

Capital Outflows And Moral Hazard

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Abstract

In this paper I build a quantitative model of international lending. As a benchmark economy I take the model of Atkeson (1991) which features a range of credit market frictions. First, lenders cannot observe the borrowing country's investment (moral hazard). Second, the borrowing country is free to repudiate its debts and face financial autarky (limited enforcement).

I solve for the optimal (state-contingent) contract and find that moral hazard friction is sufficient to explain capital outflows in low output states - a defining feature of the emerging markets business cycles. On the other hand, the model that also includes limited enforcement is inconsistent with this fact. The model with moral hazard also performs well in explaining quantitative properties of Argentina's business cycle.

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1 Introduction

With access to complete international credit markets, a typical economy is able to diversify all of its country-specific risk. In states in which resources are scarce, the economy is able to borrow to finance a stable level of consumption and investment. However, emerging economies seem to be unable to share idiosyncratic risk. First, this is reflected in the almost perfect correlation between consumption and output and consumption being at least as volatile as output. Second, emerging economies have strongly counter-cyclical current accounts, that is they experience large capital outflows when output is low. Third, emerging economies have high, volatile and counter-cyclical interest rates. These three facts put emerging markets in stark contrast with small open developed economies, *e.g.* Canada.

Recent crises in Argentina in 2001, in Russia in 1998, and in Asia in 1997 created a lot of turmoil around the world.¹ Understanding causes of these crises means being able to, at least, mitigate future crises. In this paper, I analyze the ability of moral hazard to rationalize the above observations with the main focus put on the capital flows.

The banking and the financial sectors in emerging economies are largely unfree. For example, in Argentina the government retains full ownership in four large banks, including the country's largest bank, Banco de la Nación. This suggests that a significant fraction of funds may be directed into politically motivated projects. Lender's inability to monitor the use of funds by Argentina leads to the moral hazard problem that I study in this paper.

Starting with the seminal article by Eaton & Gersowitz (1981), researchers usually model international credit market as incomplete – only a one-period bond can be traded – and subject to the risk of repudiation. Recent examples are Arellano (2005) who extends the analysis in Eaton & Gersowitz (1981) to a stochastic environment and Yue (2006) who adds a possibility of renegotiation.² Surprisingly, little attention has been devoted

¹The full list since the Mexican crisis in 1994 is: Hong Kong, Indonesia, Malaysia, and Thailand in 1997, Russia and Brazil in 1998, Chile, Colombia and Ecuador in 1999, Turkey and Argentina in 2001.

²For a list of alternative modeling approaches see Arellano & Mendoza (2003).

to the problem of moral hazard in international credit markets.³

Gertler & Rogoff (1990) study a two-period model of international lending with moral hazard. In their model a risk-neutral agent borrows to finance a risky investment project. Because investment is not observable, the creditor must provide incentives for investing by offering a spread (across states) in the borrower's net worth next period. As the authors point out

Relative to the perfect-information benchmark, ... capital flows are dampened (and possibly reversed).

Investment is also lower relative to the frictionless economy and increases with the borrower's net worth, as observed in the data.

Atkeson (1991) studies a model similar to Gertler & Rogoff (1990) that is a fully dynamic, infinite horizon model with a risk-averse borrower. Contracting between the borrower and the lender is restricted both because the lender cannot observe the use of funds by the borrower (moral hazard) and the borrower can renege on the promised repayment (limited enforcement). Atkeson (1991) shows that a binding participation constraint may trigger capital outflows in the lowest output state. In economies that are subject to the risk of repudiation, participation constraints usually bind in high output states. The presence of the moral hazard seems to reverse this result.

Atkeson (1991) also shows that the optimal allocation can be implemented by a one-period contract that depends on the borrower's net worth. Unlike in other studies on private information economies, *e.g.* Thomas & Worrall (1990), the endogenous state is not a usual promised utility but the *observed* borrower's net worth. This allows computing the external finance premium, as a function of that country's net worth.

The above results and the fact that Atkeson's (1991) is a theoretically tight model makes it a frontier work on international lending. However, his model has not been fully understood. In particular, since Atkeson

³Researchers at the IMF studied the question whether its lending generates moral hazard. Rogoff (2002) gives a short overview of the arguments advanced and concludes that evidence on the presence of the moral hazard is mixed. However, I think that the IMF is a special-status lender, whose presence is a good signal to other potential creditors.

(1991) makes assumptions about the equilibrium outcome, it is still uncertain whether capital flows are indeed possible in his model. Moreover, it is unknown whether both the moral hazard and the limited enforcement are necessary to generate capital outflows. Finally, are the capital outflows generated by the model quantitatively significant? I try to address these questions in this work.

In what follows I revisit the result in Atkeson (1991). I show that, when the limited enforcement constraint binds in the worst state of the world, capital flow have to be identically zero. The reason is that if the enforcement constraint binds in the lowest state, it also has to bind in all other states and when it binds the loan amount is enough to roll over the debt, *i.e.* there are zero capital outflows. This result contrasts with the previous finding and, most importantly, with the fact that large capital outflows are observed in the data.

The only remaining candidate to explain capital flows in this economy is thus moral hazard. Indeed, numerical simulations show that the capital outflows in the lowest output state in a model with only moral hazard can be quantitatively significant and larger than in a model which also includes limited contract enforcement.

Overall, the model with moral hazard only is consistent with the key features of business cycles in emerging economies. In particular, the presence of moral hazard limits the amount of risk-sharing and generates counter-cyclical current account imbalances and external finance premium. Importantly, the counter-cyclical current account is tightly and positively related to output persistence that is endogenous in the model.

To improve the model's quantitative properties, I depart from the Atkeson model in two ways. First, I study the case in which the borrower is less patient than the creditor. This modification is motivated by the fact that political regimes in emerging economies are relatively unstable. For example, in 2004 World Bank ranked Argentina in the 39th percentile of the distribution of political stability index.⁴ At the same time Canada was in 82nd percentile. This motivates modeling agents in the emerging economies

⁴The higher the index the more politically stable country is.

as having shorter planning horizons (or having a lower survival probability). When the borrower is less patient than the creditor, the stationary distribution of the borrower's net worth shifts towards the left tail. Thus, the borrower runs more frequently into a region where the optimal contract specifies capital outflows. Second, I introduce a persistent shock to output that is publicly observed. This helps me to match the observed persistence of output that is endogenous in the model. However, capital flows are dampened.

The plan of this paper is as follows. Section 2 documents key empirical regularities about Argentina's economy. Section 3 introduces the basic economic environment and describes two basic financial arrangements. Under the first arrangement, the borrower has access to a dynamically complete financial market. Under the second arrangement, the borrower is in financial autarky. These are useful benchmark economies and provide the upper and the lower bound on the values in other environments. Section 4 revisits the result in Atkeson (1991). Section 7 presents quantitative results. In section 8 I show how the model can be extended to a two-good setting.⁵ Finally, section 9 concludes.

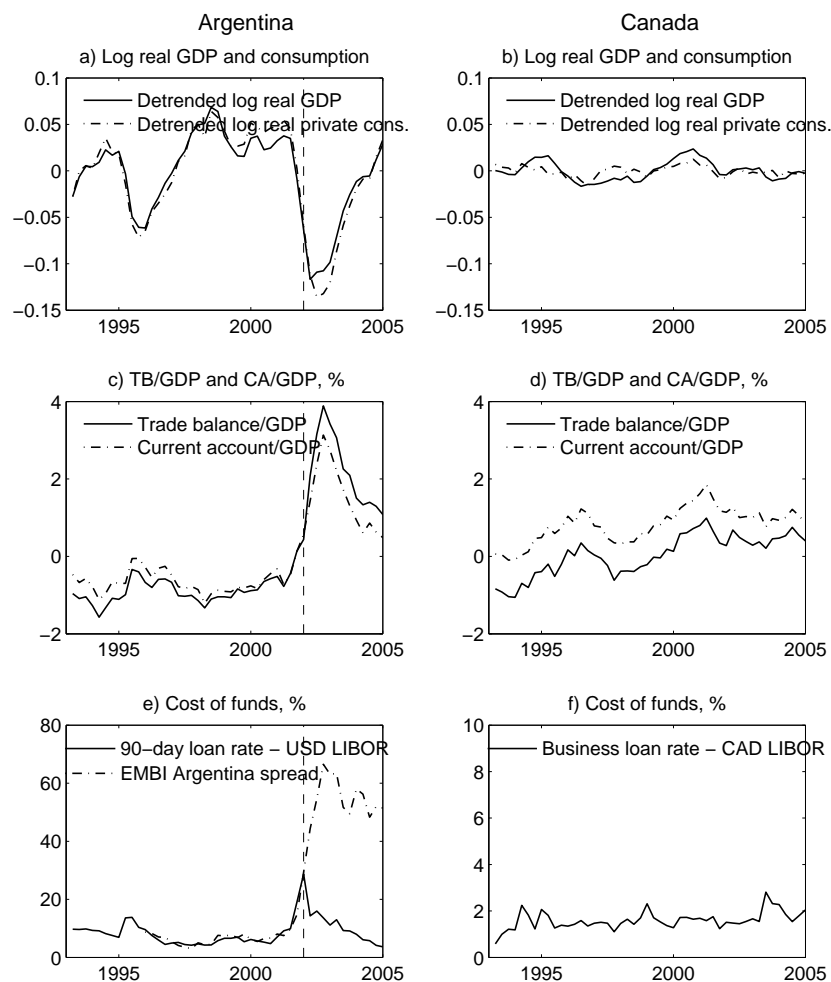
2 Empirical Facts

In this work I analyze Argentina over the period from 1993:Q1 to 2005:Q4 as a representative emerging market economy. For comparison I choose Canada as a quintessential developed small economy. Figure 2 shows selected macroeconomic series for Argentina and Canada over the chosen sample period.⁶ Vertical dashed lines on each plot in the left column mark Argentina's default in December 2001. Around this period, output and private consumption plummeted (see panel a) – just in one year, from 2001:Q1

⁵Most episodes of international crises were accompanied by significant exchange rate depreciations. The recent Tequila crisis in Mexico in 1999 and Argentinean crisis in 2001 are well known examples. Section 8.1 that the model extended to a two-good setting naturally delivers the correct exchange rate prediction.

