Stable Partnerships, Matching, and Local Public Goods

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Abstract In the presence of local public goods differences in tastes are an important determinant of the way in which partnerships are formed. Heterogeneity in tastes for private vs. public goods produces a tendency to positive assortment and partnerships of couples with similar tastes; heterogeneity in tastes for different public goods brings about partnerships of couples with similar tastes only if there is a significant overlap in the distribution of tastes of the two groups to be matched. We show that with two public goods we may get negative assortment, pure positive assortment being only one of many possibilities.

Keywords: Matching; sorting; local public goods; heterogeneity of tastes.

JEL Classification: C78; D13; D61; H41

1 Introduction

This paper analyses the role of local public goods in partnership formation. We set up a simple model of matching, and use it to examine the extent to which similar types are paired with each other. Our principal motivation is to look at partnerships between men and women who set up household together, but our model is sufficiently general that it may be applied to a wide range of forms of partnership.

The economic basis of family formation has long been a subject of academic inquiry. The pioneering work of Becker (1965, 1981) provides the starting point for most modern analysis. Becker studied the family as an economic institution that provides efficiency gains in production and consumption. It allows for specialisation according to comparative advantage, so that the combined production of a man and a woman allows both to be better off than if they were single. Building on Becker's notion of a household production function, Manser and Brown (1980) and McElroy and Horney (1981) explicitly consider the possibility of joint consumption within the household as a basis for living together rather than separately. Since these early works, there has been an extensive literature on household public goods; see the discussions in Bergstrom (1997) and Weiss (1997).

Becker also examined issues of matching, asking which men form partnerships with which women, and how can we characterise the matching (the set of partnerships) formed by a given population of men and women. Here there are two kinds of question. Firstly, which matchings, if any, are *stable*, and what are the properties of the set of stable matches? Secondly, does the outcome of partnership formation leads to *assortative mating* e.g. do more handsome men marry more beautiful women? Do richer women marry richer men? Do men who like sport on TV marry women who like to do the gardening?

Models with household public goods rest on a very simple hypothesis: cohabitation offers very substantial economies of scale in consumption. Many goods are in effect jointly consumed. Having been purchased (or produced within the household) consumption by one partner does not reduce the amount available to the other; for example, heating and lighting in the house, or the living space of the house itself. Much housework benefits both partners, such as gardening and cleaning. To cook for two barely involves more effort than cooking for one. The appearance of the house, the result of expenditure on household effects, decor, carpets, and furniture, is a good that is public to those who inhabit the house. Bergstrom (1997) cites shared automobile trips as a form of collective consumption. More substantively, and possibly touching on the one issue that makes a partnership between a man and a woman more than mere cohabitation, children are (or generate) public goods; this is the central theme of Weiss and Willis (1985). Children consume resources, but if both parents take pleasure in a child's happiness then consumption by the child is, indirectly, a local public good. The care and nurture of the child typically benefits both parents, as do resources invested to increase the child's life expectancy, future earnings and chances of a good and happy life.

The arguments above provide a microfoundation for the assertion that cohabitation allows two people to be both better off, but say nothing about who lives with whom. To look at issues of matching and assortment, we introduce two possible sources of heterogeneity. Firstly, individuals may differ in the strength of their preferences for public goods relative to private goods. For example, in a world with one private good (wine) and one public good (roses), both partners in a union may prefer, *ceteris paribus*, a garden with more rather than fewer roses but they may differ in their willingness to forego wine for roses; i.e. for the same *{wine, roses}* consumption vector, they have different marginal rates of substitution. Secondly, even if all goods are public, individuals may differ in their preferences for one public good relative to another. In a world of roses and children, one partner may prefer to devote more of the households resources to tending the garden to produce a prize crop of roses, the other would prefer to spend more on the children. In order to isolate the effects of differences in tastes, we suppress other forms of heterogeneity; in particular we assume no differences in incomes. Lam (1988) has argued that income differences combined with the possibility of collective consumption encourage positive assortment by income, as richer people will normally want more household public goods, thus benefitting their partner.

It is worth emphasising that in our model heterogeneity of tastes has bite largely because some goods are public. In any partnership, the couple must agree on an allocation of resources, which will in general be partly determined by their tastes. Most models of intra-household allocation would predict that the more one partner, e.g. the man, likes good X compared to good Y, the more of good X he will consume and the less of good Y. If X is a public good then this will directly benefit the woman; and if Y is private there is no offsetting cost to her. In this case she is better off to form a partnership with a man who has a strong preference for public rather than private goods. But if Y is a second public good, then there is an offsetting cost, and the net effect will depend on her preferences for the two goods; in this case she is better off to form a partnership with a man whose tastes for different public goods are similar to hers. Note that if both X and Y are private goods then changes in the man's tastes will have little effect on her consumption or utility.

The structure of the rest of the paper is as follows: Section 2 sets out the simplest possible analytical framework by assuming two goods, and derives some basic results using a simple model of intra-household allocation in the presence of public goods; Section 3 embeds this in a matching framework, using a theorem on the uniqueness of two-sided matching; Section 4 looks at assortative mating with one private and one public good; Section 5 examines the case when there are no private and two public goods; Section 6 discusses possible extensions of the basic model and Section 7 concludes.

