ($n_2 = 1 - \alpha - n_1$) of them observe $T_2 = \{[0,\sigma_2), [\sigma_2, 1]\} = \{A_2, B_2\}$, where $A_2 = \{[0,\sigma_2)\}$, and $B_2 = \{[\sigma_2, 1]\}$. Without loss of generality, we assume $\sigma_2 \geq \sigma_1$, and $n_1 \geq n_2$ in the benchmark case. Consumers know the asymmetric information structure ex ante, but they do not know their own information sets until period 1. Define a function $t_i : \Omega \longrightarrow T_i$, that is, when the value of the sunspot variable is realized, consumer j in group $i, i \in 1, 2$ will only know that a certain set in T_i has occurred. The probabilities of the realizations of the sunspot variable in $[0, \sigma_1)$, $[\sigma_1, \sigma_2)$ and $[\sigma_2, 1]$ are $\sigma_1, \sigma_2 - \sigma_1$, and $1 - \sigma_2$.

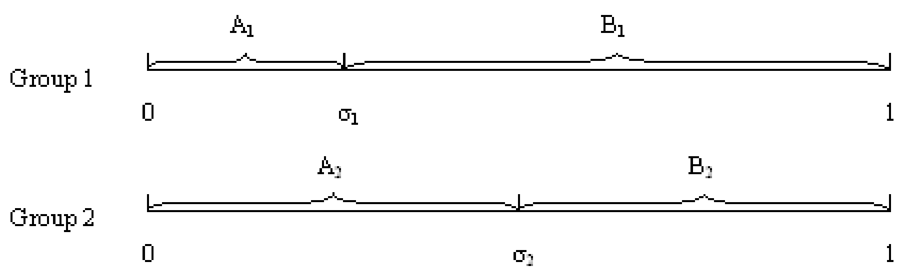

Figure 1: Information Structure

The sequence of timing is as follows:

bank announces the contract; consumers make investment decision.	$t = 0$		pre-deposit game
consumption types are revealed; information shock is realized; sunspot variable is realized; consumers make withdrawal decision.	$t = 1$	post deposit game	
bank allocates the remaining resource to the rest of the consumers.	$t = 2$		

In the post deposit game, consumer j's strategy set S_j is $S_j =${run with probability p_j, not run with probability $1 - p_j$, $p_j \in [0, 1]$}. Given a banking mechanism, a correlated equilibrium (Aumann 1987) is defined as

$$\sum_{\{\omega | t_i(\omega) = t_i\}} \pi(\omega | t_i) u_j(s_j(\omega), s_{-j}(\omega)) \geq \sum_{\{\omega | t_i(\omega) = t_i\}} \pi(\omega | t_i) u_j(s'_j(\omega), s_{-j}(\omega))$$
$$\forall s'_j(\omega) \in S_j, \forall j \textit{ in group } i$$

where $\pi(\omega|t_i)$ denotes the probability of the realization of ω conditional on observing the partition t_i. Observing one's own realization of the information partition, the consumer maximizes his expected payoff given others' strategies. In this paper, we focus on the correlated equilibria in which consumers in the same information group adopt the same pure strategy.

Definition 2.1 A banking mechanism $m \in M$ has a type 1 coordinating equilibrium (CE1) in the post deposit game if the following pure strategies construct a correlated equilibrium.

(a) patient consumers in group 1 run on the bank when observing A_1, and do not run when observing B_1;

(b) patient consumers in group 2 run on the bank when observing A_2, and do not run when observing B_2.

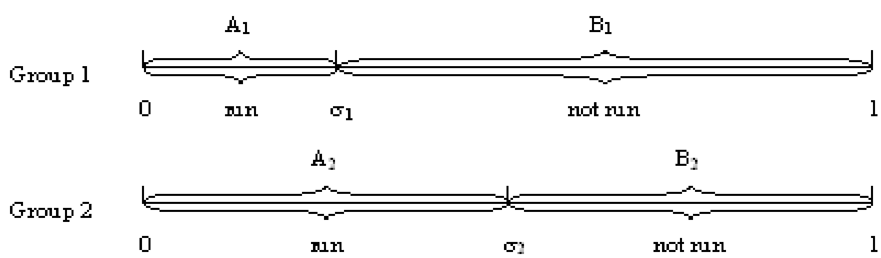

Figure 2: Type I Coordinating Equilibrium

This definition is equivalent to the following four conditions.

For a patient consumer in group 1:

$$Eu^1(\text{R}|\text{R, R}) \geq Eu^1(\text{NR}|\text{R, R}) \tag{5}$$

$$(\sigma_2 - \sigma_1)Eu^1(\text{NR}|\text{NR, R}) + (1 - \sigma_2)Eu^1(\text{NR}|\text{NR, NR}) \geq$$
$$(\sigma_2 - \sigma_1)Eu^1(\text{R}|\text{NR, R}) + (1 - \sigma_2)Eu^1(\text{R}|\text{NR, NR}) \tag{6}$$

For a patient consumer in group 2:

$$\sigma_1 Eu^2(\text{R}|\text{R, R}) + (\sigma_2 - \sigma_1)Eu^2(\text{R}|\text{NR, R}) \geq$$

$$\sigma_1 Eu^2(\text{NR}|\text{R, R}) + (\sigma_2 - \sigma_1)Eu^2(\text{NR}|\text{NR, R}) \tag{7}$$

$$Eu^2(\text{NR}|\text{NR, NR}) \geq Eu^2(\text{R}|\text{NR, NR}) \tag{8}$$

$Eu^i(.)$ is the expected utility of a patient consumer in group i. The first argument in $Eu^i(.)$ is the strategy of an individual consumer in group i. R stands for run, and NR for not run. The second argument denotes the strategy of the consumers in group 1, and the third argument is the strategy of the consumers in group 2. The expected utility depends on individual consumer's own strategy, his group members' strategies, and strategies of the consumers in the other group. The expected utilities of a patient consumer given others' strategies are specified in appendix 1.