⁶Business cycle properties for a large set of emerging economies are documented in Aguiar & Gopinath (2006).

Figure 1: Selected macroeconomic series in Argentina and Canada



to 2001:Q4, real output decreased by 10.6%. During the entire period, consumption closely followed output, exemplifying Argentina's inability to share country risk.

Fact 1. *Emerging economies are unable to share idiosyncratic risk.*

Up to fourth quarter of 2001 Argentina had also been accumulating debt

by running trade balance deficits. However, in 2001:Q4, the trade balance reversed to positive values. For the pre-default period, the correlation between detrended log real GDP and the trade balance to GDP ratio is -0.81 (see Table 1).

Fact 2. *Trade balance is strongly counter-cyclical.*

At the same time the EMBI⁷ spread over USD LIBOR, a premium at which Argentina could borrow, surged from 18 to 29 percent at the time of default and subsequently reached levels above 60%. These extraordinary spread levels effectively cut off Argentina from world credit market. Interest rates inside of Argentina were also high. The spread between rates on USD loans to private non-financial institutions, as a measure of the cost of funds in Argentina, and USD LIBOR also increased significantly at the time of default and reached 33%.

Fact 3. *Interest rates are strongly counter-cyclical.*

The above stated facts are in stark contrast with observations on developed small open economies. Table 1 summarizes important data moments for Argentina and compares them with the corresponding moments for Canada. In the table, r denotes the spread between domestic loan rates and LIBOR.

Technology Shocks?

Aguiar & Gopinath (2005) argue that the high volatility of emerging economies is due to very volatile TFP trend. However, it is known that large part of the TFP fluctuations can be explained by variations in the capacity utilization. In Argentina capacity utilization is reported monthly since January of 2002. According to the Ministry of Economy in Argentina, capacity utilization in Argentina in the beginning of 2002, *i.e.* right after the default, was 48.2%. In mid-2004 it recovered and remained above 65%. Unfortunately, there is

⁷This is the emerging markets bond index computed by JP Morgan. It is considered one of the major indicators of a country's riskiness.

Table 1: Data moments^{1,2}

Country	$\sigma(c)/\sigma(y)$	$\rho(c, y)$	$E(r)$	$\sigma(r)$	$\rho(r, y)$	$\rho(tb, y)$	$\rho(tb, r)$
Argentina 93:1–01:4	1.11	0.97	8.18	4.73	-0.58	-0.81	0.65
Argentina 93:1–05:4	1.15	0.99	7.86	4.78	-0.68	-0.82	0.30
Canada 93:1–01:4	0.55	0.62	1.51	0.33	0.23	0.27	0.21
Frictionless credit mkt	0	0	0	0	0	≈ 1	0
Argentina 1980–2005	n.a.	0.95	11.00	4.89	-0.80	-0.77	0.89

¹ Output and consumption are filtered with Hodrick-Prescott filter.

² Output, consumption and trade balance-to-GDP ratio are seasonally adjusted.

no data for the pre-crisis period; so, suppose that capacity utilization was 65% in the end of 2000, one year before the default. Under the assumption of Cobb-Douglas production function with the capital share of 1/3, such a drop in capacity utilization can rationalize the 10.6% decline in Argentina’s real GDP between 2001:Q1 and 2001:Q4 observed in the data. Figure 7 in appendix A.5 plots monthly capacity utilization and monthly economic activity indicator for the period from January 2002 to June 2006.

Gertler, Gilchrist & Natalucci (2003) also find that capacity utilization, as measured by electricity use, can explain large movements in the measured productivity during the Korean crisis.

3 Economic Environment

In this section, I describe the basic economic environment. Later sections introduce certain frictions to this basic environment and study how equilibrium allocations change.

Time is discrete and the time horizon is infinite, $t \geq 0$. There is one perishable good at each date. There is one infinitely lived borrower. The borrower at each date receives a stochastic endowment and decides how to split available resources between consumption and investment. The bor-

rower ranks alternative consumption streams according to

$$U^b = E_0 \sum_{t=1}^{\infty} \beta^t u(c_t), \beta \in (0, 1), u'(c) > 0, u''(c) < 0.$$

The borrower starts period 0 with a given net worth, Q_0 , which is a claim to Q_0 units of date-0 good. Initial net worth may be negative. Endowment at all consecutive dates depends on the previous period's investment, I . Having invested I units of date- t good, the borrower's endowment at date $t + 1$ is drawn from a distribution with pdf $g(\cdot|I)$. It is assumed that, for all investment levels, the distribution of the next period's endowment has a fixed and finite support $Y = \{Y_1, \dots, Y_n\}$ with $Y_1 < \dots < Y_n$. Following Atkeson (1991), g is specified in the following way:

$$g(Y_i|I) = \lambda(I)g_{0i} + (1 - \lambda(I))g_{1i}, \quad (3.1)$$

where $\lambda : R_+ \rightarrow [0, 1]$ is an increasing and strictly concave function.⁸ Densities $g_0 > 0$ and $g_1 > 0$ are exogenously given and satisfy:^{9,10}

Assumption A1. g_0 and g_1 satisfy monotone likelihood ratio condition.

The marginal effect of investment on the probability of state i occurring is:

$$\frac{d}{dI}g(Y_i|I) = \lambda'(I)(g_{0i} - g_{1i}) \equiv \lambda'(I)\Delta g_i = \begin{cases} < 0, & i \text{ is small;} \\ \geq 0, & i \text{ is large.} \end{cases}$$

I also need

Assumption 2. $\lim_{I \rightarrow 0^+} \lambda'(I) = \infty$.

Assumption A2 implies that production technology satisfies the Inada condition at zero because $d[\sum_{i=1}^n g_i(I)Y_i]/dI|_{I=0} = \infty$. Assumption A2 also

⁸Production technology is concave: $d^2(\sum_{i=1}^n g_i(I)Y_i)/dI^2 = \lambda''(I)\sum_{i=1}^n \Delta g_i Y_i < 0$.

⁹Distributions g_0 and g_1 are said to satisfy monotone likelihood ratio property if g_{0i}/g_{1i} is increasing in i .

¹⁰Because λ is a concave function, production technology is also concave: expected output tomorrow is a concave function of the working capital I .

implies that optimal investment is always positive. This enables me to use the relaxed first-order approach to the incentive problem in section 4.¹¹

The international credit market is represented by a sequence of overlapping generations of lenders each of which lives for two periods. The lender living in periods t and $t+1$ ranks alternative consumption streams according to

$$U_t^c = c_t + \beta E_t c_{t+1}.$$

Lenders receive endowment, $M \gg 0$, in each period they live and can invest at the risk-free rate of $1/\beta$. Presence of M emphasizes the finiteness of the world endowment of good and imposes two restrictions on equilibrium quantities:¹²

1. The amount lent cannot exceed the lender's endowment:

$$b \leq M, \tag{3.2}$$

2. The borrower's saving cannot exceed the lender's endowment:

$$-d_i \leq M, \quad \forall i. \tag{3.3}$$

3.1 Contracts

Each period, the borrower signs a contract \mathcal{C} with the creditor. In a frictionless economy (in which the borrower's investment is observed and contracts are enforceable) \mathcal{C} specifies 1) the loan amount b provided by the creditor, 2) the repayment schedule (d_1, \dots, d_n) , where d_i is the amount to be repaid by the borrower in state i , and 3) the investment, I , made by borrower. I analyze the unconstrained efficient contract in section (3.2).

The first friction that I study is private information. In a private information setting the lender is able to observe neither investment nor consumption, but output is observable. This leads to the moral hazard: the

¹¹With the relaxed first-order approach one replaces an incentive constraint with the relaxed (inequality) first-order condition to the agent's optimization problem.

¹²If the borrower were allowed to default, it would be possible to endogenize M . I do not take this route in order to keep the model simple. In what follows I will make sure that these bounds are sufficiently large. Note also that if borrower's discount factor β were sufficiently smaller than the creditor's β_c the lower bound on d_i would be irrelevant.

lender can no longer fully insure the borrower as the latter would then optimally invest nothing. Formally, private information restriction implies that investment, rather than part of the explicit terms of the contract, is part of the borrower's optimal response to the creditor's contract (b, d) .

The second friction that I study is limited enforcement. It is assumed that contracts are difficult to enforce on an international scale. If a borrower decides to renege on a contract, he is forever excluded from international credit market. Thus a contract has to be designed such that the borrower always (weakly) prefers participation in the international credit market to financial autarky. Formally, a limited enforcement restriction imposes an upper bound on repayments that the creditor can obtain from the borrower.

In what follows I study recursive economies. For the economy with private information and enforcement constraints the existence of a recursive representation is shown by Atkeson (1991) in proposition 5 (page 1083). The existence of a recursive representation for the economy with only moral hazard directly follows.

3.2 Complete Credit Market

In this section, I analyze an environment in which the borrower has access to a complete credit market. I refer to such an economy as Arrow-Debreu economy. The value function found in this section serves as an upper bound on the continuation value in the environments considered later in this work.