2 The analytical framework with two goods

We begin by considering a partnership consisting of a man and a woman, subscripted a and b respectively. There are 2 goods; good X is public, and good Y is either public or private, depending on which version of the model is being considered.

The man has a utility function

$$u_a = \alpha \log(x) + (1 - \alpha) \log(y_a)$$

and the woman has a utility function

$$u_b = \beta \log(x) + (1 - \beta) \log(y_b)$$

where x and y_i denote the consumption by i of goods X and Y respectively $(i = a, b)$.

We adopt the simplest possible approach to household bargaining and allocation. Although some writers have argued for a non-cooperative approach to household bargaining, e.g. Lundberg and Pollock (1994) and Konrad and Lommerud (1995), repeated interaction and the ability to monitor a partner's actions suggest that it is reasonable to assume a cooperative and hence efficient outcome. More precisely, we assume that the household allocation maximises $u_a + u_b$; the resultant utilities are denoted v_a and v_b, We justify this approach principally on the grounds that it provides a simple and tractable model[1]. However, it is important to note that it

[1] Alternatively we could have assumed that the couple agree that their respective utilities should be accorded equal weight in either a utilitarian Bergson-Samuelson social welfare function (SWF) $W(u_a, u_b) = u_a + u_b$ or in a multiplicative SWF of monotonically transformed utilities $\widetilde{W}(u_a, u_b) = \exp(u_a)\exp(u_b)$. For those wedded to the idea that all allocations should have a strategic or game theoretic foundation, $\widetilde{W}(u_a, u_b)$ is also the maximand in a Nash Bargain with utility functions $\exp(u_a)$ and $\exp(u_b)$ and a disagreement point of zero utilities; this in turn can be grounded in a Rubinstein (1981) framework of alternating offers without outside options and in which until agreement is reached each partner consumes nothing out of family resources and hence receives zero utility.

assumes that the household allocation, and hence v_a and v_b, are unaffected by any outside opportunities either partner may have, such as pairing up with someone else. In effect, once a partnership is formed, utility is non-transferable within the household and there is no possibility of altering the terms of the relationship.

2.1 One public good and one private good

If X is public and Y is private then the family budget constraint is

$$x + y_a + y_b = 2R, \tag{1}$$

subject to which x, y_a,and y_b jointly maximise

$$u_a + u_b = (\alpha + \beta)\log(x) + (1 - \alpha)\log(y_a) + (1 - \beta)\log(y_b) \tag{2}$$

Here R is the resource brought into the partnership by each partner. Goods X and Y can each be bought (or produced) at a constant average price (or cost), both of which by choice of units are taken to be 1. Note that this formulation assumes constant returns to scale in the production or purchase of the goods X and Y. Perhaps more importantly, there are no differences between the man and women in their ability to earn income or to produce either of the goods, so their resources can be added together to produce a single family budget constraint. The only difference between the two partners is their tastes.

The household allocation is thus given by

$$\begin{aligned} x &= (\alpha+\beta)R \\ y_a &= (1-\alpha)R \\ y_b &= (1-\beta)R \end{aligned}$$

and the resultant indirect utilities are

$$\begin{aligned} v_a(\beta) &= \alpha\log(\alpha+\beta)+(1-\alpha)\log(1-\alpha)+\log(R) \\ v_b(\alpha) &= \beta\log(\alpha+\beta)+(1-\beta)\log(1-\beta)+\log(R \end{aligned}$$

Note that v_a is increasing in β, and v_b is increasing in α. Each spouse is better off the stronger is the preference of their partner for the public good. This is illustrated in Figure 1, which shows how v_a varies with β for $\alpha = 0.3, 0.5$, and 0.9 for $R = 1$. Note that each partner is better off paired than single as long as α and β are both positive; only if either is zero is there no gain to living together.

[Figure 1 near here]

2.2 Two public goods

If X and Y are both public then $y_a = y_b = y$, and x and y jointly maximise

$$\alpha\log(x)+(1-\alpha)\log(y)+\beta\log(x)+(1-\beta)\log(y) \tag{3}$$

$$\text{subject to } \; x+y=2R$$

The household allocation is now given by

$$x = (\alpha + \beta)R$$
$$y = (2 - \alpha - \beta)R$$

and the resultant indirect utilities are

$$v_a(\beta) = \alpha \log(\alpha + \beta) + (1 - \alpha) \log(2 - \alpha - \beta) + \log(R)$$
$$v_b(\alpha) = \beta \log(\alpha + \beta) + (1 - \beta) \log(2 - \alpha - \beta) + \log(R$$

With two public goods, each individual is better off the closer are their partner's preferences to their own. To see this note that if the husband, for example, were to set x and y to maximise u_a he would choose $x = 2\alpha R$ and $y = 2(1 - \alpha)R$, which only maximises $u_a + u_b$ when $\alpha = \beta$. Thus the husband's utility, v_a, is greater the closer is β to α. This is illustrated in Figure 2, which shows, for $R = 1$, how v_a varies with β for $\alpha = 0.3, 0.5$, and 0.9. Note that there is no gain to living together only if $|\alpha - \beta| = 1$.

