Global Imbalances and Idiosyncratic Risk

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Abstract

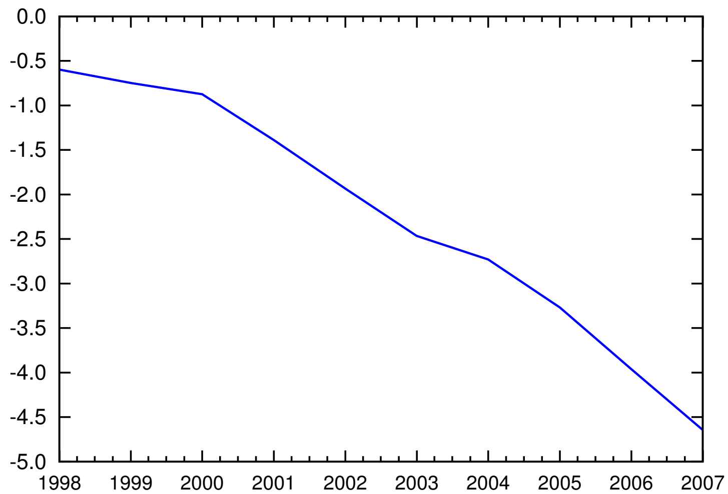
We study the role of idiosyncratic risk in generating capital flows observed between China and the U.S. We analyze a 2 country version of the model presented in Huggett (1993). The model is calibrated to match the evolution of short-term interest rates in the data. To achieve this we assume that China's income is 10% more volatile than in the U.S. As a result the U.S. net foreign asset position is -32.5% of GDP and world interest rate is depressed by 150 basis points.

1 Facts

We analyze the period from 1998 until 2007. This is the first ten years after China liberalized capital flows. These are also the years of phenomenal growth in China. While per capita income grew almost tenfold this growth was distributed very unevenly. At the same time Chinese savings, as a percentage of GDP, increased from 20 to 30 percent. This increase in savings outpaced an increase investment and contributed to a massive current account surplus. This surplus was mostly lent to the U.S. via purchases of the U.S. government debt. During the period the U.S. accumulated a net foreign asset position against China equal to -4.64% of the U.S. GDP (see figure 1).

The usual suspect is the credit market which still remains very undeveloped in China. Yet, even though there was no marked improvement in the credit market during the decade its condition certainly did not deteriorate.

Figure 1: Net foreign asset position between the U.S. and China



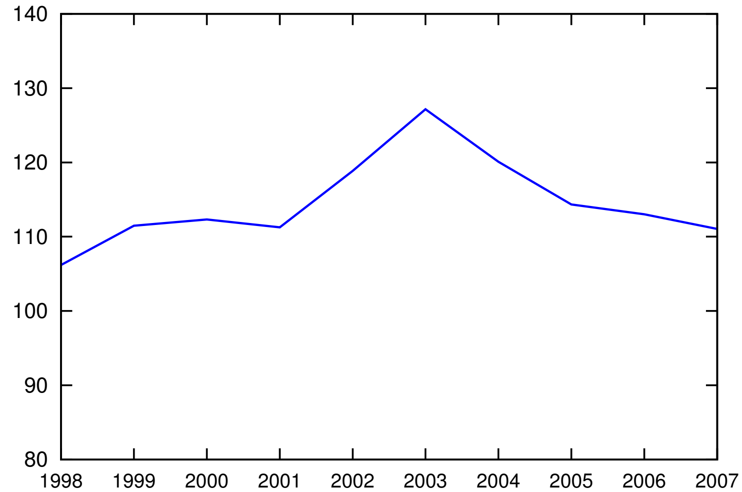
If it was the sole driving force then savings at least must not have increased. Yet we observe the reverse. While we believe that credit market imperfections are a part of the problem we aim to explain the evolution of global imbalances. Here we fix financial market development which is consistent with the evidence. A common measure of financial development, domestic credit to GDP ratio, was essentially unchanged during the period at a level of approximately 110% (see figure 2).