Let d_i be the quantity of one period Arrow securities that pay out in state i issued by the borrower and let $q(Y_i|I)$ be the price of such Arrow securities given last period's investment of I units of good. Because the lender is risk-neutral, Arrow securities are traded at

$$q(Y_i|I) = \beta g(Y_i|I). \quad (3.4)$$

Let $V_{AD}(Q)$ be the optimal value to a borrower with net worth Q . The value function $V_{AD}(Q)$ satisfies the following Bellman equation

$$V_{AD}(Q) = \max_{I,b,d} \left\{ u(Q + b - I) + \beta \sum_{i=1}^n g(Y_i|I) V_{AD}(Y_i - d_i) \right\} \quad (AD)$$

subject to lender's individual rationality condition¹³

$$b \leq \beta \sum g(Y_i|I)d_i, \quad (3.5)$$

where b is the amount of good that the borrower receives in exchange for his promises, d , to deliver goods tomorrow. Quantities b and d must also satisfy equations (3.2) and (3.3): $b \leq M$, $-d_i \leq M$. If the latter constraints were absent then the borrower would always invest I^* units of good, where I^* satisfies¹⁴

$$1 = \beta \lambda'(I^*) \sum_{i=1}^n Y_i. \quad (3.6)$$

The left-hand side of the above equation is the investment cost of one unit of good. The right-hand side is the present discounted gain from investing an additional unit of good – the marginal increase in the expected output.

The lowest sustainable net worth level in this setting is $-M$ and for convenience I denote the state space by $\mathcal{Q} = [-M, \infty)$. As shown in the appendix A.2, the optimal contract can be summarized as follows (thresholds Q_1, Q_2 are provided in the appendix A.2):

1. If $Q < Q_1$ then the agent is borrowing constrained. The borrower receives maximal loan of M and agrees to a repayment schedule that delivers state *uncontingent* net worth $Q' \in (Q, Q_1]$ next period: $d_i = Y_i - Q'$. Investment is increasing in the borrower's net worth in this region.
2. If $Q \in [Q_1, Q_2]$ then the borrower receives a contract under which the borrower's net worth is constant across states and time: $d_i = Y_i - Q$, $b = \beta \sum_i g(Y_i|I^*)d_i$. Investment is equal to the unconditionally optimal level I^* .
3. If $Q > Q_2$ then the borrower's saving decision is constrained. Under the optimal contract, in low output states the agent receives (repayment is negative) from the creditor the maximal amount M . Mathematically, there exists $k \leq n$ such that $d_i = -M, \forall i \leq k$ and $d_i =$

¹³According to this condition the lender's gross rate of return is no less than $1/\beta$.

¹⁴ I^* is also the level of investment that would be chosen by the lender if he or she were allowed to operate the investment technology directly.

$Y_i - Q', \forall i > k$ for some $Q' \in [Q_2, Q)$. Like in the first region, investment is increasing in the borrower's net worth.

Figure 2: Optimal investment in an Arrow-Debreu economy

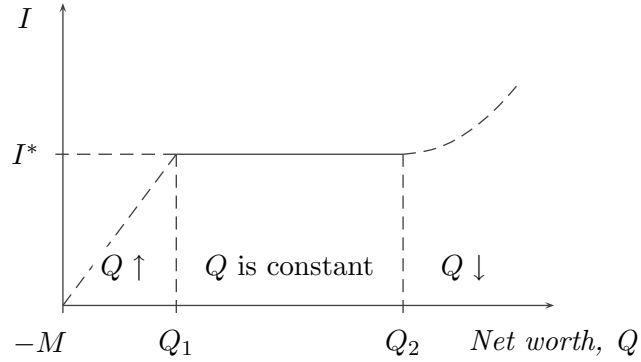


Figure 2 summarizes the above description of the equilibrium in the Arrow-Debreu economy. Note that irrespective of the initial level of net worth Q_0 the borrower eventually ends up with $Q \in [Q_1, Q_2]$, which corresponds to the region 2 described above. Thus, investment and consumption is also eventually constant, both across time and states. Capital flows each period are $b - d_i = (1 - \beta)Q + \beta \sum_{i=1}^n g(Y_i|I)Y_i - Y_i$. Thus, the correlation between the ratio of current account balance to output and log-output is $\rho((d_i - b)/Y_i, \ln(Y_i)) = -\rho(1/Y_i, \ln(Y_i)) \approx 1$. The presence of the borrowing limit only affects the support of the sustainable net worth levels, but not the limit behavior of the economy.

3.3 Autarky

Suppose the borrower has no access to the international credit market (financial autarky) and the investment technology is the only means available for intertemporal consumption smoothing. Let $V_{aut}(Q)$ be the optimal value for a borrower with Q units of good living in autarky. This value function is the lower bound on the values achieved in other environments. It satisfies

the following Bellman equation

$$V_{aut}(Q) = \max_{I \in [0, Q]} \left\{ u(Q - I) + \beta \sum_{i=1}^n g(Y_i | I) V_{aut}(Y_i) \right\} \quad (\text{Aut})$$

The optimal investment made by the agent is an increasing function of Q . Note also that $Q < 0$ is not sustainable in autarky.

4 Moral Hazard and Limited Enforcement

In this section I consider the setup in Atkeson (1991). Contracting between the borrower and the lender is restricted both because the lender cannot observe the use of funds by the borrower (moral hazard) and the borrower can renege on the promised repayment (limited enforcement). Thus, the creditor cannot perfectly insure the borrower as then no investment will be made. In addition, the creditor has a limited choice of repayment schedules as a large repayment may induce the borrower to default.

As shown in Atkeson (1991), the optimal allocation can be implemented by a one-period contract that depends on the borrower's net worth. Let $V_{Atk}(Q)$ be the optimal value to the borrower with net worth Q under the optimal contract. The value function $V_{Atk}(Q)$ satisfies the following Bellman equation

$$V_{Atk}(Q) = \max_{b, d} \left\{ u(Q + b - I) + \beta \sum_{i=1}^n g(Y_i | I) V_{Atk}(Y_i - d_i) \right\} \quad (\text{ATK})$$

subject to

$$b \leq \beta \sum_{i=1}^n g(Y_i | I) d_i \quad (4.1)$$

$$I \in \arg \max_{\hat{I} \in [0, Q+b]} \left\{ u(Q + b - \hat{I}) + \beta \sum_{i=1}^n g(Y_i | \hat{I}) V_{Atk}(Y_i - d_i) \right\} \quad (4.2)$$

$$V_{aut}(Y_i) \leq V_{Atk}(Y_i - d_i), \quad \forall i \in S. \quad (4.3)$$

According to the constraint (4.1) the loan amount never exceeds the present discounted value of the repayments. The constraint (4.2) is the borrower's

incentive compatibility constraint. It states that given the contract (b, d) the borrower chooses investment optimally. The constraints (4.3) are the borrower's participation constraints. They say that the borrower in every state weakly prefers honoring his debt to exclusion from the credit market (autarky). Note that the participation constraints now do the work of the restriction $b \leq M$ by setting the lower bound on sustainable net worth levels Q .

Risk-neutral borrower

It is useful to study the special case of risk-neutral borrower. Despite its simplicity it conveys a basic intuition about the effect of moral hazard. The creditor's problem is to choose contract (b, d) to maximize the borrower's value given by

$$V_{Atk}(Q) = \max_{b,d} \max_{I \in [0, Q+b]} \left\{ Q + b - I + \beta \sum_{i=1}^n g(Y_i|I) V_{Atk}(Y_i - d_i) \right\}$$

subject to $b \leq \beta \sum_{i=1}^n g(Y_i|I) d_i$. To single out the effect of the moral hazard, suppose that the participation constraints do not bind. It is easy to verify that $V_{Atk}(Q) = Q + K$ for some constant K . Then the optimal investment choice satisfies

$$1 = \beta \lambda'(I) \sum_{i=1}^n \Delta g_i \cdot (Y_i - d_i).$$

Note that when $d_i = d, \forall i$ then the borrower chooses the unconditionally optimal level of investment I^* as defined by equation (3.6). Thus the creditor trades insurance for efficiency. In this simple example, providing no insurance at all is optimal as it entails no welfare cost to the risk-neutral agent.¹⁵ When the agent is risk-averse, the case I turn to next, the creditor must balance the provision of insurance with that of incentives.

Risk-averse borrower: first-order approach

Constraint (4.2) is not tractable as it is. Replacing it with the first-order condition is a viable alternative. But, first, it has to be shown that the

¹⁵This also shows that risk-aversion is a crucial ingredient of the model.

first-order approach is valid in this setting.¹⁶

Following Rogerson (1985) I replace constraint (4.2) with

$$-u'(Q + b - I) + \beta\lambda'(I) \sum_{i=1}^n \Delta g_i V_{Atk}(Y_i - d_i) \geq 0. \quad (4.4)$$

I need to show that maximizing (ATK) with respect to (4.4) instead of (4.2) leads to the same solution. For this I need to prove that for any feasible contract (b, d) the following hold

$$\begin{aligned} -u'(Q + b - I) + \beta\lambda'(I) \sum_{i=1}^n \Delta g_i V_{Atk}(Y_i - d_i) &= 0, \\ u''(Q + b - I) + \beta\lambda''(I) \sum_{i=1}^n \Delta g_i V_{Atk}(Y_i - d_i) &< 0. \end{aligned}$$

The second inequality implies that the borrower's objective is strictly concave in investment. And when the objective is strictly concave the first-order condition is a sufficient condition for an (interior) solution.

Proposition 1. *Replacing constraint (4.2) with the relaxed first-order condition (4.4) does not change the solution to the problem (ATK).*

Proof. Fix a contract (b, d) . Because $\lim_{I \downarrow 0} \lambda'(I) = \infty$, the contract for which $\sum_{i=1}^n \Delta g_i V_{Atk}(Y_i - d_i) \leq 0$ holds is not optimal because it implies zero investment. In the relaxed program such a contract is simply rendered infeasible because it would violate (4.4).