[Figure 2 near here]

2.3 Preferences and the no crossing condition

The indirect utility functions v_α and v_β allow us to analyse how an individual might rank, and be ranked by, different potential partners. Whether we consider one or two public goods, these functions have the following implication: consider two men, m_1 and m_2, and two women, w_1 and w_2, with coefficients on X of $\alpha_1, \alpha_2, \beta_1$, and β_2

respectively. Assume without loss of generality that $\alpha_1 \leq \alpha_2$ and $\beta_1 \leq \beta_2$. Then we can rule out the possibility that m_1 prefers w_2 and at the same time m_2 prefers w_1. Figure 3 illustrates, and suggests that this property may be described as a *no crossing condition.* The preferences of the women obey a similar condition, *mutatis mutandis.*

[Figure 3 near here]

In the case of one public good, the source of this result is obvious, as both men will prefer w_2. With two public goods, the function v_α embodies an increasing disutility the further are the tastes of a man's partner from his own. Formally, if m_1 prefers w_2 then the difference $v_{\alpha_1}(\beta_2) - v_{\alpha_1}(\beta_1)$ must be positive, and since $dv_\alpha/d\beta$ is increasing in the parameter α then *a fortiori* m_2 must prefer w_2.

3 Matching

The question of two-sided matching has long excited economists, and a number of results have been established (for a comprehensive survey see Roth and Sotomayor, 1990). In the standard formulation, there are two finite and disjoint sets, M (men) and W (women), both of size n. Each man has preferences over all the women in W and would rather be matched with any woman than remain single; similarly each woman has preferences over all the men in M and would rather be matched with any man than remain single. These preferences are strict (so that no-one is indifferent between two members of the opposite sex) and transitive. Thus we can think of the preferences of each man (resp. woman) as a ranking of the elements of the set W (resp. M).

A *matching* μ of M and W is a one-to-one function from the set $M \cup W$ onto itself such that (i) $m = \mu(w)$ if and only if $w = \mu(m)$; (ii) if $m \in M$ then $\mu(m) \in W$ and if $w \in W$ then $\mu(w) \in M$ (no same sex matches). A matching μ can be *blocked* by a pair (p, q) for whom $p \neq \mu(q)$ if p prefers q to $\mu(p)$ and q prefers p to $\mu(q)$. A matching μ is *stable* if it cannot be blocked by any pair.

For the matching problem outlined above a stable matching always exists (see Gale and Shapley, 1962). However, without imposing any further structure on the problem, there is no reason to suppose that there is a unique stable matching. Indeed, a considerable amount of effort has been spent exploring the set of stable matchings seen as a *lattice*. One way to provide further structure is to specify agents' preferences in more detail, and of course this is exactly the subject of Section 2. What emerges from that analysis is that the preferences of Section 2 satisfy a no crossing condition. Formally:

Definition 1 *A population $P = M \cup W$ satisfies the No Crossing Condition if there exists an ordering $\{m_1, m_2, ..., m_n\}$ of M and an ordering $\{w_1, w_2, ..., w_n\}$ of W such that for $i < j$ and $k < l$*

(i) *if w_i prefers m_l to m_k then w_j prefers m_l to m_k*

(ii) *if w_j prefers m_k to m_l then w_i prefers m_k to m_l*

(iii) *if m_k prefers w_j to w_i then m_l prefers w_j to w_i*

(iv) *if m_l prefers w_i to w_j then m_k prefers w_i to w_j.*

We now have:

Theorem 1 *If preferences satisfy the no crossing condition, there is a unique stable matching.*

Proof. See Clark (2002). ■

An important component of the proof is this: for $m \in M$ and $w \in W$, if m prefers w out of all the women in W and w prefers m out of all the men in M, then m and w must be partners in all stable matchings of M and W i.e. they are a *fixed pair* of the population $M \cup W$. It follows that any stable matching μ of M and W must consist of m being matched with w, plus some stable matching μ' of $M' = M \setminus \{m\}$and $W' = W \setminus w$. It is always possible to find a fixed pair for any population whose preferences satisfy the no crossing condition. Hence one can find the unique stable matching by finding a fixed pair (m, w) for the population $M \cup W$, then finding a fixed pair (m', w') for the population $M' \cup W'$, and so one until the population is exhausted (perhaps literally). This constructive procedure will prove useful in the next section.