If the realization of the sunspot variable falls in $[0, \sigma_1]$, a patient consumer in group 1 knows for sure that group 2 observes partition A_2. For a type 1 coordinating equilibrium, a patient consumer in group 1 should find "run" to be the best response given that the other group and his own group run on the bank for sure. Therefore, (5) should hold. If the sunspot variable takes any value in $[\sigma_1, 1]$, a group 1 member knows that with probability $\frac{\sigma_2 - \sigma_1}{1 - \sigma_1}$, people in group 2 observe partition A_2, and that with probability $\frac{1 - \sigma_2}{1 - \sigma_1}$, the other group observes partition B_2. Given his group members' strategies, that is, "not run" on the bank, and group 2's strategies, that is, "run" if A_2, and "not run" if B_2, a patient consumer in group 1 should find "not run" the best response which maximizes his expected utility according to the definition of type 1 coordinating equilibrium. Thus, equation (6) should hold. Similarly, we have equation (7) and (8) for a patient consumer in group 2.

Let M^{CE1} denote the set of banking mechanism $m \in M$ which satisfies (5)-(8). (5) is identical to the condition for the existence of a run equilibrium, and (8) is identical to the least incentive compatibility constraint. In addition to these, we have (6) and (7) which can be interpreted as the incentive compatibility constraints given the possibility that some consumers run on the bank. Therefore, M^{CE1} is a subset of the $M \backslash M^{RP}$.

For a banking mechanism $m \in M$, it is possible that m neither allows a type 1 coordinating equilibrium nor is run proof. It occurs if $m \in M \backslash (M^{CE1} \cup M^{RF})$. In this case, we move back to the original Diamond-Dybvig world in which a contract has a run as well as a no run equilibrium. If such a contract is offered, according to the traditional Diamond-Dybvig interpretation, consumers would either accept it believing no run equilibrium will take place or do not accept it believing the run equilibrium will occur at $t = 0$.

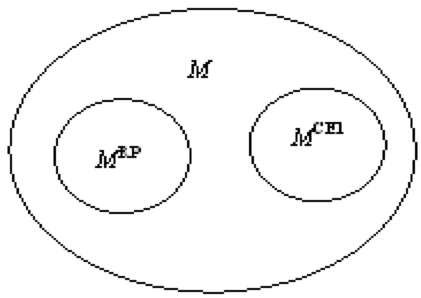

Figure 3: Partitions of M

2.2 Numerical examples:

In this section, we will give some examples showing that among all feasible banking mechanisms, some are run proof, some are consistent with type 1 coordinating equilibrium, and some are neither run proof nor consistent with type 1 coordinating equilibrium.

Let $u(c) = \frac{ac}{1+bc}$, $a = 4$, $b = 2$. $R = 9$, $\alpha = 0.4$, $n_1 = n_2 = 0.3$.

1. $\sigma_1 = 0.02$, $\sigma_2 = 0.05$

In this example, M, M^{RP}, M^{CE1} and $M \backslash (M^{CE1} \cup M^{RF})$ are as follows (summarized by c^1):

Banking Mechanism	c^1
M	$[0, 2.14]$
M^{RP}	$[0, 1]$
M^{CE1}	$(1, 2.11]$
$M \backslash (M^{CE1} \cup M^{RF})$	$(2.11, 2.14]$

2. $\sigma_1 = 0.15$, $\sigma_2 = 0.20$

In this example, M, M^{RP}, M^{CE1} and $M \backslash (M^{CE1} \cup M^{RF})$ are as follows (summarized by c^1):

Banking Mechanism	c^1
M	$[0, 2.14]$
M^{RP}	$[0, 1]$
M^{CE1}	$(1, 2.06]$
$M \backslash (M^{CE1} \cup M^{RF})$	$(2.06, 2.14]$

3. $\sigma_1 = 0.25$, $\sigma_2 = 0.40$

In this example, M, M^{RP}, M^{CE1} and $M \backslash (M^{CE1} \cup M^{RF})$ are as follows (summarized by c^1):

Banking Mechanism	c^1
M	$[0, 2.14]$
M^{RP}	$[0, 1]$
M^{CE1}	$(1, 1.45]$
$M \backslash (M^{CE1} \cup M^{RF})$	$(1.45, 2.14]$

2.3 Optimal m

A run proof contract m^{RP} is a contract such that a patient consumer will weakly prefer to wait even if everyone else withdraws the deposits. In the demand deposit contract framework, a banking mechanism is run proof if and only if $c^1 \leq 1$. It is equivalent to the autarky when $c^1 = 1$. Since the coefficient of relative risk aversion is less than -1, the banking contract that consumers will accept ex ante should satisfy $c^1 \geq 1$. Thus the only acceptable run proof contract requires $c^1 = 1$, which results in the same allocation as under autarky.

The highest level of welfare under run proof contract is

$$\hat{W}(m) = \alpha u(c^1) + (1-\alpha)u(\frac{1-\alpha c^1}{1-\alpha}R) = \alpha u(1) + (1-\alpha)u(R) \tag{9}$$

Given m which allows a run equilibrium, from game theory's point of view, the game has equilibria in which consumers in the same group adopt different strategies or consumers can adopt strictly mixed strategies. Here we focus on the case in which consumers in the same information group adopt the same pure strategy as we want to show how the sunspot variable coordinates people's behavior.

Before we start the general welfare analysis, let us first specify the strategies of an individual consumer in the post-deposit game. We start with a banking mechanism $m \in M$.

1. If $m \in M^{CE1}$, that is, m allows a type 1 coordinating equilibrium, consumers start with the beliefs that the partitions related with the lower values of ω, that is A_i, are bad, and they will withdraw deposits. The partitions related with the higher values of ω, that is B_i, are good, and the patient consumers wait until the last period if they observe such a partition.

2. If $m \in M^{RP}$, patient consumers will not run regardless of the realization of the sunspot variable.

3. If $m \in M \backslash (M^{CE1} \cup M^{RF})$, the contract neither is consistent with type 1 coordinating equilibrium nor proves runs on the bank. In this case, we can not predict consumers' decision ex ante. We will follow Diamond-Dybvig's interpretation, that is, if consumers accept the contract, they will foresee the no-run equilibrium at $t = 1$. Otherwise, consumers anticipate the run equilibrium at $t = 1$, and do not deposit at the bank ex ante.