When $\sum_{i=1}^n \Delta g_i V_{Atk}(Y_i - d_i) > 0$ then the borrower's objective must be strictly concave because $u'', \lambda'' < 0$. Hence, the first order condition (4.4) must hold with equality. \square

As shown in the appendix A.3, the optimal contract must satisfy the following optimality condition

$$V'_{Atk}(Q) = V'_{Atk}(Y_i - d_i) \left(1 + \gamma_i + \mu \frac{\lambda'(I) \Delta g_i}{g(Y_i|I)} \right) + \phi - \psi_i, \quad (4.5)$$

¹⁶Note that availability of the first-order approach also simplifies computation of the optimal contract.

where $\mu \geq 0$ is the Lagrange multiplier associated with the borrower's incentives constraint and $\gamma_i \geq 0$ is the scaled Lagrange multiplier associated with limited enforcement constraint for state i . The Lagrange multipliers on the constraints (3.2) and (3.3) are $\phi, \psi_i \geq 0$ respectively.

Suppose that V_{Atk} is a strictly concave function and, hence, V'_{Atk} is a strictly decreasing function.¹⁷ When neither of the participation constraints bind ($\gamma_i = 0$) then it is easy to show that $Y_i - d_i$ is increasing in i . An increasing profile of the continuation net worth reflects desire to provide incentives to invest. The slope of this profile is proportional to $\lambda'(I)\Delta g_i/g_i(I)$. This expression shows relative importance of effort (*i.e.* investment) and luck, and it is increasing in state i by assumption A1. When the participation constraint in state i binds, then optimal repayment is determined from $V_{Atk}(Y_i - d_i) = V_{aut}(Y_i)$. Since $\lambda'(I)\Delta g_i/g_i(I)$ is negative for low i , it may be the case that in the lowest-output state 1 the participation constraint must bind. According to Atkeson (1991) (page 1086), in this economy the pattern of the binding constraints is reversed relative to the setup in which investment is publicly observed and only the participation constraints are present (see *e.g.* Worrall (1990)). That is one should observe binding participation constraint in the lowest output state while the participation constraints in high output states may never bind. The pattern of binding constraints is important from the perspective of the capital outflows, as the following lemma shows.

Lemma 1.

Suppose that and $V_{Atk}(Q) = V_{aut}(Y_j)$ for some j . Then

*A) If $V_{Atk}(Y_i - d_i(Q)) = V_{aut}(Y_i)$ for some state i and some sustainable level of net worth Q , then $b(Y_i - d_i(Q)) \leq d_i(Q)$, *i.e.* an optimal contract specifies capital outflows.*

*B) When $V_{Atk}(Y_i - d_i(Q)) = V_{aut}(Y_i), \forall i$ then $b(Y_i - d_i(Q)) = d_i(Q)$, *i.e.* an optimal contract specifies zero capital flow.*

¹⁷I cannot prove that the optimal value function must be concave. However, under a wide range of parameters the numerically computed value function turned out to be indeed strictly concave.

Proof. Direct comparison of (ATK) and (Aut) gives

$$\begin{aligned} V_{Atk}(Q) &= \max_{I \in [0, Q+b]} \left\{ u(Q+b-I) + \beta \sum_{i=1}^n g_i(I) V_{Atk}(Y_i - d_i) \right\} \\ &\geq \max_{I \in [0, Q+b]} \left\{ u(Q+b-I) + \beta \sum_{i=1}^n g_i(I) V_{aut}(Y_i) \right\} = V_{aut}(Q+b), \end{aligned}$$

where b is the optimal amount lent to the borrower with net worth Q . Since $V_{Atk}(Q) = V_{aut}(Y_j)$, then $V_{aut}(Y_j) \geq V_{aut}(Q+b)$ by the above inequality. Because $Q = Y_j - d_j$ where d_j is the repayment that borrower agreed to in the previous period, it must be true that

$$d_j \geq b.$$

The proof of part B is straightforward repetition of the above argument. \square

By part A of lemma 1, one should observe capital outflows after the realization of output in which the participation constraint binds. Now suppose that, like in Worrall (1990), the binding participation constraint in the lowest-output state implies that the participation constraint must bind in all the other states $2, \dots, n$. Then, by part B of lemma 1, capital flow should be *zero*.

Atkeson (1991) shows that if $1 + \mu \lambda'(I) \Delta g_i / g_i(I) < 0$, which can happen only for low output states, then it must be the case that $\gamma_i > 0$, that is participation constraint must bind. Then, by lemma 1, a non-positive capital flow is observed *next period* in the state i . In what follows I extend the argument in Atkeson (1991) by showing that his economy inherits the pattern of binding constraints from Worrall (1990).

I suggest to consider the level of net worth \underline{Q} that satisfies the following relation:

$$\underline{Q} = Y_1 - d_1(\underline{Q}). \tag{4.6}$$

\underline{Q} is the lowest level of the borrower's net worth that could be observed after period 1.¹⁸ The borrower with the net worth \underline{Q} must also experience the

¹⁸In such the case, the lowest level of the borrower's net worth \underline{Q} must be preceded by the lowest-output realization. It is also the lowest level of net worth that can be reached under an optimal contract, by equation (4.6).

largest capital outflows. The next proposition shows that if the participation constraint were binding in the lowest-output state, then an optimal contract specifies the maximal repayment in each state.

Proposition 2. *Consider the borrower with the net worth level \underline{Q} living in the Atkeson economy. If the participation constraint binds in the lowest-output state then (b, d) such that $V_{Atk}(Y_i - d_i) = V_{aut}(Y_i)$ and $b = d_1$ is an optimal contract.*

Proof. Suppose that the borrower's net worth is equal to \underline{Q} and the enforcement constraint binds in state 1. To show that (b, d) such that $b = d_1, V_{Atk}(Y_i - d_i) = V_{aut}(Y_i)$ is the solution to the optimal contracting problem, I need to show that a) the constraints (4.1) and (4.2) are satisfied and b) the suggested contract achieves the optimal value $V_{Atk}(\underline{Q}) = V_{aut}(Y_1)$.

To this end

$$\begin{aligned} V_{Atk}(\underline{Q}) = V_{aut}(Y_1) &= \max_{\hat{I}} \left\{ u(Y_1 - d_1 + d_1 - \hat{I}) + \beta \sum_{i=1}^n g_i(\hat{I}) V_{aut}(Y_i) \right\} \\ &= \max_{\hat{I}} \left\{ u(\underline{Q} + d_1 - \hat{I}) + \beta \sum_{i=1}^n g_i(\hat{I}) V_{Atk}(Y_i - d_i) \right\} \end{aligned}$$

Thus, under the suggested contract the borrower receives the optimal value equal to $V_{aut}(Y_1)$. It remains to be shown that $b = d_1 \leq \beta \sum_{i=1}^n g_i(I_{aut}(Y_1)) d_i$, which must be true in view of lemma 1. \square

Thus after the lowest-output realization, if the participation constraint is binding, capital flow must be zero. This is at odds with the fact that large capital outflows are observed in the data. However, the argument above does not exclude capital outflows in the lowest output states in the Atkeson economy altogether. But if such happen, it must be for reasons other than limited enforcement. This also suggests that it must be the moral hazard that drives capital outflows. To show that this indeed is the case I consider the economy in which investment is not observed but there exists a third party that can enforce contracts. In section 7 I compute optimal contracts for the two economies (with and without participation constraints).

Output loss after default. After default the borrowing country usually suffers extra costs due to negotiation with creditors. These costs are estimated to amount to 2% of output (see Sturzenegger (2002)). So let me assume that in the period immediate after default the borrower suffers a cost which is a fraction δ of output. Thus, the borrower's outside option is now

$$V_{aut}(Y_i) = \max_{I \in [0, (1-\delta)Y_i]} \left\{ u(Y_i - \delta Y_i - I) + \beta \sum_{i=1}^n g(Y_i|I) V_{aut}(Y_i) \right\}.$$

By the argument in lemma 1, $V_{Atk}(Y_i - d_i) \geq V_{aut}(Y_i + \delta Y_i - d_i + b)$, where b is the optimal amount lent to the borrower with net worth $Y_i - d_i$. Now suppose that participation constraint in state i binds: $V_{Atk}(Y_i - d_i) = V_{aut}(Y_i)$. Then capital outflow must be at least fraction δ of output

$$\frac{d_i - b}{Y_i} \geq \delta.$$

Thus in this modified environment, the borrower indeed suffers capital outflow after realization of output in which participation constraint binds. However, this does not mean that presence of the moral hazard reverses the pattern of binding participation constraints. Proposition 1 adapted to this environment still holds and, thus, participation constraints are binding in high rather than low output states.

4.1 External Finance Premium

Even though the creditor's gross return is $1/\beta$, the borrower's internal rate of return R_b is always larger

$$R_b(I) = \lambda'(I) \sum_{i=1}^n \Delta g_i Y_i > 1/\beta.$$

The internal rate of return R_b , when evaluated at the unconditionally optimal level of investment I^* , equates both sides of the above equation.

External finance premium is the difference between internal rate of return and the external cost of funds

$$premium(I) = \lambda'(I) \sum_{i=1}^n \Delta g_i Y_i - 1/\beta > 0. \quad (4.7)$$

The external finance premium is larger for low levels of net worth Q when investment approaches zero. In the Arrow-Debreu environment without borrowing constraints the external finance premium is eventually zero.

5 Limited Enforcement: No Capital Outflows

In this section I analyze the model with the limited enforcement only. Models with limited enforcement are well-understood; however, the setting is non-standard because the distribution of the shock is endogenous. This setup was also studied in Atkeson (1988); so I skip many of the details and directly consider possibility of capital outflows in this setting. I show below that only capital *inflow* is possible after a binding enforcement constraint.