We believe that the driving force is an increase in idiosyncratic uncertainty in China and other fast growing emerging market economies. [EVIDENCE]

An increase in idiosyncratic uncertainty increases precautionary demand for savings in China. To keep the world financial market in equilibrium the interest rate declines to contain the savings demand in China. The lower interest rate, however, leads the U.S. into a position of a permanent debtor. Figure 3 plots the real interest rate in the US.[1] In 1998 the real interest rate was 3.35% and in 2007 it declined to 1.99%. Decline of the short term real

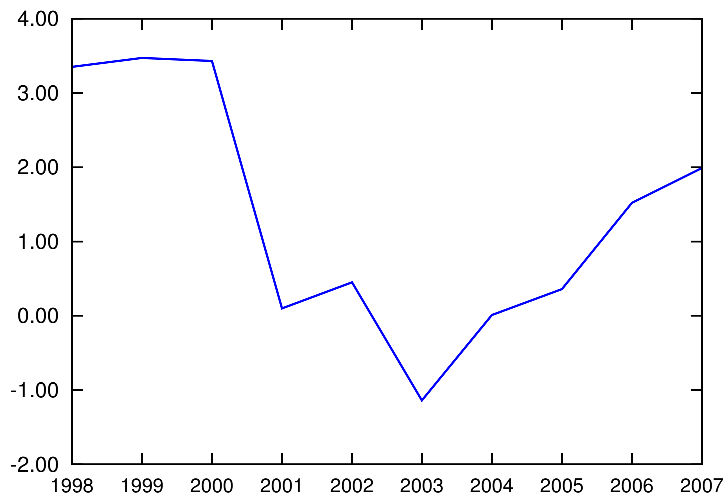
[1]It is computed as the difference between return (constant maturity) on a 1-year T-Bill rate and year-on-year change in the consumer price index (all items).

Figure 2: Domestic credit to GDP ratio, %



interest rate is consistent with the model.

Figure 3: 1-year Treasury bill real rate (constant maturity)



2 Model

The model described in this paper is a two country version of the model in Huggett (1993). The model can be easily extended to the case with an arbitrary but finite number of economies. The world consists of two economies each populated by a large number (a continuum) of heterogeneous agents. Economies are indexed by $j \in 1, 2$. Population in country j is denoted by $L^j > 0$.

Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$.

Uncertainty and endowments. There is no aggregate uncertainty. The only source of uncertainty is household's income. Income shocks are household and country specific. Income of household h in country j is an AR(1) process:

$$ln(y_t^{hj}) = \mu_j(1 - \rho_j) + \rho_j ln(y_{t-1}^{hj}) + \sigma^j e_t^{hj}, \quad e_t^{ij} \sim i.i.d.\ \mathrm{N}(0, 1). \tag{1}$$

μ_j is the expected individual log-income, which is also country specific.

Preferences of household h in country j are represented by a standard time-separable welfare function:

$$W(c^{hj}) = E\Big[\sum_{t=0}^{\infty} \beta^t u(c_t^{hj})\Big|\mathcal{I}_0\Big], \quad \beta \in (0, 1), \tag{2}$$

where $\mathcal{I}_t$ denotes the date-t information set. Expectation is over sequences of income realizations.

Financial markets are incomplete. Households in each country can borrow and lend at (gross) interest R. Flow of funds between countries is unrestricted; so, the interest rate is equalized across borders. Individuals in country j can borrow at most $B^j > 0$.

Budget constraint of household h in country j is:

$$c_t + a_{t+1}/R = y_t + a_t, \quad \forall t.$$

2.1 Recursive formulation

All households in each country are identical ex ante and ex post differ only by the paths of realized income. Let $V^j(a, y|R, \sigma)$ be the life-time utility of a household in country j that behaves optimally, has a bank account balance a, current income y. Arguments R, σ reflect the fact that these values are parameters of the individual optimization problem. The value function must satisfy the following Bellman equation:

$$V^j(a, y|R) = \max_{c \geqslant 0, a' \geqslant -B^j} \left[u(c) + \beta \int V(a', y'|R) dF(y'|y) \right] \tag{3a}$$

subject to

$$c + a'/R = a + y. \tag{3b}$$

The first-order necessary and sufficient condition for the above optimization problem is:[2]

$$u'(c) = \beta R \int u'(c') dF(y'|y) + \mu R, \tag{4}$$

where μ is the Lagrange multiplier on the borrowing constraint. The Lagrange multiplier must satisfy the following complementarity condition:

$$\mu(a' - B^j) = 0.$$

For a given interest rate R, we denote the solution to the country j household's optimization problem by $\rho^j(a, y|R) \in (\mathcal{A} \times \mathcal{Y})^{\mathcal{A} \times R_+}$:[3]

$$a' = \rho^{aj}(a, y|R), \tag{5a}$$

$$c = \rho^{cj}(a, y|R). \tag{5b}$$

2.2 Equilibrium

Let $F^j(\bar{a}, \bar{y}|R)$ be a distribution over financial wealth-income pairs in country j when the interest rate is R:

$$F^j(a, y|R) = prob(a^{hj} < \bar{a} \text{ and } y^{hj} < \bar{y}). \tag{6}$$

[2]It will be later shown that the objective is strictly concave. Hence, the above conditions are also sufficient. The budget constraint and the complementary slackness conditions were left out for clarity.