In the last two scenarios, the sunspots do not matter in the post-deposit game, as the consumers' strategies are independent of the partitions of the sunspot variable.

Definition 2.2 (Full Bank Runs) In the post deposit game, if all consumers withdraw deposits, full bank runs occur.

Definition 2.3 (Partial Bank Runs) In the post deposit game, if some, not all patient consumers withdraw deposits, partial bank runs occur.

Definition 2.4 (No Bank Run) In the post deposit game, if all patient consumers do not withdraw deposits in period 1, there is no bank run in this post deposit game.

If the contract allows a type 1 coordinating equilibrium, the probabilities of full bank runs, partial bank runs and no run are decided by the information structure. The probabilities of full runs, partial runs and no run are σ_1, $\sigma_2 - \sigma_1$, and $1 - \sigma_2$.

If no run occurs, the welfare, denoted by $W^{no-run}(m)$, will be

$$W^{no-run}(m) = \alpha u(c^1) + (1-\alpha)u(\frac{1-\alpha c^1}{1-\alpha}R).$$

If partial runs occur, the welfare, denoted by $W^{partial-run}(m)$, will be

$$W^{partial-run}(m) = \begin{cases} \frac{1}{c^1}u(c^1), & \text{if } (n_1+\alpha)c^1 \geq 1, (n_2+\alpha)c^1 \geq 1, \\ (\alpha+n_2)u(c^1) + n_1 u(\frac{1-(n_2+\alpha)c^1}{1--n_2-\alpha}R), & \text{otherwise.} \end{cases}$$

If full runs occur, the welfare, denoted by $W^{run}(m)$ will be

$$W^{run}(m) = \frac{1}{c^1}u(c^1).$$

Given these probabilities, the best contract which allows a type 1 correlated equilibrium the bank will offer to the consumers is

$$\hat{W}(m) = \max_{c^1} \, (1-\sigma_2)W^{no-run}(m) + (\sigma_2 - \sigma_1)W^{partial-run}(m) + \sigma_1 W^{run}(m)$$

$$s.t. \; m \in M^{CE1}.$$

In the pre-deposit game, the bank decides what to offer and the consumers decide whether to deposit in the bank or not. We impose an assumption that if the contract yields expected payoff equal to that under autarky, people still deposit in the bank. With this assumption, the bank can at least offer the run proof contract to the consumers.

Definition 2.5 Given a mechanism $m \in M$, the predeposit game has a type 1 coordinating equilibrium if there is a subgame perfect Nash equilibrium in which (i) consumers are willing to deposit, and (ii) the post deposit game has a type 1 coordinating equilibrium.

Similar to Peck and Shell (2003), we have the following proposition and the proof.

Proposition 1 Given a mechanism m, if for some realization of the sunspot variable ω, there is a subgame perfect Nash equilibrium where all patient consumers wait until period 2 and the welfare is higher than that under autarky, the predeposit game has a type 1 coordinating equilibrium if and only if the post deposit game has a type 1 coordinating equilibrium.

Proof. It is easy to see that if the predeposit game has a type 1 coordinating equilibrium, the post deposit game has a type 1 coordinating equilibrium as the post deposit game is a subgame of the predeposit game.

Let the post deposit game have a type 1 coordinating equilibrium under the mechanism m. Construct the predeposit game as follows. First, all consumers deposit their endowments. Next, for $\omega \in [\sigma_2, 1]$, group 1 observes B_1, and group 2 observes B_2. As the post deposit game has a type 1 coordinating equilibrium, patient consumers in both groups will wait. For $\omega \in [\sigma_1, \sigma_2)$, group 1 observes A_1, and group 2 observes B_2. The post deposit game has a type 1 coordinating equilibrium in which patient consumers in group 1 run when observing A_1, and patient consumers in group 2 do not run when observing B_2. For $\omega \in [0, \sigma_1)$, group 1 observes A_1, and group 2 observes A_2. As the post deposit game has a type 1 coordinating equilibrium in which patient consumers in both groups run when observing $A_i, i = 1, 2$, all patient consumers withdraw deposits. Finally, for sufficiently small β and γ, consumers are willing to deposit. The ex ante welfare is $W^{CE1} = (1 - \sigma_2)W^{no-run}(m) + (\sigma_2 - \sigma_1)W^{partical-run}(m) + \sigma_1 W^{run}(m)$. If $W^{no-run}(m) > W^{AUT}$, for sufficiently small σ_1 and σ_2, W^{CE1} still exceeds welfare under autarky by the continuity of W^{CE1}. Therefore, consumers still want to deposit ex ante. ■

If $m \in M \backslash (M^{CE1} \cup M^{RF})$, the best outcome one can hope for is that all consumers anticipate the no-run equilibrium in the post deposit game, so they make deposits ex ante and that they do not run at $t = 1$. Thus the welfare is given by

$$\hat{W}(m) = \max \alpha u(c^1) + (1 - \alpha) u(\frac{1 - \alpha c^1}{1 - \alpha} R)$$
$$s.t. m \in M \backslash (M^{CE1} \cup M^{RF}).$$

A sufficient condition that an optimal banking mechanism allows a type 1 coordinating equilibrium is that the welfare of the optimal $m \in M^{CE1}$ is higher than that of the optimal run proof contract and that of the best outcome of the best m in $M \backslash (M^{CE1} \cup M^{RF})$. Similarly, a run proof contract will be the best if the welfare is higher than that of the optimal m in M^{CE1} and the best outcome of the optimal contract in $M \backslash (M^{CE1} \cup M^{RF})$. However, it is hard to state the necessary and sufficient condition for having a run proof contract or a contract which allows a type 1 coordinating equilibrium as we do not know which strategy the consumers will adopt if bank offers neither of these two contracts to them.

Proposition 2 In some economy, an optimal demand deposit banking contract without suspension of convertibility tolerates a type 1 coordinating equilibrium.