4 Matching with one public good and one private good

We begin by considering a population of n men and n women, each with preferences of the type analysed in Section 2.1. Let the men and women be labelled $m_1, m_{2,}...m_n$ and $w_1, w_2, ...w_n$ such that $\alpha_1 < \alpha_2 < ... < \alpha_n$,and $\beta_1 < \beta_2 < ... < \beta_n$. Each man would most like to be matched with w_n and each woman would most like to be matched with m_n. Thus m_n and w_n are a fixed pair for the population $M \cup W$; similarly, m_{n-1} and w_{n-1} are a fixed pair for the population $(M \setminus \{m_n\}) \cup (W \setminus \{w_n\}), m_{n-2}$ and w_{n-2} are a fixed pair for the population $(M \setminus \{m_{n-1}, m_n\}) \cup (W \setminus \{w_{n-1}, w_n\})$, and so on. Thus the unique stable matching of M and W pairs m_i with w_i, for $i = 1, 2, ...n$. This is as clear an example of positive assortment as one could wish for, but note that it arises from a desire to be matched not with someone of similar tastes, but with someone of extreme tastes.

5 Matching with two public goods

We again consider a population of n men and n women, now with preferences of the type analysed in Section 2.2. We continue to assume that $\alpha_1 < \alpha_2 < ... < \alpha_n$,and $\beta_1 < \beta_2 < ... < \beta_n$. The conditions of Theorem 1 still hold so that there is a unique stable matching, and the constructive proof shows how to find it. However, there are no simple results on the nature of the stable matching comparable to those of the previous section. To fix ideas, suppose that $n = 2$, with $\alpha_1 = 0.25$, $\alpha_2 = 0.5$, $\beta_1 = 0.5$, $\beta_2 = 0.75$. Then m_2 and w_1 form a fixed pair, leaving m_1 and w_2 as a couple, even though they have extremely different tastes. Figure 4 illustrates.

[Figure 4 near here]

Figure 4 shows negative assortment, even though preferences are for partners with similar tastes. If we change the women's preferences so that $\beta_1 = 0.35$ and $\beta_2 = 0.6$ then m_1 and w_1 form a fixed pair, as do m_2 and w_2. Figure 5 illustrates.

[Figure 5 near here]

5.1 A parameterisation of the distribution of tastes

The two examples above suggest that the stable matching is likely to be sensitive to the precise distribution of tastes among men and women. To explore this further when there are more than two couples, we put more structure on the distribution of preferences.

Let $A = \{\alpha_1, \alpha_2, ... \alpha_n\}$, and $B = \{\beta_1, \beta_2, ... \beta_n\}$; and let N be a finite number such that $N > n - 1$.We now assume that $\alpha_i = \alpha_1 + \frac{i-1}{N}$ and $\beta_i = \beta_1 + \frac{i-1}{N}, i = 2, ..n$; thus α and β are *evenly distributed.* Since $\beta_i = \alpha_i + d$, where $d = \beta_1 - \alpha_1$, we can think of A and B as displacements of each other, overlapping only if $0 \leq d \leq \alpha_n - \alpha_1 = (n-1)/N$ or $0 \leq -d \leq \beta_n - \beta_1 = (n-1)/N$.

If $d = 0$, then m_i is paired with w_i $(i = 1, 2, ..., n)$ and we have perfect positive assortment. Figure 6 illustrates when $n = 5, N = 10$, and $\alpha_1 = \beta_1 = 0.3$.

[Figure 6 near here]

Figure 7 shows what happens of we displace B by an amount $d = 0.3$.

[Figure 7 near here]

In this case the fixed pairs are (m_4, w_1) and (m_5, w_2); once these are paired off, it is straightforward to see that matches are made by (m_3, w_3), (m_2, w_4), and (m_1, w_5).Now we have both positive and negative assortment. Within the overlapping subsets of A

and B, there is positive assortment; elsewhere there is negative assortment; the man who likes good X the least is matched with the woman who likes it the most. As we might expect, when there is no overlap between A and B then we have perfect negative assortment, as illustrated by Figure 8. Here, m_i is matched with w_{6-i}.

[Figure 8 near here]

It is straightforward to generalise these examples to cover any values of d, n, and N (as long as d is no greater than$1-\frac{n-1}{N}$, and is an integer multiple of the gap $1/N$). If the two distributions coincide ($d=0$), then we have perfect positive assortment; if there is no overlap at all ($d>(n-1)/N$) then we have perfect negative assortment. Otherwise we have a mix of positive and negative assortment. In these cases what is the overall level of assortment? One measure of this is the coefficient of variation between α and $\mu(\alpha)$, which for the case of the distributions considered here is also a measure of rank correlation:

$$R(\alpha,\mu(\alpha))=\frac{Cov(\alpha,\mu(\alpha))}{\sqrt{Var(\alpha)\;Var(\mu(\alpha))}}$$

where

$$Cov(\alpha,\mu(\alpha))=\frac{1}{n}\sum\left(\alpha_i-\overline{\alpha}\right)\left(\mu(\alpha_i)-\overline{\beta}\right),$$

$$Var(\alpha)=\frac{1}{n}\sum\left(\alpha_i-\overline{\alpha}\right)^2,$$

and

$$Var(\mu(\alpha))=Var(\beta)=\frac{1}{n}\sum\left(\beta_i-\frac{1}{n}\overline{\beta}\right)^2.$$

The precise value of $R(\alpha, \mu(\alpha))$ will in general depend on d, n, and N. For ease of exposition, let us consider the continuous approximation to the limit of the matching as n and N tend to infinity, keeping $\alpha_n - \alpha_1 = \beta_n - \beta_1 = (n-1)/N$ fixed at some value s. In effect we are taking α and β to be uniformly distributed on the intervals $M = [\alpha_1,\, \alpha_1 + s]$ and $W = [\alpha_1 + d,\, \alpha_1 + d + s]$ respectively, as illustrated in Figure 9. The matching μ, which takes an element α of M and returns $\mu(\alpha)$ in W, can be represented by the function graphed in Figure 10.

