Ani Guerdjikova
Department of Economics
Cornell University

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Problem Set 3

Problem 7 *(Dominance, iterated elimination of dominated strategies)*

For each of the games from problem 1, determine

a) whether the players have dominant strategies;
b) whether the players have dominated strategies;
c) whether the game has an equilibrium in dominant strategies;
d) whether there is an equilibrium after iterative elimination of dominated strategies.

Problem 8* *(Vickrey-Auction)*

Consider a Vickrey-Auction ("second-price sealed bid auction"), in which each of the bidders $i \in \{1 \ldots 10\}$ submits her bid $(\hat{\pi}_i)$ in sealed envelope. The bidder with the highest bid wins the auction and pays the second highest bid. If two players (i and j) bid equal amounts, the good is randomly assigned to one of them. The true evaluation of player i is denoted by π_i.

a) Represent the game in strategic form.
b) Derive the best-responses of the players and draw them in a diagram.
c) Determine the equilibrium in dominant strategies.

Problem 9 *(Clarke-Groves Mechanism)*

The University faces a decision of whether to build a new parking lot on its grounds $a = 1$ or to preserve the existing rose garden, $a = 0$. The deans of the colleges $i \in \{1...I\}$ are asked to submit their evaluations $\hat{\pi}_i = \hat{\pi}_i(1) - \hat{\pi}_i(0)$. The president of the University then decides according to the following rule:

$$a(\hat{\pi}_1 ... \hat{\pi}_I) = \begin{cases} 1 & if \quad \sum_{i=1}^{I} \hat{\pi}_i \geq 0 \\ 0 & if \quad \sum_{i=1}^{I} \hat{\pi}_i < 0 \end{cases} .$$

After the decision has been made, each of the colleges has to contribute to the project the amount:

$$t_i(\hat{\pi}_1 ... \hat{\pi}_I) = \sum_{j \neq i} \hat{\pi}_j (a(\hat{\pi}_1 ... \hat{\pi}_I)) - \max_{a \in \{0;1\}} \sum_{j \neq i} \hat{\pi}_j (a).$$

The true evaluation of dean i is $\pi_i = \pi_i(1) - \pi_i(0)$.

a) Represent the game in strategic form.

b) Derive the best-responses of the players and draw them in a diagram.

c) Determine the equilibrium in dominated strategies.

<u>Problem 10*</u> *(Iterated elimination of dominated strategies, rationalizability)*

Consider an economy with 2 households which consume a private and a public good. The preferences of the households are identical and are given by the utility function:

$$u(q; x_i) = q x_i,$$

where q denotes the quantity of the public good and x_i is the quantity of the private good available to household i. Each household has an initial endowment of x^0 units of the private good ($x_0 > 0$) and none of the public good. The public good is produced from the private according to the following technology:

$$q = \sum_{i=1}^{2} (x^0 - x_i).$$

Each household has to decide how much of the private good to contribute to the production of the public good.

a) Determine the strategic form of the game.

b) Suppose that $x_2 = x^0$. Derive the optimal contribution of household 1. What is the optimal contribution of household 1 for the case $x_2 = 0$? Derive the best-responses of the players and draw them in a graph.

c) Derive the set of undominated strategies of both players in the first round of an iterative elimination of dominated strategies.

d) Continue the process of iterative elimination of dominated strategies. Show that for each player, the set of undominated strategies converges to a point and find the equilibrium of the game.

e) Determine the set of rationalizable strategies in this game. Compare your result to the equilibrium you derived in part d).

<u>Problem 11</u> *(Iterative elimination of dominated strategies)*

4 Pirates must agree on how to divide 400 golden coins. In order of seniority, each of the pirates can make an offer of how to divide the coins. The pirates vote on the offer made. If at least half of the pirates vote in favor of the offer, it is implemented and the game ends. Otherwise, the pirate who made the offer receives nothing and is killed and the next-oldest pirate makes an offer.

a) Determine the equilibrium strategies in this game using backward induction.

(Hint: Consider first the stage of the game in which pirate 4 is the last one left. What are the resulting payoffs? Now, go one step back: only pirates 3 and 4 are left. How much is 3 going to offer to 4? Is pirate 4 going to vote for or against the proposal? Proceed further: how many votes does pirate 2 need to implement his proposal? Whom is he going to "bribe"? How much is he going to offer?)

b) What is the relationship between backward induction and iterative elimination of dominated strategies in games with perfect information? Determine the allocation in equilibrium after iterative elimination of dominated strategies.

c) What is the allocation if the number of pirates becomes 10?

d) What information about the game should the players have so that the equilibrium after iterative elimination of dominated strategies appears to be a sensible solution concept?

<u>Problem 12</u> *(Cournot duopoly, iterative elimination of dominated strategies)*

Firm 1 and firm 2 are the only suppliers of a homogenous good. The market demand for this good is given by:

$$Q(p) = \max\{a - p; 0\},$$

where p denotes the price. The production of q_i units of output costs firm i $C_i(q_i) = cq_i$, $c < a$.

Both firms determine their output simultaneously.

a) Determine the strategic form of the game.

b) Derive the optimal quantity produced by a firm which is a monopolist in the market described above.

c) Show that any strategy which satisfies $q_i > q_m$ is dominated by the strategy q_m. Illustrate your argument in a diagram.

d) Assume that none of the two firms produces more than q_m. Show that each strategy $q_i < \frac{a-c}{4}$ is dominated by the strategy $q = \frac{a-c}{4}$.

e) Continue the iterative elimination of dominated strategies and derive the equilibrium of the game.