Proof. see the examples in next sub-section. ■

2.4 The example continued

We calculate the welfare of the optimal contracts in M^{CE1}, M^{RP} and $M \backslash (M^{CE1} \cup M^{RF})$.

1. $\sigma_1 = 0.02$, $\sigma_2 = 0.05$

m in	c^{1*}	$\hat{W}(m)$
M^{RP}	1	1.67
M^{CE1}	1.35*	1.68
$M \backslash (M^{CE1} \cup M^{RF})$	$\rightarrow$ 2.11	1.64

The optimal contract in this example is $c^{1*} = 1.35$, which yields a welfare level of 1.68. It is better than the best run proof contract. If m in $M \backslash (M^{CE1} \cup M^{RF})$, in the best situation, that is, consumers accept the contract and do not run ex post, the highest welfare level is 1.64, which is still lower than that under M^{CE1}. Therefore, the optimal m is in M^{CE1} in this example.

2. $\sigma_1 = 0.15$, $\sigma_2 = 0.20$

m in	c^{1*}	$\hat{W}(m)$
M^{RP}	1*	1.67
M^{CE1}	$\rightarrow$ 1	1.61
$M \backslash (M^{CE1} \cup M^{RF})$	$\rightarrow$ 2.06	1.65

In this economy, the run proof contract is the optimal one.

3. $\sigma_1 = 0.25$, $\sigma_2 = 0.40$

m in	c^{1*}	$\hat{W}(m)$
M^{RP}	1	1.67
M^{CE1}	$\to 1$	1.56
$M \backslash (M^{CE1} \cup M^{RF})$	$\to 1.56$	1.71

In this economy, providing a contract which is neither run proof nor consistent with type 1 coordinating equilibrium might be the best.

We plot the welfare levels of run-proof contracts and contracts consistent with type 1 coordinating equilibrium for any combinations of σ_1 and σ_2 in the appendix.

In a similar way, we can define other types of coordinating equilibria.

Definition 2.6 A banking mechanism $m \in M$ has a type 2 coordinating equilibrium (CE2) in the post deposit game if the following pure strategies construct a correlated equilibrium.

(a) patient consumers in group 1 run on the bank when observing A_1, and do not run when observing B_1;

(b) patient consumers in group 2 run on the bank when observing B_2, and do not run when observing A_2.

Definition 2.7 A banking mechanism $m \in M$ has a type 3 coordinating equilibrium (CE3) in the post deposit game if the following pure strategies construct a correlated equilibrium.

(a) patient consumers in group 1 run on the bank when observing B_1, and do not run when observing A_1;

(b) patient consumers in group 2 runs when observing B_2, and do not run when observing A_2.

Definition 2.8 A banking mechanism $m \in M$ has a type 4 coordinating equilibrium (CE4) in the post deposit game if the following pure strategies construct a correlated equilibrium.

(a) patient consumers in group 1 run when observing B_1, and do not run when observing A_1;

(b) patient consumers in group 2 run when observing A_2, and do not run when observing B_2.

3 Partial Suspension with Aggregate Uncertainty in the Economy

3.1 Model Set Up

Banking mechanism will be generalized in this section. We discuss the economy which bears aggregate uncertainty and let the contract be contingent on the consumers' positions in the line. To keep the illustration simple, a discrete case will be analyzed here. There are N consumers in the economy, among them there are α number of impatient consumers, where $\alpha \leq N$ is a random number with probability density function $f(\alpha)$. Each consumer is endowed with y units of consumption good. Impatient consumers' utility function is denoted by $u(c^1)$, and the patient consumers' utility function is by $v(c^1 + c^2)$. u and v are strictly increasing, strictly concave and twice continuously differentiable. The coefficients of relative risk aversion of u and v are less than -1.

A patient consumer updates the probabilities of α using Bayes' rule. The ex post probability of α, contingent on a consumer's being patient is

$$f_p(\alpha) = \frac{[1 - (\alpha/N)]f(\alpha)}{\Sigma_{\alpha'=0}^{N-1}[1 - (\alpha'/N)]f(\alpha')}$$

for $\alpha = 0, 1, ..., N$.

The technology is the same as in the demand deposit case. Following Peck and Shell (2003), let $c^1(z)$ denote the period 1 withdrawal of consumption by the consumer in arrival position z. The resource constraint is

$$c^2(\alpha^1) = \frac{Ny - \Sigma_{z=1}^{\alpha^1} c^1(z)}{N - \alpha^1} R, \; c^1(N) = Ny - \sum_{z=1}^{N-1} c^1(z). \tag{10}$$

A banking mechanism m is described by the vector

$$\begin{aligned} m \;=\; & (c^1(1), ..., c^1(z), ..., c^1(N), c^2(0), ..., c^2(N-1)) \text{ and} \\ & (c^1(1), ..., c^1(z), ..., c^1(N), c^2(0), ..., c^2(N-1)) \text{ satisfies (10).} \end{aligned}$$

Consumers do not know their positions in the line when they make the withdrawal decision. They assume that they have equal chance to be in any position in the line.

The specification of the information structure is the same as in the previous framework with $\sigma_2 \geq \sigma_1$. Consumers do not know which group they will be in ex ante. Group i has N_i number of consumers, where $i = 1, 2$, and $N_1 + N_2 = N_1$. N_1 and N_2 are known ex ante. Consumers have probability N_1/N to be in group 1, and probability N_2/N to be in group 2. For each α, let α_i $(0 \leq \alpha_i \leq \min\{N_i, \alpha\})$ be the number of impatient consumers in group i, $i = 1, 2$, and $\alpha_1 + \alpha_2 = \alpha$. Conditional on α, the ex ante probability of having α_1 impatient consumers in group 1, and α_2 in group 2 is

$$g(\alpha_1, \alpha_2 | \alpha) = prob(\alpha_1, \alpha_2 | \alpha) = \frac{C_{N_1}^{\alpha_1} C_{N_2}^{\alpha_2}}{C_N^{\alpha}},$$

where $\alpha_1 = 0, 1, ... \min\{N_1, \alpha\}$, and $\alpha_2 = \alpha - \alpha_1$.