[Figure 9 near here]

[Figure 10 near here]

Elementary integration now reveals that

$$R(\alpha, \mu(\alpha)) = \frac{(s-\widetilde{d})^3}{s^3} - \frac{\widetilde{d}^3}{s^3} - \frac{3(s-\widetilde{d})s\widetilde{d}}{s^3} \tag{4}$$

$$= 2(1 - \frac{\widetilde{d}}{s})^3 - 1 \tag{5}$$

where $\widetilde{d} = \min(|d|, s)$. The three terms in 4 reflect (i) the positive covariance due to the subsets of M and W that are positively assorted; (ii) the negative covariance due to the subsets of M and W that are negatively assorted; (iii) the negative covariance between the means of the subsets of M and W induced by the matching μ (which is evident from Figure 10). Note that $R(\alpha, \mu(\alpha))$ depends only on the ratio $\widetilde{d}/s$

For $\widetilde{d}/s = 1, R(\alpha, \mu(\alpha)) = -1$; so when there is no overlap between the sets M and W, there is a perfect negative correlation between α and $\mu(\alpha)$. When $\widetilde{d} = 0$, $R(\alpha, \mu(\alpha)) = 1$, so when the sets M and W coincide there is perfect positive correlation. Note that $R(\alpha, \mu(\alpha)) = 0$ when $\widetilde{d}/s \approx \frac{1}{5}$, so a relatively small displacement between M and W will generate a zero measure of assortment, even though $\frac{4}{5}$ of the population are matched with partners of identical tastes.

If the set M representing men's tastes is a displacement of the set W, what general conclusions can be drawn about which individuals benefit from the unique stable matching? Figure 10 shows that those with 'extreme' tastes are paired with partners with very different preferences; the more 'moderate' one's tastes, the closer the tastes of one's partner; and if one is in the modal group of the population $M \cup W$ then one is perfectly matched. This suggests that households will be of two types: perfectly matched couples who agree completely on the allocation of resources, and imperfectly matched couples, who disagree. Of the latter group, the disagreement varies from the positive but mild (new wallpaper in the drawing room vs. a mobile phone for little Billy) to the potentially bitter (bigger house with a garden vs. a sibling for little Billy). Of course, even the badly matched are better off than if they remain single, but the benefits of the partnership are slight. In a more complete model of matching and separation, these would be the most vulnerable partnerships.

From a slightly different perspective, we can ask how a particular individual, e.g. a male m_i with a preference parameter α_i is affected by a shift in the distribution of women's preferences (i.e. by a change in the displacement parameter d). Clearly, if there is some woman w_j such that $\beta_j = \alpha_i$ then m_i and w_j are matched. But if $|d|$ is large enough then such a perfect match is not possible. Let $\alpha_i = \alpha_1 + t_i$ and recall that $\mu(\alpha_i)$ denotes the preference parameter of m_i's partner. Ideally m_i would like

$\mu(\alpha_i) - \alpha_i = 0$. It is simple to derive the following:

$$\mu(\alpha_i) - \alpha_i = \begin{cases} d + s - 2t_i & \text{if } d < t_i - s \\ 0 & \text{if } t_i - s < d < t_i \\ d + s - 2t & \text{if } d > t_i \end{cases} \quad (6)$$

which is illustrated in Figure 11. This shows that one cannot hope for 'almost compatibility': as d moves away from 0, there is at some point a jump from perfect compatibility to a limited but positive level of conflict of tastes.

[Figure 11 near here]

5.2 More general allocations

With two public goods, the analysis above shows that with a common maximand of $u_a + u_b$ firm results are available if we assume two even distributions of taste differing only by a displacement parameter d. Then the equilibrium matching and its degree of assortment depends in a straightforward way on the degree of overlap between the two distributions and we get negative assortment if $|d|$ is sufficiently large.