Let $V_{LE}(Q)$ be the value to the the borrower with net worth Q of being in the contractual relationship with the creditor. Then V_{LE} must solve the following functional equation:

$$V_{LE}(Q) = \max_{I,d} \left\{ u(Q + \beta \sum_{i=1}^n g_i(I)d_i - I) + \beta \sum_{i=1}^n g_i(I)V_{LE}(Y_i - d_i) \right\}$$

subject to enforcement constraints

$$V_{aut}(Y_i) \leq V_{LE}(Y_i - d_i), \quad \forall i.$$

The optimality condition for this optimization problem is

$$V'_{LE}(Q) = V'_{LE}(Y_i - d_i) \left\{ 1 + \frac{\gamma_i}{\beta g_i(I)} \right\}, \quad (5.1)$$

where γ_i is the Lagrange multiplier on the state- i enforcement constraint. When the enforcement constraint is slack, then the optimal repayment in state i solves $V'_{LE}(Q) = V'_{LE}(Y_i - d_i)$, otherwise d_i solves $V_{LE}(Y_i - d_i) = V_{aut}(Y_i)$.

It is easy to see that if the enforcement constraint in state i binds then it must bind in state $i + 1$.¹⁹ Assuming that the constraint in the lowest-output state binds, every enforcement constraint must bind: $V_{LE}(Y_i - d_i) =$

¹⁹It follows directly from the optimality condition above and concavity of V_{LE} .

$V_{aut}(Y_i), \forall i$. Then

$$\begin{aligned} V_{LE}(Q) &= u(Q + b - I) + \beta \sum_{i=1}^n g_i(I) V_{aut}(Y_i) \\ &\leq \max_{\hat{I}} \left\{ u(Q + b - \hat{I}) + \beta \sum_{i=1}^n g_i(\hat{I}) V_{aut}(Y_i) \right\} = V_{aut}(Q + b), \end{aligned}$$

where $b := \beta_c \sum_{i=1}^n g_i(I) d_i$ and (I, b, d) is the optimal contract between the lender and the borrower. Thus when the enforcement constraint in the lowest output state binds it must be true that

$$V_{aut}(Y_i) = V_{LE}(Y_i - d_i) \leq V_{aut}(Y_i - d_i + b(Y_i - d_i))$$

which, by monotonicity of V_{aut} , implies that no capital outflows are possible: $d_i \leq b(Y_i - d_i)$. This proves

Proposition 3. *In the economy with limited enforcement only, for low levels of net worth such that the limited enforcement constraint in state 1 binds only capital inflow is possible in the lowest-output state.*

In particular, there can be no capital outflows when the enforcement constraint in the lowest output state binds. Proposition 3 suggests that an explanation of capital outflows must come from a friction other than limited enforcement.

6 Patient Creditors, Impatient Borrower

Emerging economies usually have very unstable political situations which suggest that agents (governments) may have shorter planning horizons or, equivalently, may be less patient. Indeed, Argentina had a very high government turnover during the considered period of 1993-2001. This gives a reason to think of Argentinean government as of agent with low discount factor, which is indeed a product of the original discount factor and the survival probability of the government.

Secondly, since I consider a small open economy, there is no reason to restrict the discount factor of lender to be equal to that of the borrower. In

fact, different discount factors are more in the spirit of small open economy modeling, where a lender's discount factor is proxied by exogenous (world) interest rate, which may or may not be equal to the internal discount factor. So let me briefly analyze a situation in which the creditor is more patient than the borrower.

Let β_c and β_b be, respectively, the discount factor of the creditor and the borrower where $\beta_c > \beta_b$. In this case unconditionally optimal level of investment is a function of the lender's discount factor β_c only: $\beta_c \lambda'(I^*) \sum_{i=1}^n \Delta g_i Y_i = 1$. The equivalent of the optimality condition (A.4) in the appendix A.3 is

$$V'_{Atk}(Q) = \frac{\beta_b}{\beta_c} V'_{Atk}(Y_i - d_i) \left(1 + \gamma_i + \mu \frac{\lambda'(I) \Delta g_i}{g(Y_i|I)} \right) + \phi - \psi_i.$$

When the optimal value function is concave, the difference between β_b and β_c creates downward pressure on the borrower's net worth. This means that optimal contract must ask for the repayments that are larger relative to the case with $\beta_b = \beta_c$.

7 Quantitative Analysis

For the rest of this paper I make the following assumptions about the functional forms of utility

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}, \quad (7.1)$$

and investment

$$\lambda(I) = \min((I/\bar{I})^\eta, 1). \quad (7.2)$$

I would also like the effect of the finite lender's endowment, M , to have minimal impact on the results. Hence, I modify the constraints:

$$b \leq M, \quad -d_i \leq M_s, \forall i, \quad M_s \gg M.$$

However, when the borrower is sufficiently impatient (relative to the lender) then the constraint $-d_i \leq M_s$ becomes irrelevant.

Table 2 lists the parameter values chosen for the baseline simulation. In particular I choose the coefficient of the relative risk aversion γ to be 0.5 and

the lender’s discount factor $\beta_c = 0.95$, which corresponds to a 5.26% annual return on a risk-free asset. A low discount factor is needed for the following reason. If the discount factor is close to unity then optimal contract is very close to implementing the unconstrained efficient allocation, which fails to replicate the data facts mentioned in section 2. The discount factor of the borrower is $\beta = 0.90$. Such a low discount factor is still significantly larger than those used in *e.g.* Aguiar & Gopinath (2006), Arellano (2005) and Yue (2006).²⁰

Table 2: Baseline model parameters

Parameter		Value
borrower’s discount factor	β	0.90
lender’s discount factor	β_c	0.95
coef. of relative risk aversion	γ	0.50
the creditor’s endowment	M_b	4.45
the borrower’s output	Y	{ 9.35, 10.0, 10.7 }
investment technology	(ν, \bar{I})	(0.550, 0.755)

I choose to discretize output process using only three values. The reason is that with more than 3 states equilibrium distribution of output is likely to have 2 modes. However, the empirical distribution of output is single peaked as shown in the appendix A.6. Another reason is that for n states I need to estimate $2n - 1$ probabilities that describe the transition density.

²⁰For example, Arellano (2005) uses 0.953 as the quarterly discount factor, Yue (2006) uses 0.873 as the quarterly discount factor. Both of these correspond to the discount factor which is less than 0.825 at the annual frequency.

Distributions g_0 and g_1 are²¹

$$g_0 = \begin{bmatrix} 0.05 \\ 0.35 \\ 0.60 \end{bmatrix}, \quad g_1 = \begin{bmatrix} 0.60 \\ 0.35 \\ 0.05 \end{bmatrix}. \quad (7.3)$$

The three output states are Hermite nodes, scaled to match the observed volatility of log-output, 0.0540. To compute the optimal value functions I approximate them by cubic splines on a uniform grid on interval $[-M_b, Y_n + M_s]$ with 4000 points.²² All value functions are computed (by iterating on the Bellman operator) with L_1 precision of at least 10^{-4} .

Figure 3 plots trade balance for the moral hazard and the Atkeson economy. Vertical dashed lines denote the lowest bounds on the *observed* borrower's net worth in each economy. As one can see from the figure, in the moral hazard economy current account at the lowest level of net worth is about 2.5%. It is smaller than, observed after the 2001 default, 3.7% capital outflow but is still a significant number. On the other hand in the Atkeson economy capital flow is zero at the lowest level of net worth in the lowest output state, which is consistent with the result of proposition 2.

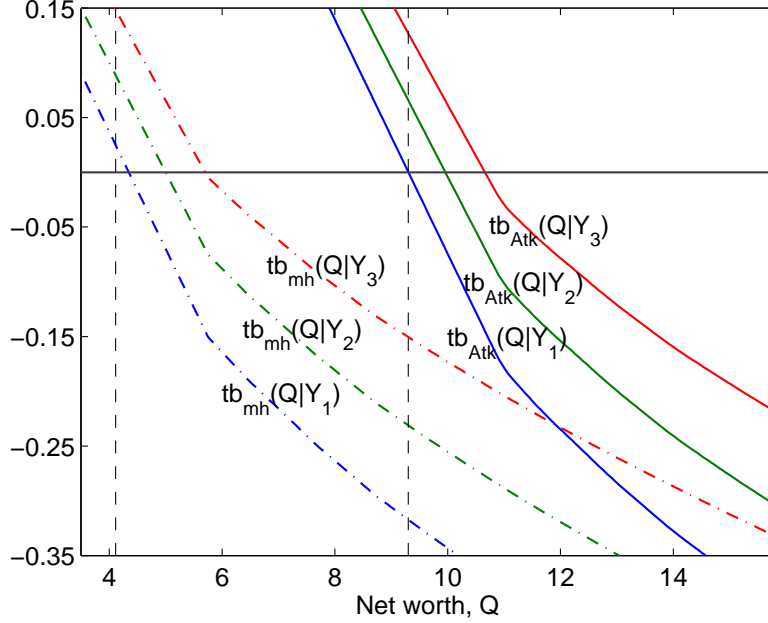
Numerical result 1. *For a range of low net worth levels optimal contract in the moral hazard economy specifies capital outflows in state Y_1 , while optimal contract in the economy with both moral hazard and limited enforcement prescribes only capital inflows in state Y_1 .*

Figure 4 plots the optimal contract for the Atkeson economy. Note that the participation constraint in the lowest output state stops to bind for low values of the borrower's net worth, while the other two participation

²¹Thus I am choosing $\nu, \bar{I}, g_0, g_1, \beta, Mb$ – a total of six parameters if g_0 and g_1 are anti-symmetric – to match $\sigma(c)/\sigma(y), \rho(c, y), \rho(tb, y), \rho(r, y), \rho(tb, r), E(r), \sigma(r), E(d/Y)$ – the total of eight moments. I confine the set of parameters to a grid and then search over the set of parameters to minimize the moment criterion. The moment values used in the criterion are those reported in table 3.

²²It is expected that a variable mesh grid could reduce approximation error; however, having a variable grid introduces certain complications in storing and comparing results across models.