The ex ante probability that there are α number of impatient consumers, α_1 of them in group 1, and α_2 of them in group 2 is

$$h(\alpha_1, \alpha_2, \alpha) = f(\alpha) g(\alpha_1, \alpha_2 | \alpha).$$

We denote the marginal distribution of α_1 by $h^1(\alpha_1)$, and the marginal distribution of α_2 by $h^2(\alpha_2)$.

After the consumption shock and information shock are realized, the patient consumers update the probabilities of $\alpha's$ by Bayes rule conditional on their consumption type and information type.

$g_p^1(\alpha_1, \alpha_2|\alpha) = prob(\alpha_1, \alpha_2|\alpha$ and patient consumer i is in group 1$) =$

$$\begin{cases} \frac{C_{N_1-1}^{\alpha_1} C_{N_2}^{\alpha_2}}{\Sigma_{m=0}^{\min\{N_1-1,\alpha\}} C_{N_1-1}^{m} C_{N_2}^{\alpha-m}}, & \text{if } \alpha_1 \leq N_1 - 1, \alpha_2 \leq N_2, \text{ and } \alpha_1 + \alpha_2 = \alpha. \\ 0, & \text{otherwise.} \end{cases}$$

Similarly,

$g_p^2(\alpha_1, \alpha_2|\alpha) = prob(\alpha_1, \alpha_2|\alpha$ and patient consumer i is in group 2$) =$

$$\begin{cases} \frac{C_{N_1}^{\alpha_1} C_{N_2-1}^{\alpha_2}}{\Sigma_{m=0}^{\min\{N_2-1,\alpha\}} C_{N_1}^{\alpha-m} C_{N_2-1}^{m}}, & \text{if } \alpha_2 \leq N_2 - 1, \alpha_1 \leq N_1, \text{ and } \alpha_1 + \alpha_2 = \alpha. \\ 0, & \text{otherwise.} \end{cases}$$

The ex post probability that there are α number of impatient consumers, and among them α_1 are in group 1, and α_2 are in group 2 for a patient consumer in group i is

$$h_p^i(\alpha_1, \alpha_2, \alpha) = f_p(\alpha) g_p^i(\alpha_1, \alpha_2|\alpha)$$

A consumer has equal chance to be in any position in the line in period 1. If there are α^1 consumers withdrawing the deposits, the probability of getting $c^1(z)$ will be $\frac{1}{\alpha^1}$ for $z = 1, 2, ...N$. Therefore, the expected utility for a patient consumer if he withdraws the deposit in period 1 is $\frac{1}{\alpha^1} \sum_{z=1}^{\alpha^1} v(c^1(z))$.

The least incentive compatibility constraint requires

$$\sum_{\alpha=0}^{N-1} f_p(\alpha) v(\frac{Ny - \Sigma_{z=1}^{\alpha} c^1(z)}{N - \alpha} R) \geq \sum_{\alpha=0}^{N-1} f_p(\alpha) [\frac{1}{\alpha+1} \sum_{z=1}^{\alpha+1} v(c^1(z))]. \tag{11}$$

The set of feasible banking mechanisms M is defined as

$$M = \{m \in R_+^{2n} : (10) - (11) \text{ hold for all } \alpha\}.$$

A run proof contract requires

$$v((Ny - \Sigma_{z=1}^{N-1} c^1(z))R) \geq \frac{1}{N} \sum_{z=1}^{N} v(c^1(z)). \tag{12}$$

The set of run proof banking mechanisms M^{RP} is defined as

$$M^{RP} = \{m \in M : (12) \text{ hold for all } \alpha\}.$$

We continue to use the definition of coordinating equilibrium as in the previous section.

Definition 3.1 A banking mechanism $m \in M$ has a type 1 coordinating equilibrium (CE1) in the post deposit game if the following pure strategies construct a correlated equilibrium.

(a) patient consumers in group 1 run on the bank when observing A_1, and do not run when observing B_1;

(b) patient consumers in group 2 run on the bank when observing A_2, and do not run when observing B_2.

The corresponding restrictions on the banking mechanism which allows a type 1 coordinating equilibrium are:

For a patient consumer in group 1:

$$\frac{1}{N}\sum_{z=1}^{N} v(c^1(z)) \geq v([Ny - \Sigma_{z=1}^{N-1} c^1(z)]R) \tag{13}$$

$$(\sigma_2 - \sigma_1)\sum_{\alpha=0}^{N-1}\sum_{\alpha_1=0}^{\min\{N_1-1,\alpha\}} h_p^1(\alpha_1, \alpha_2, \alpha) v(\frac{Ny - \Sigma_{z=1}^{N_2+\alpha_1} c^1(z)}{N_1 - \alpha_1} R) +$$
$$(1-\sigma_2)\sum_{\alpha=0}^{N-1} f_p(\alpha) v(\frac{Ny - \Sigma_{z=1}^{\alpha} c^1(z)}{N-\alpha} R) \geq$$
$$(\sigma_2 - \sigma_1)\sum_{\alpha=0}^{N-1}\sum_{\alpha_1=0}^{\min\{N_1-1,\alpha\}} \frac{h_p^1(\alpha_1, \alpha_2, \alpha)}{N_2 + \alpha_1 + 1}\sum_{z=1}^{N_2+\alpha_1+1} v(c^1(z)) + (1-\sigma_2)\sum_{\alpha=0}^{N-1}\frac{f_p(\alpha)}{\alpha+1}\sum_{z=1}^{\alpha+1} v(c^1(z)) \tag{14}$$