Negative assortment when individuals wish ideally to be matched with someone of similar tastes is a counter-intuitive result. It is therefore worth asking whether it is the consequence of the particular assumptions we make about matching and allocation within the household. We now drop the assumption that matched couples maximise $u_a + u_b$, and allow more general allocations, perhaps influenced by what is happening or available outside the partnership. We are not concerned here with a complete characterisation of all possible equilibria in this more general case, and we impose (rather than derive) the mild restriction that if m_i is matched with w_j and $\alpha_i \neq \beta_j$ then the share of the family budget spent on good X, σ_{ij}, lies strictly between α_i and β_j; i.e. $\sigma_{ij} = \alpha_i$ if and only if $\alpha_i = \beta_j$ (thus all partnerships in which tastes differ involve some compromise, although possibly infinitessimally little; for what might happen if there is not some compromise, see Footnote 2). The conclusions we reached above must now be qualified, but are not overturned. Suppose $d = \lambda/N$, where λ is a positive integer. Then for $1 + \lambda \leq i \leq n$, $\alpha_i = \beta_{i-\lambda}$; consider any matching and allocation in which m_i is not paired with $w_{i-\lambda}$. This allocation can be blocked by m_i and $w_{i-\lambda}$, since if they were paired they could set $\sigma_{i,i-\lambda} = \alpha_i = \beta_{i-\lambda}$, and hence both be better off. Thus in the region of overlap between the two distributions, like is matched with like and we still have positive assortment. But this implies, for the case where d is positive, that when we come to the remainder of the population we are left, as before, with a subset of men $M' = \{m_1, m_2, ..., m_\lambda\}$ with weak preferences for good X and a subset of women $W' = \{w_{n-\lambda+1}, w_{n-\lambda+2}, ..., w_n\}$ with strong preferences such that $\alpha_\lambda < \beta_{n-\lambda+1}$.

In the case where matched couples maximise $u_a + u_b$, these two subsets sort negatively, so $m_i \in M'$ matches with $w_{n+1-i} \in W'$. Moreover for each couple the share spent on good X is a simple average of the two preference parameters; hence $\sigma_{i,n+1-i} = (\alpha_i + \beta_{n+1-i})/2 = (\alpha_1 + \beta_n)/2$ for all $i \leq \lambda$. Somewhat surprisingly, then, these couples all spend the same share of household income on good X. Thus no-one would be worse off if M' and W' matched in some other way but all pairs still chose a share $(\alpha_1 + \beta_n)/2$. In particular, if M' and W' sorted positively, so that $m_i \in M'$ matched with $w_{n-\lambda+i} \in W'$, and set $\sigma_{i,n-\lambda+i} = (\alpha_1 + \beta_n)/2$ then no unmatched pair could depart from this and make either partner better off; such a matching is illustrated in Figure 12. More generally, no matching of M' and W' in which all pairs spend the same share, $(\alpha_1 + \beta_n)/2$, on good X can be blocked.

We can now put upper and lower bounds on the degree of sorting between the sets M and W as measured by $R(\alpha, \mu(\alpha))$ in the limiting case as n and N tend to infinity. Where there is overlap, like is matched with like so the first term of (4) remains the same. So does the third term, because (for positive d) men with weak preferences for good X, M', match with women with strong preferences, W'. There exists an equilibrium matching, shown in Figure 10, in which these two subsets sort negatively, which accounts for the negative sign of the second term of (4); but there is also an equilibrium matching, shown in Figure 12, in which they sort positively, thus changing the sign of the second term in (4). Overall, then, the effect of allowing couples to depart from a simple rule of maximising $u_a + u_b$ is that (4) should be replaced with

$$\frac{(s-\widetilde{d})^3}{s^3} - \frac{\widetilde{d}^3}{s^3} - \frac{3(s-\widetilde{d})s\widetilde{d}}{s^3} \leq R(\alpha, \mu(\alpha)) \leq \frac{(s-\widetilde{d})^3}{s^3} + \frac{\widetilde{d}^3}{s^3} - \frac{3(s-\widetilde{d})s\widetilde{d}}{s^3} \tag{7}$$

or

$$2(1-\frac{\widetilde{d}}{s})^3 - 1 \leq R(\alpha, \mu(\alpha)) \leq 1 - 6\frac{\widetilde{d}}{s} + 6\frac{\widetilde{d}^2}{s}\ . \tag{8}$$

Note that all values of $R(\alpha, \mu(\alpha))$ in this range can be achieved with no change in individual utility levels. It need be only the matching, and hence the degree of assortment, that changes; no individual necessarily experiences a change in his or her consumption of the two public goods. But if there any such changes, then they cannot take $R(\alpha, \mu(\alpha))$ beyond the upper and lower bounds in (8): there can be no change in matching (and hence expenditure shares and utility) in the region of overlap, given our assumption that all partnerships must involve some compromise. And however the sets M' and W' match and sort, they cannot go beyond complete negative or complete positive assortment.[2]

6 Extensions of the basic model

6.1 Uneven populations

The analysis is readily adapted to populations with unequal numbers of men and women. Without loss of generality, suppose that there are n' men and n women, with $n' > n$. The case of one public and one private good is straightforward: it will be the $n' - n$ men with the weakest preference for the public good who remain unmatched. To analyse the case of two public goods, we confine ourselves to the case where couples maximise $u_a + u_b$; we assume that both men and women's taste parameters are evenly distributed, with a gap $1/N$ between types, such that $\beta_1 = \alpha_1 + \lambda/N$ for some integer

[2] Dropping the 'some compromise' restriction opens up interesting possibilities. For example, consider any matching in which all men get their preferred allocation, so $\sigma_{ij} = \alpha_i$ for all i. The matching cannot be blocked by any unmatched pair, if this requires that both man and woman be made better off. Of course, this begs the question of why men should appear to have such favourable treatment. It is clearly an interesting area for further research.