[3]We denote the set of all functions mapping X into Y by X^Y.

Let $A^j(R)$ denote the aggregate savings in the economy j when the interest rate equals R. $A^j(R)$ is a result of household decisions:

$$A^j(R) = \int \rho^j(a, y|R) dF^j(a, y|R). \tag{7}$$

*Definition. A **competitive equilibrium** is an interest rate R and policy functions $\rho^1, \rho^2 \in (\mathcal{A} \times \mathcal{Y})^{\mathcal{A} \times R_+}$ such that:*

a) Given the interest rate $\rho^j(a, y)$ is the optimal policy of a household living in country j with assets a and current income level y;

b) Financial markets clear:

$$L^1 A^1(R) + L^2 A^2(R) = 0. \tag{8}$$

2.3 Properties of aggregate savings

The aggregate saving function $A(R, \sigma)$ is decreasing in R and σ for all $R \in [0, 1/\beta)$ and $\sigma > 0$.

Proposition 1. *Let R^j denote the autarkic interest rate in country j. If*
a) $\sigma^1 > \sigma^2$ and $B^1 = B^2$ or
b) $\sigma^1 = \sigma^2$ and $B^1 < B^2$
then $R^1 < R < R^2$ and $A^1 > 0 > A^2$.

Proof. See appendix. □

3 Benchmark Simulation

We think of country 1 as representing the U.S. The stochastic process for individual income in country 1 is that estimated by Heaton and Lucas (1995) for a representative U.S. household. Households in country 1 can borrow up to the average annual income. The same borrowing limit was assumed by Huggett (1993). The risk aversion parameter γ was set to a standard value of 2. We choose the discount factor β such that the interest rate in country 1 were 3.35% in the absence of financial linkages with country 2. That is our parameters are set to mimic the situation when China had strict financial account restrictions set in place and any imbalances between the U.S. and China were absent.

Table 1: Benchmark parameters

Parameter	Value	Source
β	0.940	Implies $r = 3.35\%$ in a closed economy
γ	2.000	–
$E(y^1)$	1.000	Normalization
$E(y^2)$	0.110	PWT 6.3
L_1	1.000	Normalization
L_2	4.530	PWT 6.3
ρ_1	0.530	Heaton and Lucas (1995)
σ_1	0.296	Heaton and Lucas (1995)
ρ_2	0.530	Assumed $= \rho_1$
σ_3	0.049	Assumed to satisfy $cv(y_1) = cv(y_2)$
B_1	$1 \times E(y^1)$	Huggett 1993
B_2	$1 \times E(y^2)$	Assumed to satisfy $B_1/E(y_1) = B_2/E(y_2)$

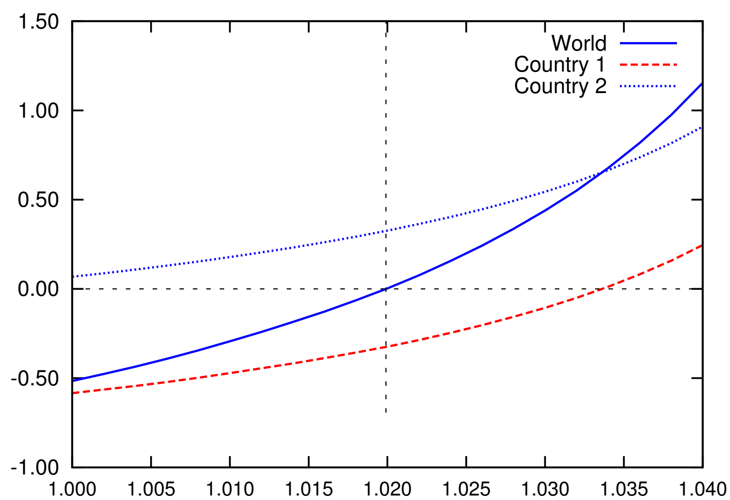
Preference parameters in country 2 are the same as in country 1. Country 2 also faces the same *relative* borrowing limit that equals the average period income. Income process in country 2 is also the same with the exception that it is *relatively more volatile*. If individual income had the same coefficient of variation as in country 2 then there would be no trade. In the latter case one country would be just a scaled replica of the other. We set the coefficient of variation for country 2 to 0.4459 (compare to 0.2960 for country 1) so that the interest rate in the stationary equilibrium is 1.99% as observed in 2007.