For a patient consumer in group 2:

$$\frac{\sigma_1}{N}\sum_{z=1}^{N-1} v(c^1(z)) + (\sigma_2 - \sigma_1)\sum_{\alpha=0}^{N-1}\sum_{\alpha_2=0}^{\min\{N_2-1,\alpha\}} \frac{h_p^2(\alpha_1, \alpha_2, \alpha)}{N_2 + \alpha_1}\sum_{z=1}^{N_2+\alpha_1} v(c^1(z)) \geq$$
$$\sigma_1 v([Ny - \Sigma_{z=1}^{N-1} c^1(z)]R) + (\sigma_2 - \sigma_1)\sum_{\alpha=0}^{N-1}\sum_{\alpha_2=0}^{\min\{N_2-1,\alpha\}} h_p^2(\alpha_1, \alpha_2, \alpha) v(\frac{Ny - \Sigma_{z=1}^{N-\alpha_1-1} c^1(z)}{\alpha_1 + 1} R) \tag{15}$$

$$\sum_{\alpha=0}^{N-1} f_p(\alpha) v(c^2(\frac{Ny - \Sigma_{z=1}^{\alpha} c^1(z)}{N-\alpha} R)) \geq \sum_{\alpha=0}^{N-1} f_p(\alpha)[\frac{1}{\alpha+1}\sum_{z=1}^{\alpha+1} v(c^1(z))] \tag{16}$$

The set of banking mechanisms which are consistent with type 1 coordinating equilibrium is defined as

$$M^{CE1} = \{m \in M : (13) - (16) \text{ hold for all } \alpha\}.$$

3.2 A numerical example

Let us take an example similar to that in Peck and Shell (2003). Two consumers. One in each group. The probability of being in either group is equal for both of them ex ante. Let $u(x) = \dfrac{Ax^{1-a}}{1-a}$, $v(x) = \dfrac{x^{1-b}}{1-b}$, $A = 7$, $a = b = 1.01$, $R = 1.1$, $y = 3$. A consumer is impatient with probability p, $p = 0.4$.

As the coefficients of relative risk aversion of u and v are less than -1, the banking contract should at least provide $c^1(1) \geq y = 3$ to induce people to deposit ex ante.

In this simple example, there is only one choice variable, which is $c^1(1)$. We use c to simplify the notation.

The conditions for having a type 1 coordinating equilibrium are

$$0.5v(c) + 0.5v(2y-c) \geq v((2y-c)R) \tag{13'}$$

$$(\sigma_2 - \sigma_1)v((2y-c)R) + (1-\sigma_2)\ [(1-p)v(yR) + pv((2y-c)R)] \geq$$

$$(\sigma_2 - \sigma_1)[0.5v(c) + 0.5v(2y-c)] + (1-\sigma_2)\ [(1-p)v(c) + p(0.5v(c) + 0.5v(2y-c))] \tag{14'}$$

$$\sigma_1[0.5v(c) + 0.5v(2y-c)] + (\sigma_2 - \sigma_1)[(1-p)v(c) + p(0.5v(c) + 0.5v(2y-c))] \geq$$

$$\sigma_1 v((2y-c)R) + (\sigma_2 - \sigma_1)[(1-p)v(yR) + pv((2y-c)R)] \tag{15'}$$

$$(1-p)v(yR) + pv((2y-c)R) \geq (1-p)v(c) + p(0.5v(c) + 0.5v(2y-c)) \tag{16'}$$

Let $\sigma_1 = 0.001$, $\sigma_2 = 0.010$. We have the following descriptions for the sets of banking mechanisms.

m in	c
M	$[0, 3.2937]$
M^{RP}	$[0, 3.2852]$
M^{CE1}	$[3.2928, 3.2936]$
$M\backslash(M^{CE1} \cup M^{RF})$	$(3.2852, 3.2928) \cup (3.2936, 3.2937]$

3.3 Optimal m^{CE1}

We can calculate the optimal $m \in M^{CE1}$ which allows a type 1 coordinating equilibrium in the way we do in section two. σ_1, $\sigma_2 - \sigma_1$ and $1 - \sigma_2$ are the probabilities of full runs, partial runs and no run respectively. Let $W^{run}(m), W^{partial-run}(m)$ and $W^{no-run}(m)$ denote the welfare if full runs, partial runs or no run occurs under $m \in M^{CE1}$, where

$$\begin{aligned}
W^{run}(m) &= \sum_{\alpha=0}^{N} f(\alpha)\{\sum_{z=1}^{N}[\frac{\alpha}{N}u(c^1(z)) + \frac{N-\alpha}{N}v(c^1(z))]\} \\
W^{partial-run}(m) &= \sum_{\alpha_1=0}^{\min\{N_1,\alpha\}} h^1(\alpha_1)\{\sum_{z=1}^{N_2+\alpha_1}[\frac{\alpha}{N_2+\alpha_1}u(c^1(z)) + \frac{N_2+\alpha_1-\alpha}{N_2+\alpha_1}v(c^1(z))] + \\
&\quad (N_1-\alpha_1)v(c^2(N_2+\alpha_1))\} \\
W^{no-run}(m) &= \sum_{\alpha=0}^{N} f(\alpha)[\sum_{z=1}^{\alpha} u(c^1(z)) + (N-\alpha)v(c^2(\alpha))]
\end{aligned}$$

The best banking mechanism which allows a type 1 coordinating equilibrium solves

$$\begin{aligned}
W(m) &= \max_{c^1(z)} \sigma_1 W^{run}(m) + (\sigma_2 - \sigma_1)W^{partical-run}(m) + (1-\sigma_1)W^{no-run}(m) \\
&\quad s.t.\ m \in M^{CE1}.
\end{aligned}$$

The best run proof m^{RP} contract solves

$$\begin{aligned}
W(m^{RP}) &= \max_{c^1(z)} \sum_{\alpha=0}^{N} f(\alpha)[\sum_{z=1}^{\alpha} u(c^1(z)) + (N-\alpha)v(\frac{Ny - \Sigma_{z=1}^{\alpha}c^1(z)}{N-\alpha}R)] \\
&\quad s.t.\ m \in M^{RP}.
\end{aligned}$$

As in the previous section, set M^{CE1} is a subset of M. If a contract is neither in M^{CE1} nor in M^{RP}, we will compute the welfare given the best outcome. We compare the best

$m \in M^{CE1}$, the best $m \in M^{RP}$, and the best $m \in M \backslash (M^{CE1} \cup M^{RF})$ given the best outcome, and give a condition for adopting a $m \in M^{CE1}$ or $m \in M^{RP}$.