λ. Then if $\lambda \leq 0$, so women tend to prefer good Y more than men do, it will be the $n' - n$ men with the strongest preference preference for good X who are unmatched. If $0 < \lambda < n' - n$, all women can find a perfect match, leaving unmatched the $n' - n$ men with 'extreme' parameters $\alpha_1, \alpha_2, ..., \alpha_\lambda$ and $\alpha_{\lambda+n+1}, \alpha_{\lambda+n+2}, ..., \alpha_{n'}$. If $\lambda > n' - n$, it will be the $n' - n$ men with the weakest preference for good X who remain unmatched.

6.2 More than two goods

Generalising the analysis to more than two goods presents a number of problems, which we do not attempt to solve here. With Cobb-Douglas utility functions and k goods, the vector of coefficients in each agent's utility function is an element of the $(k-1)$ dimensional simplex. Each individual is still able to rank all the members of the opposite sex, but there is a new tension not present with only two goods. A potential partner may attach a high weight to public goods in aggregate, but within the subset of public goods have tastes that are very different. If heterogeneity of tastes is more than one dimensional (e.g. private vs. public *and* public 1 vs. public 2) then it is more difficult to order the sets M and W. Even with simple distributions of tastes this seems likely to result in much a richer range of possible matchings than those discussed in this paper, and opens up interesting new questions. However, by focusing on the simplest case, we have isolated determinants of the degree of assortment which should survive generalisation to more than two goods.

7 Conclusions

The central conclusion of this paper is that in the presence of local public goods tastes are an important determinant of the way in which partnerships are formed. In the case of heterogeneity in tastes for private vs. public goods, the gains from partnership are greatest when both partners have a strong preference for the public good; this produces a tendency to positive assortment and partnerships of couples with similar tastes. This suggests that the strongest unions (and hence those least likely to break up) will be those with a high consumption of the local public good relative to private consumption.

By contrast, heterogeneity in tastes for different public goods brings about partnerships of couples with similar tastes only if there is a significant overlap in the distribution of tastes of the two groups to be matched. The strongest unions will be those in which couples agree, and only those individuals endowed with tastes that are in some sense not extreme can hope to find a partner with similar preferences. We have shown that with two public goods, we may well get negative assortment, contrary to Becker's argument that preferences are likely to be positively sorted because joint consumption encourages the matching of persons with similar preferences (Becker, 1991, pp.123-4). While Becker's assertion may find a theoretical basis in other models (for example where there exist returns to scale in the production or purchase of consumption goods), we find that pure positive assortment may be only one of many possibilities, and can sometimes be ruled out altogether.

References

Becker, G. (1965), "A theory of the allocation of time", Economic Journal, 75, 493-517.

Becker, G. (1991), A treatise on the family (enlarged edition) (Harvard University Press, Cambridge, MA).

Bergstrom, T. 1997, "A survey of theories of the family", in: Rosenzweig, M. and O. Stark, (eds), Handbook of population and family economics (Elsevier, The Netherlands).

Clark, S. (2002), "Uniqueness of equilibrium in two-sided matching", Discussion Paper, Edinburgh University. http://www.econ.ed.ac.uk/academic_02discuss.htm

Gale, D. and L. Shapley, (1962), "College admissions and the stability of marriage", American Mathematical Monthly, 69, 9-15.

Konrad, K. and K. Lommerud, (1995), "Family policy with non-cooperative families", Scandinavian Journal of Economics, 97, 581-601.

Lam, D. (1988), "Marriage markets and assortative mating with household public goods," Journal of Human Resources, 23, 462-487.

Lundberg, S. and R. Pollock (1993), "Separate spheres bargaining and the marriage market", Journal of Political Economy, 101, 988-1011.

Manser, M. and M. Brown (1980), “Marriage and household decision theory - a bargaining analysis”, International Economic Review, 21, 21-34.

McElroy, M. and M. Horney (1981), Nash-bargained decisions: towards a generalization of the theory of demand”, International Economic Review, 22, 333-349.

Roth, A. and M. Sotomayor (1990), Two-sided matching (Cambridge University Press, Cambridge).

Rubinstein, A. (1982), “Perfect equilibrium in a bargaining model”, Econometrica, 50, 97-109.

Weiss, Y. (1997), “The formation and dissolution of families: why marry? who marries whom? and what happens upon divorce?”, in: Rosenzweig, M. and O. Stark, (eds), Handbook of population and family economics (Elsevier, The Netherlands).

Weiss, Y. and R. Willis (1985), “Transfers amongst divorced couples: evidence and interpretation”, Journal of Labor Economics, 11, 629-679.