Figure 3: Trade balance to output ratio, tb , in the moral hazard economy (dash-dotted) and the Atkeson economy (solid). Dashed vertical lines are lower bounds on the borrower’s net worth.)



constraints continue to bind. This is again consistent with the result in proposition 2.

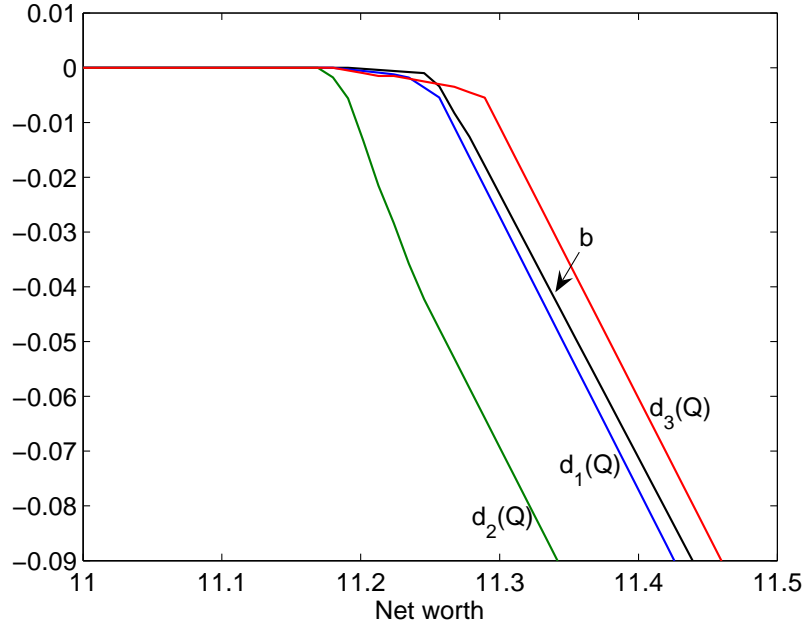
The question that arises is whether the assumption in Atkeson (1991) that leads to the “capital outflow in the lowest output state” result holds.²³ This assumption is that for some levels of net worth the following holds

$$1 + \mu\lambda'(I)\frac{\Delta g_1}{g_1(I)} < 0, \quad (7.4)$$

where μ is the Lagrange multiplier on the incentives constraint. Figure 5 plots the above expression as a function of the borrower’s net worth, Q . As can be seen from the figure, the above condition does not hold. In fact it is very close to unity. This is intuitive as the expression in (7.4) controls the

²³Atkeson (1991) imposes two assumptions. The first (assumption 6) is to insure that the first-order approach applies. But proposition 1 proves that it does apply in this setting.

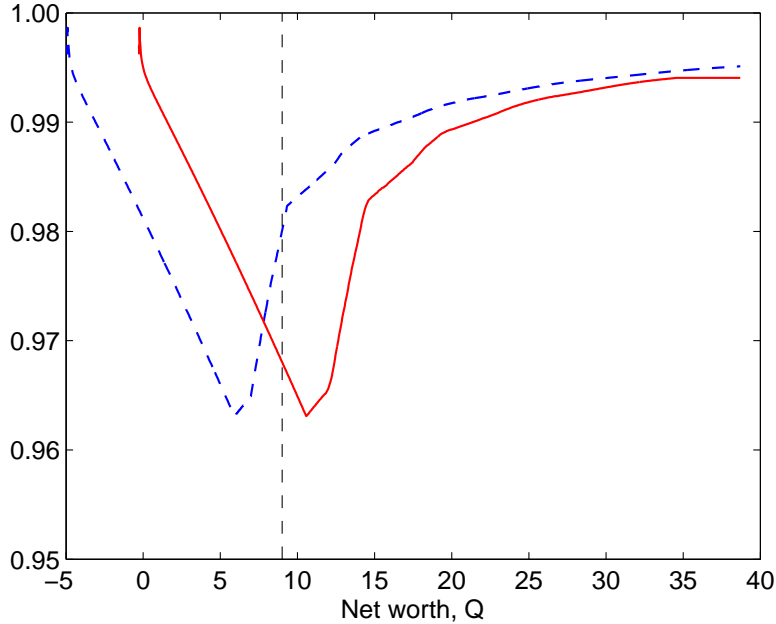
Figure 4: Optimal contract in the economy with both moral hazard and limited enforcement



spread in the borrower's continuation net worth. If this value were zero, the spread in the borrower's net worth would be infinite, which clearly cannot happen given that the borrower is risk averse.

Table 3 presents model moments. In this table r denotes external finance premium, and $E(d/Y)$ equals to debt-output ratio. First of all, note that the moral hazard model is able to replicate the data moments well. In particular, in the economy with moral hazard (and in the Atkeson economy) log-output and log-consumption are highly correlated and log-consumption is as volatile as log-output. This shows that moral hazard may significantly restrict the insurance motive. Next, the external finance premium in both economies is very high and volatile. They are also almost perfectly correlated with output. This is a consequence of the full capital depreciation. Because capital in the model fully depreciates each period, the borrower would never invest more than a difference in expected output under the two extreme

Figure 5: The effect of moral hazard: the expression in (7.4) for the moral hazard (dashed line) and the Atkeson (solid line) economy.



distributions:

$$I < \sum_{i=1}^n (g_0(Y_i) - g_1(Y_i)) Y_i \approx 0.76,$$

which is approximately 7.6% of output. Even though the level of investment in both economies is below 4% I get

Numerical result 2. In both models *A) investment volatility and volatility of the external finance premium are high; B) the external finance premium is strongly counter-cyclical.*

Correlation of trade balance with log-output is negative in the moral hazard economy. At the same time, in the Atkeson economy correlation between trade balance and output is positive. Correlation of the external finance premium with log-output is positive and significant in the economy with moral hazard only, while it is positive in the Atkeson economy. The

external finance premium is large and volatile in both economies. It is larger and more volatile than in the data because the creditor's discount factor is assumed to be 0.95. However, the reported number may be, in fact, close to reality as many of (high) finance premia are often not observed. Finally the debt to output ratio in the moral hazard economy is 52.7% which is close to the observed in the data 47.8%, while in the Atkeson economy it is close to zero. Had I chosen to match this number perfectly for the Atkeson economy I would need a much lower discount factor for the borrower. This would further increase (already large) level and volatility of the external finance premium.

Table 3: Model moments

Moment	Data	MH model	ATK model	MH model 2
$\sigma(c)/\sigma(y)$	1.1135	0.9835	0.9779	0.9235
$\rho(c, y)$	0.9710	0.9994	1.0000	0.9445
$\rho(y)$	0.9365	0.0812	0.0687	0.6712
$E(r)$	11.00%	7.63%	6.21%	8.03%
$\sigma(r)$	4.89%	3.22%	2.81%	3.41%
$\rho(r, y)$	-0.8014	-0.9994	-1.0000	-0.9601
$\rho(tb, y)$	-0.7728	-0.2103	0.6032	-0.1008
$\rho(tb, r)$	0.8930	0.2442	-0.6033	0.2667
$E(d/Y)$	0.4649	0.4693	0.0054	0.4705

Finally, note that the coefficient of autocorrelation of the output is poorly replicated by both economies. To fix this I compute the optimal contract for the economy in which g_0 and g_1 are state dependent. With two shock values the two distributions are

$$g_0 = \begin{bmatrix} 0.50 \\ 0.50 \end{bmatrix}, \quad g_1 = \begin{bmatrix} 0.80 \\ 0.20 \end{bmatrix}, \quad \text{if state } Y_1 \text{ is realized,}$$

$$g_0 = \begin{bmatrix} 0.20 \\ 0.80 \end{bmatrix}, \quad g_1 = \begin{bmatrix} 0.50 \\ 0.50 \end{bmatrix}, \quad \text{if state } Y_2 \text{ is realized.}$$

All the other parameters remain unchanged. As can be seen this modification significantly increases persistence of output. However, correlation

between the trade balance and the log-output is dampened.

Approximate Implementation

As one can see in figure 6 in the appendix A.4 the optimal contract provides very little insurance. Thus, I study an approximate solution – state un-contingent repayment.

Let $V_{\text{MHR}}(Q)$ be optimal value to borrower under optimal contract in the economy with moral hazard only and un-contingent debt when his net worth is Q . Then V_{MHR} must satisfy the following Bellman equation

$$V_{\text{MHR}}(Q) = \max_{I,d} \left\{ u(Q + \beta d - I) + \beta \sum_{i=1}^n g(Y_i|I) V_{\text{MHR}}(Y_i - d) \right\} \quad (\text{MHR})$$

To compensate the borrower for non-optimal insurance contract, discount rate of the creditor has to be lowered by 0.10%. Given that the largest loans to emerging economies are above 50bn USD, 0.10% is a significant cost of using suboptimal contract design. This number may be also viewed as the benefit of “completing” the credit market.

8 Extensions

8.1 Two-good economy

Often capital outflows in the emerging economies are accompanied by large exchange rate depreciations. Good examples are Mexico in 1995 and Argentina in 2002. In what follows I show how the model can be extended to a two good setting. If all the lending is in terms of a tradeable good, after a low output realization the amount of tradable good decreases and with it exchange rate (of home vs foreign currency) must depreciate.²⁴

Consider the setup in which consumption good is produced from two intermediate goods – tradeable Y and non-tradeable L – according to CRS

²⁴Kocherlakota & Pistaferi (2006) consider a model with heterogeneous agents having private information regarding their endowments. This model is shown to solve the Backus-Smith puzzle.

technology

$$C = F(Y, L).$$

Borrower has initial endowment of tradeable good Q_0 and receives exogenous stochastic endowment $\{L_t\}$ of non-tradeable input. I assume that endowment of the non-tradeable input is a discrete first-order Markov process with transition matrix $\Pi = \{\pi_{kj}\}$ with $prob(L_{t+1} = L_j | L_t = L_k) = \pi_{kj}$. Each period the borrower decides how much of final good to invest and how much to consume. Investing I units of tradeable good at date t yields Y_{t+1} units of tradeable good tomorrow, where $Y_{t+1}|I \sim g$ and g is specified in equation (3.1). Creditor receives non-stochastic endowment of tradeable good M each period, which can be lent to the borrower.