Proposition 1 in section 2 still holds.

Proposition 3 In some economy, an optimal banking mechanism with partial suspension of convertibility tolerates a type 1 coordinating equilibrium.

Proof. Show by example. See the example in appendix 3. ∎

3.4 The example continued

The best $m \in M^{CE1}$ in the example is by solving

$$\begin{aligned} W(m) &= \max_{c^1(z)} \sigma_1 W^{run}(m) + (\sigma_2 - \sigma_1) W^{partical-run}(m) + (1-\sigma_2) W^{no-run}(m) \\ & \quad s.t.\ m \in M^{CE1}. \end{aligned}$$

that is

$$\begin{gathered} \max_{c^1} \sigma_1 [p^2 (u(c) + u(2y-c)) + (1-p)^2 (v(c) + v(2y-c)) + \\ 2p(1-p)(0.5v(c) + 0.5v(2y-c) + 0.5u(c) + 0.5u(2y-c))] + \\ (\sigma_2 - \sigma_1)[p^2(u(c) + u(2y-c)) + (1-p)^2 (v(c) + v((2y-c)R)) + \\ p(1-p)(0.5v(c) + 0.5v(2y-c) + 0.5u(c) + 0.5u(2y-c)) + \\ p(1-p)(u(c) + v((2y-c)R))] + (1-\sigma_2)[p^2(u(c) + u(2y-c)) + \\ 2p(1-p)(u(c) + v((2y-c)R)) + 2(1-p)^2 v(yR)] \\ s.t.\ 3.2928 \le c \le 3.2936 \end{gathered}$$

The best $m \in M^{RP}$ in the example is by solving

$$W(m) = \max_{c^1} p^2 (u(c) + u\,(2y-c)) + p(1-p)(u(c) + v((2y-c)R)) + 2(1-p)^2 v(yR)$$

$$s.t.\ 3 \le c \le 3.2852$$

The welfare is re-normalized to be $W + 673$. The welfare in each partition of M is calculated as follows:

m in	$c^{1*}(1)$	$\hat{W}(m)$
M^{RP}	3.2852	0.787
M^{CE1}	3.2936	0.791
$M \backslash (M^{CE1} \cup M^{RF})$	3.2937	0.793

The welfare under autarky is $W = 0.5427$.

In this example, a run proof contract is a bad idea. A contract which allows a type 1 coordinating equilibrium might be the best. In the appendix, we will see examples with three consumers in which a contract allowing a type 1 coordinating equilibrium is the best.

With the increase in $\sigma_2 - \sigma_1$, the set of m^{CE1} shrinks. In the extreme case that nobody can infer anything substantial from the sunspot variable, the coordinating device fails. There will be no equilibrium in the post deposit game which indeed uses the sunspot variable as a guide to choose the strategy under any banking mechanism. People will be coordinated by a new coordinating device. However, this is not the focus of the paper. The point made in this paper is that given an imperfect information technology, the optimal contract might tolerate people's heterogenous decisions since it is the best choice of the society.

4 Discussions

If a contract neither allows a type 1 coordinating equilibrium nor proves bank runs, the description of the equilibrium needs more discretion. From the game theory point of view, at least one Nash equilibrium exists. If we accept Diamond-Dybvig's logic, such a contract can be offered on a trial base. If consumers accept the offer and make deposits at the bank, no-run equilibrium is the only pure strategy Nash equilibrium in the post deposit game. If they do not accept the offer, consumers predict a run equilibrium ex post, so they would prefer to stay in autarky. If this situation occurs, the bank should change the offer. The bank should either offer a run proof contract or a contract allowing the coordinating equilibrium, depending on the welfare levels that the two contracts would provide.

However, this argument sends us back to the question that why we actually observe panic runs. Empirical studies (Boyd, et al, 2001) show that the most banking crises cannot be explained by the weakness in the fundamentals. Maybe, the economy falls in the set of M^{CE1}, which provides sufficiently optimal contract for the economy. This could happen when σ_1 is sufficiently small, and σ_1 and σ_2 are close enough.

Second plausible approach is to introduce mixed strategies into the banking game. There are two reasons that we do not follow this approach. First, it requires a large amount of calculations for a N-player game. Second, if we allow mixed strategies, we implicitly add a coordinating device to the economy, such as a "coin", which is a potential conflict with the sunspot coordinating device we introduce in the first place.

Another possible way to deal with the problem is to simply rule out participation if m is in $M\backslash(M^{CE1}\cup M^{RF})$. This can be done with the assumption of ambiguity aversion. Consumers will simply stay away from the bank if it provides a contract which admits a run equilibrium but the consumers do not know for sure the probability of bank runs. Recent work on ambiguity aversion (Easley and O'Hara, 2005) provides an explanation for the statistically significant market effect after the "too big to fail" doctrine was established in 1985. This can be the future research direction.

5 Conclusions

We extend the Peck-Shell model to the environment where consumers cannot observe the sunspot perfectly. We show that in an economy with certain information structure, an optimal banking mechanism might tolerate full runs and partial runs. The run proof banking mechanism is not the best as it sacrifices too much welfare. In our paper, we use different frameworks for different classes of banking mechanism, and we get the same results.

There are banking mechanisms which are neither run-proof nor consistent with coordinating equilibrium if sunspots are imperfectly observed. It is hard to describe the equilibrium without further discussion in game theory or further assumptions on consumers' preference. Therefore, our analysis gives a sufficient condition for using a run proof banking mechanism or a mechanism which is consistent with the coordinating equilibrium.