The relative size of the two economies were chosen to equal the relative size of China versus the U.S. in 1998. We used the data in Penn World Tables version 6.4. In 1998 the U.S. per capita income was 9 times that in China but China 4.53 times more populous at the time. Note that without differences in (relative) volatility of individual income country size differences play no role in generating financial imbalances.

Parameter values are summarized in table 1.

Figure 4 demonstrates the solution to the model. It plots aggregate savings of each country as a function of the interest rate R. The equilibrium interest rate is such that world savings is zero. In such an equilibrium country with less risk (lower coefficient of variation of income)[4] will be a net debtor and the other economy a net creditor. Under the parametrization summarized in table 1 country 1 borrows an (astonishing) 32.5% of its GDP from country 2 and the equilibrium net interest rate equals the target 1.99%.

Figure 4: Aggregate savings and equilibrium interest rate

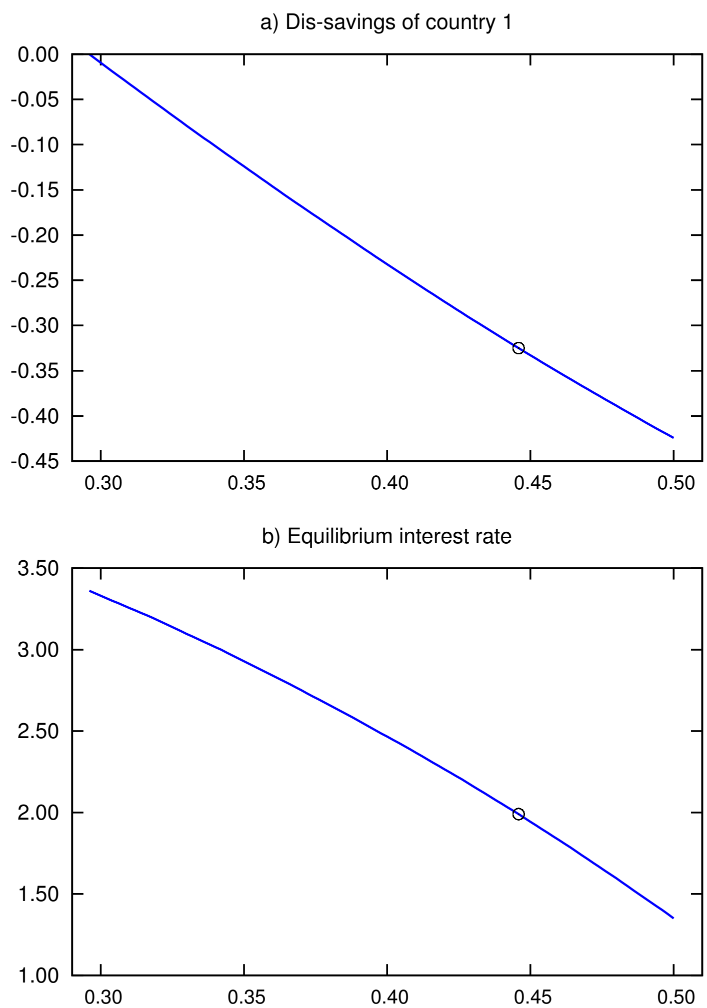


Since we do not have direct evidence on volatility of income in China, we consider next how the size of global imbalance changes with σ^2. Figure 5 panel (a) and (b) plot respectively the country 2's savings and the equilibrium interest rate as a function of σ^2. Benchmark calibration is marked with an 'o' sign. This figure shows that global imbalances accumulate quickly. In 2007 the US net foreign asset position was -24% of GDP. To obtain such an imbalance as an equilibrium outcome we need to assume that coefficient of

[4]It could also be a country with a more developed financial system as reflected in a more generous borrowing limit.

variation of individual income in China is 0.40, 35% higher than that in the U.S.

Figure 5: Global imbalances and world interest rate



4 Conclusions

We used a 2-country version of the model presented in Huggett (1993) to show that global imbalances can easily arise from difference in idiosyncratic risk. Thus, if income volatility is China were only 10% higher than in the U.S. we would observe global imbalances of 7% of the U.S. GDP. The world interest rate would be depressed by 40 basis points. However, we need to assume that income volatility in China is 35% higher to match the evolution of the short term interest rate. In this case global imbalances are 32.5% of the U.S. GDP, larger than 24% observed in the data.

References

Huggett, M. (1993), 'The risk-free rate in heterogeneous-agent incomplete-insurance economy', *Journal of Economic Dynamics and Control* (17).

Appendix