Let V_{2g} be the optimal value to the borrower with net worth Q living in the two-good economy. Then it must satisfy the following Bellman equation

$$V_{2g}(Q, k) = \max_{b,d} \left\{ u(F(Q+b-I, L_k)) + \beta \sum_{j=1}^m \pi_{kj} \sum_{i=1}^n g(Y_i|I) V_{Atk}(Y_i - d_{ij}, L_j) \right\} \quad (8.1)$$

subject to

$$\begin{aligned} b &\leq \beta \sum_{i=1}^n g(Y_i|I) d_i \\ 0 &\leq -u_1(F(Q+b-I, L_k)) + \beta \lambda'(I) \sum_{j=1}^m p_{kj} \sum_{i=1}^n g(Y_i|I) V_{Atk}(Y_i - d_{ij}, L_j) \\ d_{ij} &\leq Y_i - V_{Atk}^{-1}(V_{aut}(Y_i, j)), \quad \forall i \in S. \end{aligned}$$

Let world price of tradeable good be normalized to unity and let p_L be price of non-tradeable good L . Then it must satisfy

$$p_L = \frac{F_L(Y, L)}{F_Y(Y, L)} \equiv f(Y/L), f' > 0. \quad (8.2)$$

Given normalization $p_Y = 1$, real exchange rate equals price of consumption good, which, in turn, equals unit production cost

$$e := p_C/p_Y = p_C = \min_{K,L} Y + p_L L \quad \text{subject to} \quad F(Y, L) = 1.$$

According to the envelope theorem, price of final good p_C is an increasing function of p_L .²⁵

In general it is a difficult model to analyze; so to convey intuition let us study the case when $L_t = L, \forall t$ – that is the borrower receives constant endowment of non-tradeable input. If the borrower is hit with an adverse shock and has little of the tradeable good, ratio of tradeable to non-tradeable goods used in production, $(Q + b - I)/L$, declines, and so does p_L (see equation 8.2). Consequently, real exchange rate depreciates as it is positively related to p_L .²⁶

8.2 Other Extensions

For the future research I consider the following extensions of the model.

Long-term capital. With long-term capital the model may be able to generate larger capital outflows. The reason is that low investment leads to lower capital stock which has a long lasting affect on output. Thus creditor will want to punish low realizations of low output more to avoid costly cycle in the borrower’s production.

Labor. It may be meaningful (and more tractable) to assume that output is produced using unobserved labor effort and capital. With this formulation the model would be more comparable with other RBC models, *e.g.* Aguiar & Gopinath (2005).

Alternative punishment. Alternative punishment in which the borrower is excluded from the market only temporarily will be somewhere in between the moral hazard and the Atkeson economy. If, in addition, after default the borrower suffers immediate output loss the relative attractiveness of default in different states may be changed. If default is made more attractive in the low output states, as it is in Arellano (2005), the model performance would improve.

²⁵By envelope theorem $dp_C/dp_L = L^* > 0$.

²⁶For an example with CES production technology see appendix.

9 Conclusions

In this paper, I analyze theoretically and numerically how capital flows are affected by moral hazard. As a starting point, I choose the economy modeled in Atkeson (1991) which includes both moral hazard and limited enforcement. Then I show that, at the lowest point of the stationary distribution, if the participation constraint binds in the lowest-output state then it must bind in all other states. This in turn implies that capital flow in low-output state should be identically zero. This result suggests that enforcement constraints play limited role in generating capital outflows in the model. Thus, I turn to analyzing model with moral hazard only.

In numerical simulations I show that the capital outflows in the lowest output state in a model with only moral hazard can be quantitatively significant and larger than in a model which also includes limited contract enforcement the borrower always experiences capital inflow. Overall, the model with moral hazard only is consistent with the key features of business cycles in emerging economies. In particular, the presence of moral hazard limits the amount of risk-sharing and generates counter-cyclical current account imbalances and external finance premium. The model endogenously generates significant, but small as compared to the data, persistence of output. To fix this, I allow the distribution of output to depend not only on the level of investment but also on the state of the economy today. This modification allows to bring the output persistence closer to the level observed in the data; however, the capital flows are dampened.

I also study the economy in which the creditor is restricted to use state-independent repayment. To compensate the borrower for the welfare loss resulting from such a suboptimal contracting the lender's, required rate of return should decrease by 0.10%.

As a policy implication, this paper suggests that the lenders (including the IMF) should be more concerned with how their funds are spent and not with securing their repayment. Thus, increased surveillance of the execution of the loans is expected to increase the borrower's welfare.

A Appendix

A.1 Useful lemmas

In this section I prove a set of lemmas that are used to prove main results in this paper.

Lemma 2. *Let $b(Q)$ be the loan specified under optimal contract in the Atkeson economy when the borrower's net worth is Q . Then $V_{Atk}(Q) \geq V_{aut}(Q + b(Q))$ for all sustainable Q . Exact equality holds only when all enforcement constraints bind at $Q : V_{Atk}(Y_i - d_i(Q)) = V_{aut}(Y_i), \forall i \in \{1, \dots, n\}$ implies $V_{Atk}(Q) = V_{aut}(Q + b(Q))$.*

Proof. The result follows directly from comparison of the two programming problems. So let $(b(Q), d(Q))$ be optimal contract in the Atkeson economy (with moral hazard and limited enforcement) when agent's net worth is Q . Next observe that

$$\begin{aligned} V_{Atk}(Q) &= \max_I \left\{ u(Q + b(Q) - I) + \beta \sum_{i=1}^n g_i(I) V_{Atk}(Y_i - d_i(Q)) \right\} \\ &\geq \max_I \left\{ u(Q + b(Q) - I) + \beta \sum_{i=1}^n g_i(I) V_{aut}(Y_i) \right\} = V_{aut}(Q + b(Q)). \end{aligned}$$

To prove the second part of the claim, note that when all enforcement constraints bind at Q then values in the first and the second lines are equal. \square

Lemma 3. *If $V_{Atk}(Q) = V_{aut}(Q')$ for some sustainable levels of net worth Q, Q' then $c_{Atk}(Q) \leq c_{aut}(Q')$. Exact equality holds only when all enforcement constraints bind at $Q : V_{Atk}(Y_i - d_i(Q)) = V_{aut}(Y_i), \forall i \in \{1, \dots, n\}$.*

Proof. Suppose on the contrary that consumption in autarky is lower than in the economy with moral hazard and limited enforcement: $c_{Atk}(Q) > c_{aut}(Q')$. Then for $V_{Atk}(Q) = V_{aut}(Q')$ to hold it must be true that $I_{aut}(Q') > I_{Atk}(Q)$. The latter implies

$$Q + b(Q) - I_{Atk}(Q) = c_{Atk}(Q) > c_{aut}(Q') = Q' - I_{aut}(Q')$$

or equivalently

$$Q + b(Q) - Q' > I_{Atk}(Q) - I_{aut}(Q') > 0,$$

where $b(Q)$ is the optimal loan in the Atkeson economy. But lemma 2 implies that $V_{aut}(Q + b(Q)) \leq V_{Atk}(Q) = V_{aut}(Q')$ leading to $Q + b(Q) \leq Q'$. A contradiction. \square

Lemma 4. *Suppose $V_{Atk}(Q) = V_{aut}(Q')$ for some sustainable Q, Q' . Then $V_{Atk}(Q + x) > V_{aut}(Q' + x)$ for all $x > 0$.*

Proof. First, $c_{Atk}(Q) \leq c_{aut}(Q')$ by lemma 2. Then, by concavity of utility function u and the envelope theorem $V'_{aut}(Q') = u'(c_{aut}(Q')) \leq u'(c_{Atk}(Q)) < V'_{Atk}(Q)$.²⁷ Then it must be true that $V_{Atk}(Q + x) > V_{aut}(Q' + x)$ for all $x > 0$ as otherwise there must exist $y \in (0, x]$ such that $V'_{aut}(Q' + y) \leq V'_{Atk}(Q + y)$ and $V_{aut}(Q' + y) = V_{Atk}(Q + y)$. A contradiction. \square

Define \bar{d}_i to be the maximum repayment in state i :

$$V_{Atk}(Y_i - \bar{d}_i) = V_{aut}(Y_i) \quad (\text{A.1})$$

Lemma 5. $\bar{d}_i < \bar{d}_{i+1}, \forall i$.

Proof. Suppose, on the contrary, that $\bar{d}_i \geq \bar{d}_{i+1}$ for some i . Then using lemma 4 it follows that

$$V_{aut}(Y_{i+1}) < V_{Atk}(Y_{i+1} - \bar{d}_i) \leq V_{Atk}(Y_{i+1} - \bar{d}_{i+1}) = V_{aut}(Y_{i+1}).$$

A contradiction. \square

Proposition 4. *No borrowing can be sustained in the Atkeson economy: $\bar{d}_i = 0, \forall i$.*

Proof. Consider $Q_m = \inf\{Q : Q + b(Q) \geq 0\}$. Note that $\lim_{Q \rightarrow Q_m^+} c_{Atk}(Q) = \lim_{Q \rightarrow Q_m^+} I_{Atk}(Q) = 0$ and $\lim_{Q \rightarrow Q_m^+} d_i(Q) = \bar{d}_i$. Note also that $\sum_{i=1}^n \Delta g_i \bar{d}_i > 0$ cannot hold as it would be possible to provide the borrower with a loan above $\beta \sum_{i=1}^n g_i(0) \bar{d}_i$ and improve the borrower's welfare because $\lambda'(0) = +\infty$.²⁸ But $\sum_{i=1}^n \Delta g_i \bar{d}_i = 0$ can hold only when $\bar{d}_i = d, \forall i$. But this in turn contradicts lemma 5 unless $d = 0$. \square

²⁷By the envelope theorem $V'_{Atk}(Q) = u'(c) - \mu u''(c) > u'(c)$, where $\mu \geq 0$ is the Lagrange multiplier on the incentives constraint. In equilibrium $\mu > 0$ for observed levels of net worth.