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Appendix 1 - Expected utility of individual consumers given others' strategies

If $(n_1+\alpha)c^1 \geq 1, (n_2+\alpha)c^1 \geq 1$, for an individual in group 1 or 2:

	$Eu^i(.\vert\text{R, R})$	$Eu^i(.\vert\text{R, NR})$	$Eu^i(.\vert\text{NR, R})$	$Eu^i(.\vert\text{NR, NR})$
$Eu^i(\text{Run}\vert.)$	$\frac{1}{c^1}u(c^1)$	$\frac{1}{(n_1+\alpha)c^1}u(c^1)$	$\frac{1}{(n_2+\alpha)c^1}u(c^1)$	$u(c^1)$
$Eu^i(\text{Not Run}\vert.)$	$u(0)$	$u(0)$	$u(0)$	$u(\frac{1-\alpha c^1}{1-\alpha}R)$

If $(n_1+\alpha)c^1 \geq 1, (n_2+\alpha)c^1 < 1$, for an individual in group 1 or 2:

	$Eu^i(.\vert\text{R, R})$	$Eu^i(.\vert\text{R, NR})$	$Eu^i(.\vert\text{NR, R})$	$Eu^i(.\vert\text{NR, NR})$
$Eu^i(\text{Run}\vert.)$	$\frac{1}{c^1}u(c^1)$	$\frac{1}{(n_1+\alpha)c^1}u(c^1)$	$u(c^1)$	$u(c^1)$
$Eu^i(\text{Not Run}\vert.)$	$u(0)$	$u(0)$	$u(\frac{1-(n_2+\alpha)c^1}{1-n_2-\alpha}R)$	$u(\frac{1-\alpha c^1}{1-\alpha}R)$

If $(n_1+\alpha)c^1 < 1, (n_2+\alpha)c^1 < 1$, for an individual in group 1 or 2:

	$Eu^i(.\vert\text{R, R})$	$Eu^i(.\vert\text{R, NR})$	$Eu^i(.\vert\text{NR, R})$	$Eu^i(.\vert\text{NR, NR})$
$Eu^i(\text{Run}\vert.)$	$\frac{1}{c^1}u(c^1)$	$u(c^1)$	$u(c^1)$	$u(c^1)$
$Eu^i(\text{Not Run}\vert.)$	$u(0)$	$u(\frac{1-(n_1+\alpha)c^1}{1-n_1-\alpha}R)$	$u(\frac{1-(n_2+\alpha)c^1}{1-n_2-\alpha}R)$	$u(\frac{1-\alpha c^1}{1-\alpha}R)$

Appendix 2 - Plot of welfare levels for $m \in M^{CE1}$ and $m \in M^{RP}$

We plot the welfare levels for run-proof contracts and contracts consistent with the beliefs for any combinations of σ_1 and σ_2 in the example in section 2.

Dark Blue: $W(m^{CE1}) < W(m^{RP})$; Green: $W(m^{CE1})$ is optimal. Light Blue: $W(m^{RP})$; Red: $W(m^{CE1})$ is optimal.

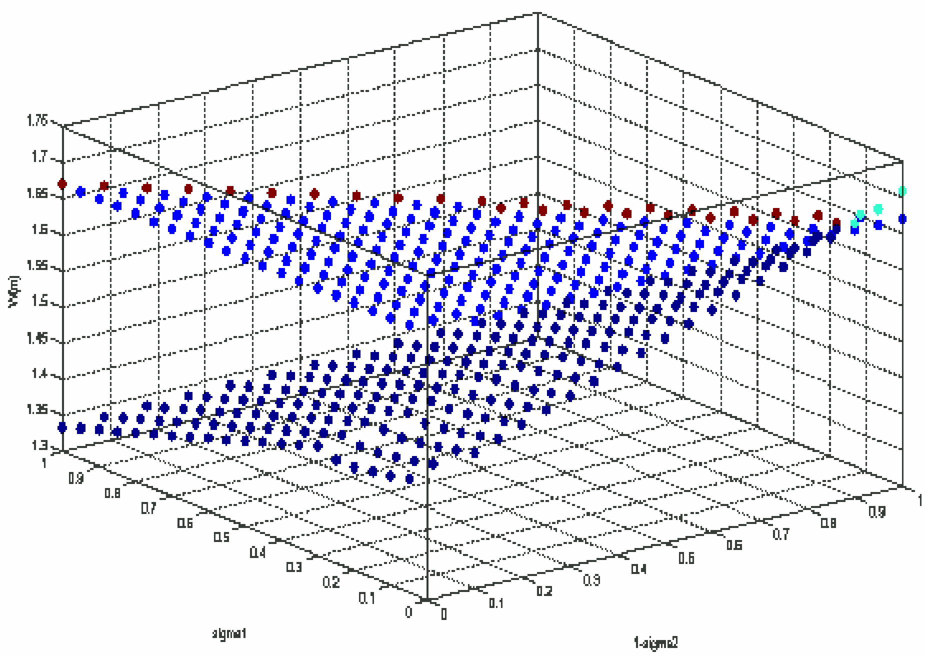

Figure 4: $W(m^{RP})$ and $W(m^{CE1})$:

Appendix 3 - An example of three consumers

We will give an example of an economy with three consumers. Each consumer has probability 0.5 to be impatient. $u(x) = \dfrac{Ax^{1-a}}{1-a}$, $v(x) = \dfrac{x^{1-b}}{1-b}$, $A = 10$, $a = b = 1.01$, $R = 2$, $y = 3$. There are two choice variables here, $c^1(1)$ and $c^1(2)$. Welfare are renormalized to be $W + 1629$.

(1) $\sigma_1 = 0.0001$, $\sigma_2 = 0.001$

The highest ex ante welfare levels the best contracts in each subset can achieve are:

m in	$c^{1*}(1)$	$c^{1*}(2)$	$\hat{W}(m)$
M^{RP}	4.7340	2.9479	0.3280
M^{CE1}	4.8681*	3.1101*	0.3930*
$M \backslash (M^{CE1} \cup M^{RF})$	4.7475	3.0718	0.3756

(2) $\sigma_1 = 0.0008$, $\sigma_2 = 0.001$

The highest ex ante welfare levels the best contracts in each subset can achieve are:

m in	$c^{1*}(1)$	$c^{1*}(2)$	$\hat{W}(m)$
M^{RP}	4.7340	2.9479	0.3280
M^{CE1}	4.8678	3.1097	0.3911
$M \backslash (M^{CE1} \cup M^{RF})$	4.9643*	3.0865*	0.3912*