²⁸Recall that $\frac{d[\beta \sum_i g_i(I) d_i]}{dI} = \beta \lambda'(I) \sum_i \Delta g_i d_i$.

A.2 Optimal contract in Arrow-Debreu economy

To solve optimization problem (AD) one has to maximize Lagrangean

$$\begin{aligned} \mathcal{L} = & u(Q + b - I) + \beta \sum_{i=1}^n g(Y_i|I) V_{AD}(Y_i - d_i) \\ & + \kappa \cdot (\beta_c \sum_{i=1}^n g(Y_i|I) d_i - b) + \phi \cdot (M - b) + \beta \sum g(Y_i|I) \psi_i(d_i + M) \end{aligned}$$

with respect to current controls (I, b, d) and minimize with respect to Lagrange multipliers κ, ϕ, ψ .

First order necessary conditions and the envelope theorem imply the following optimality condition

$$u'(C) = V'_{AD}(Q) = V'_{AD}(Y_i - d_i) - \phi + \psi_i. \quad (\text{A.2})$$

Unconstrained decisions: $\phi = \psi_i = 0$. Equation (A.2) implies $Y_i - d_i = Q, \forall i$, *i.e.* the borrower maintains constant net worth. Investment is equal to the unconditionally optimal level I^* . Borrower issues $d_i = Y_i - Q$ Arrow securities paying off in state i at price $\beta g(Y_i|I^*)$ and obtains loan $b = \beta \sum_i g(Y_i|I^*) d_i = \beta (\sum_i g(Y_i|I^*) Y_i - Q)$. Optimal value to the borrower is

$$V_{AD}(Q) = u((1 - \beta)Q + \beta \sum_{i=1}^n g(Y_i|I^*) Y_i - I^*). \quad (\text{A.3})$$

Now it is possible to solve for the subset of \mathcal{Q} on which decisions are unconstrained, *i.e.* $\phi_b = \phi_{s_i} = 0$. Lower bound Q_1 is the smallest level of net worth for which $b \leq M$

$$b = \beta \left(\sum g(Y_i|I^*) Y_i - Q_1 \right) = M \quad \rightarrow \quad Q_1 = \sum g(Y_i|I^*) Y_i - M/\beta.$$

Upper bound Q_2 is the largest level of net worth for which $d_1 \geq -M$

$$d_1 = Y_1 - Q_2 = M \quad \rightarrow \quad Q_2 = Y_1 + M.$$

Next I solve for optimal decisions outside $[Q_1, Q_2]$.

Constrained borrowing: $\phi > 0, \psi_i = 0$.

For $Q < Q_1$, loan amount is constrained at M . Agent's net worth is constant across states $Y_i - d_i = Q', \forall i$, some $Q' \in (Q, Q_1]$. Given this optimal repayment schedule must satisfy

$$d_i = Y_i - \sum_{j=1}^n g(Y_j|I) Y_j + M/\beta.$$

Optimal investment solves

$$I = \arg \max_{I \in [0, Q+M]} u(Q + M - I) + \beta V_{AD} \left(\sum_{j=1}^n g(Y_j|I) Y_j - M/\beta \right),$$

and is an increasing function of the borrower's net worth. From first-order condition to the above problem it follows that $I(Q) < I^*$. In turn, the borrower's net worth monotonically increases over time and asymptotically approaches Q_1 .

Constrained savings: $\phi = 0, \psi_i > 0$ for some or all i .

Suppose that for some $k \geq 1, \psi_i > 0$ for all $i \leq k$. Then agent $d_i = -M, \forall i \leq k$ and $d_i = Y_i - Q, \forall i > k$. Optimal investment solves

$$I = \arg \max_{I \in [0, Q+b]} \left\{ u(Q + b - I) + \beta \sum_{j=1}^k g(Y_j|I) V_{AD}(Y_j + M) \right\} / \left(1 - \beta \sum_{j=k+1}^n g(Y_j|I) \right)$$

subject to

$$b = \beta M \sum_{j=1}^k g(Y_j|I) + \beta \sum_{j=k+1}^n g(Y_j|I) (Y_i - Q).$$

From first-order condition to the above problem it follows that optimal investment $I(Q)$ is an increasing function of net worth and is greater than I^* . In this region, the borrower's net worth monotonically decreases over time and asymptotically approaches Q_2 .

A.3 Optimality condition in the Atkeson economy

To solve optimization problem (ATK) one has to maximize Lagrangean

$$\begin{aligned} \mathcal{L}_{Atk} = & u(Q + b - I) + \beta \sum_{i=1}^n g(Y_i|I) V_{Atk}(Y_i - d_i) \\ & + \kappa \left(\beta \sum_{i=1}^n g(Y_i|I) d_i - b \right) + \phi \cdot (M - b) + \beta \sum g(Y_i|I) \psi_i \cdot (d_i + M) \\ & + \mu \left(-u'(Q + b - I) + \beta \lambda'(I) \sum_{i=1}^n \Delta g_i V_{Atk}(Y_i - d_i) \right) \\ & + \beta \sum_{i=1}^n g(Y_i|I) \gamma_i \left(V_{Atk}(Y_i - d_i) - V_{aut}(Y_i) \right) \end{aligned}$$

with respect to current controls (b, d_1, \dots, d_n) and minimize with respect to Lagrange multipliers $\kappa, \mu, \gamma, \phi, \psi \geq 0$.

The first order necessary conditions and the envelope theorem imply

$$\begin{aligned} V'_{Atk}(Q) &= u'(Q + b - I) - \mu u''(Q + b - I) \\ &= V'_{Atk}(Y_i - d_i) \left(1 + \gamma_i + \mu \frac{\lambda'(I) \Delta g_i}{g(Y_i|I)} \right) + \phi - \psi_i. \end{aligned} \quad (\text{A.4})$$

The optimality condition in the economy with moral hazard is

$$\begin{aligned} V'_{MH}(Q) &= u'(Q + b - I) - \mu u''(Q + b - I) \\ &= V'_{MH}(Y_i - d_i) \left(1 + \mu \frac{\lambda'(I) \Delta g_i}{g(Y_i|I)} \right) + \phi - \psi_i. \end{aligned} \quad (\text{A.5})$$

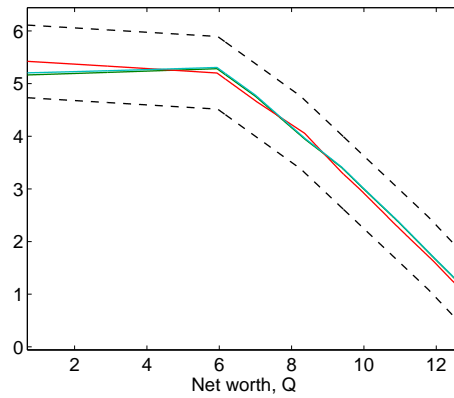
The optimality condition in the economy with only limited enforcement is

$$V'_{LE}(Q) = u'(Q + b - I) = V'_{LE}(Y_i - d_i)(1 + \gamma_i) + \phi - \psi_i. \quad (\text{A.6})$$

A.4 Optimal Repayment Schedule in Moral Hazard Economy

This plot demonstrates that there is very little state contingency built into the optimal contract. Hence, it may be reasonably well approximated by a state-independent amount.

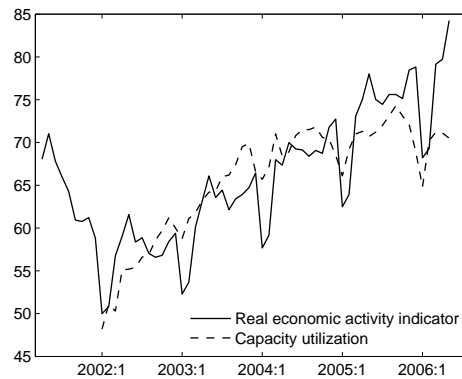
Figure 6: Optimal repayment schedule in the economy with moral hazard. Dashed lines denote the upper and lower bound on repayment in the complete markets economy.



A.5 Capacity Utilization

Figure 7 plots capacity utilization and real activity indicator (at constant 1993 prices). The activity indicator is scaled to equal 50% in January 2002.

Figure 7: Capacity utilization and economic activity indicator.



A.6 Distribution of Output

Figure 8 shows distribution of detrended log real GDP in Argentina during 1980:Q1–2001:Q4. The very long left tail is an indication of non-normal distribution. Indeed Jarque-Bera test of normality rejects the null of normal distribution at a 1% significance level.²⁹ This plot demonstrates that the distribution of output is skewed. Skewed output distribution is also obtained in the economy with moral hazard.

²⁹The sample contains only 88 observations; thus, as a check I also used Lilliefors test and obtained the same result.

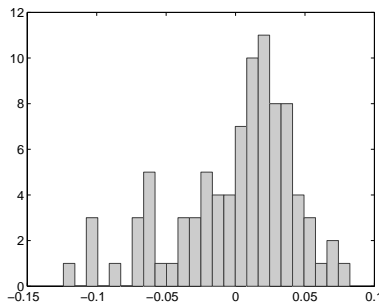


Figure 8: Distribution of log-output in Argentina, 1980-2001

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