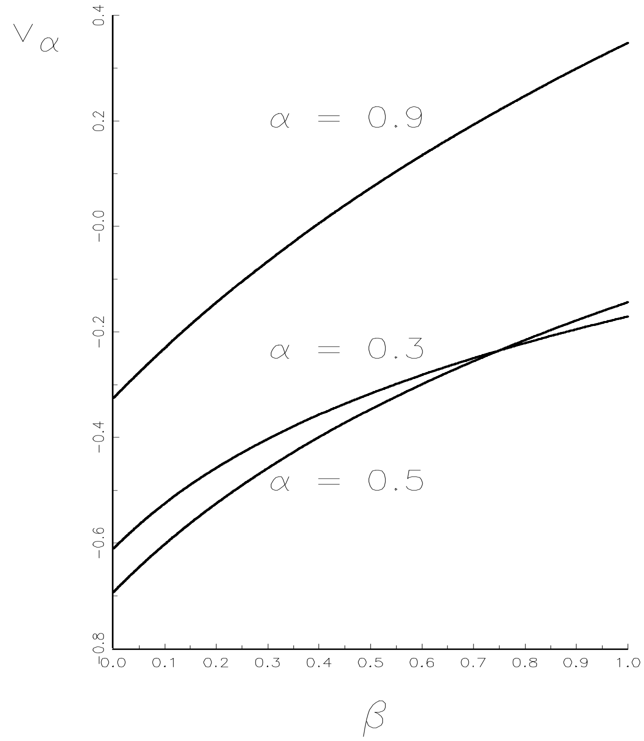

Figure 1: The effect on v_a of changes in β when there is one public good

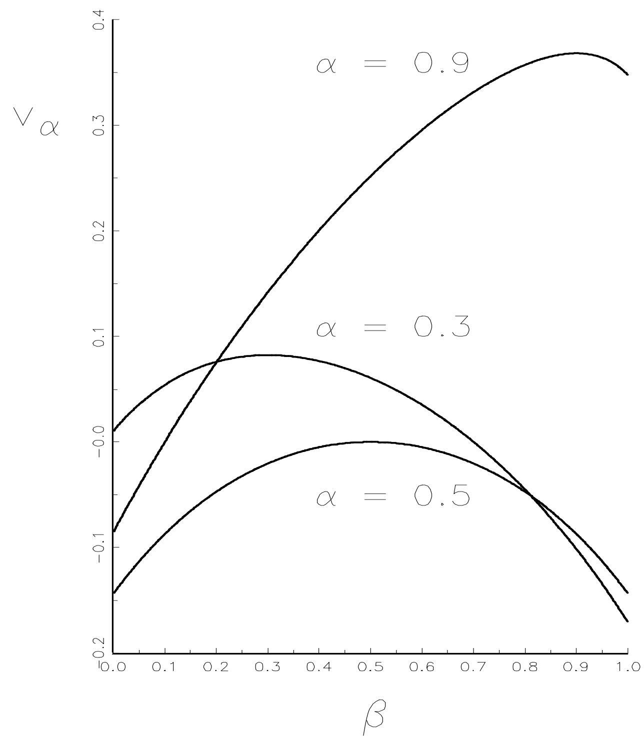

Figure 2: The effect on v_a of changes in β when there are two public goods

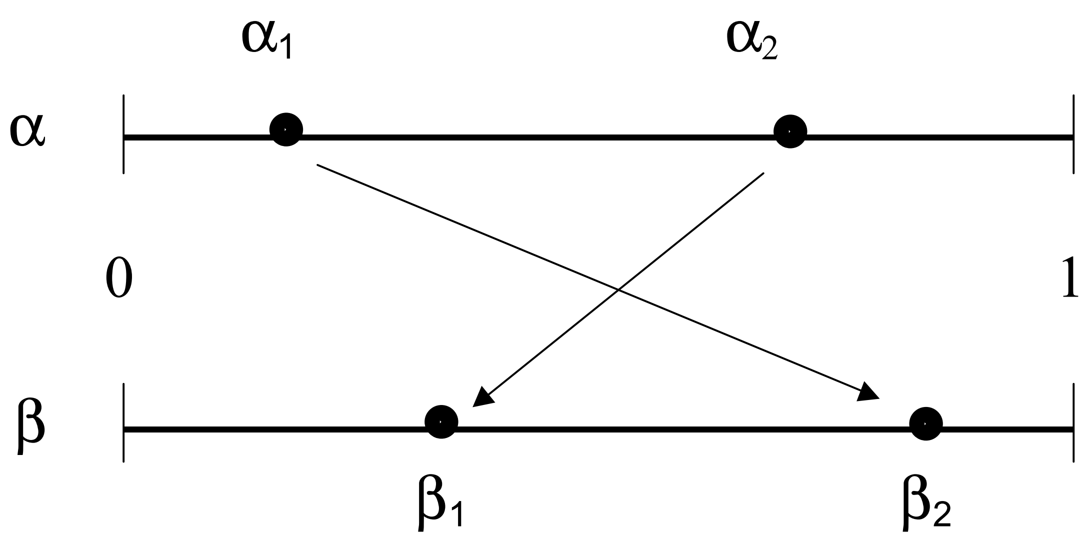

Figure 3: Preferences ruled out by the no crossing condition when $\alpha_1 \leq \alpha_2$ and $\beta_1 \leq \beta_2$. The arrows from $\alpha_1$to β_2 and from α_2 to β_1 denote that m_1 prefers w_2 and m_2 prefers w_1.

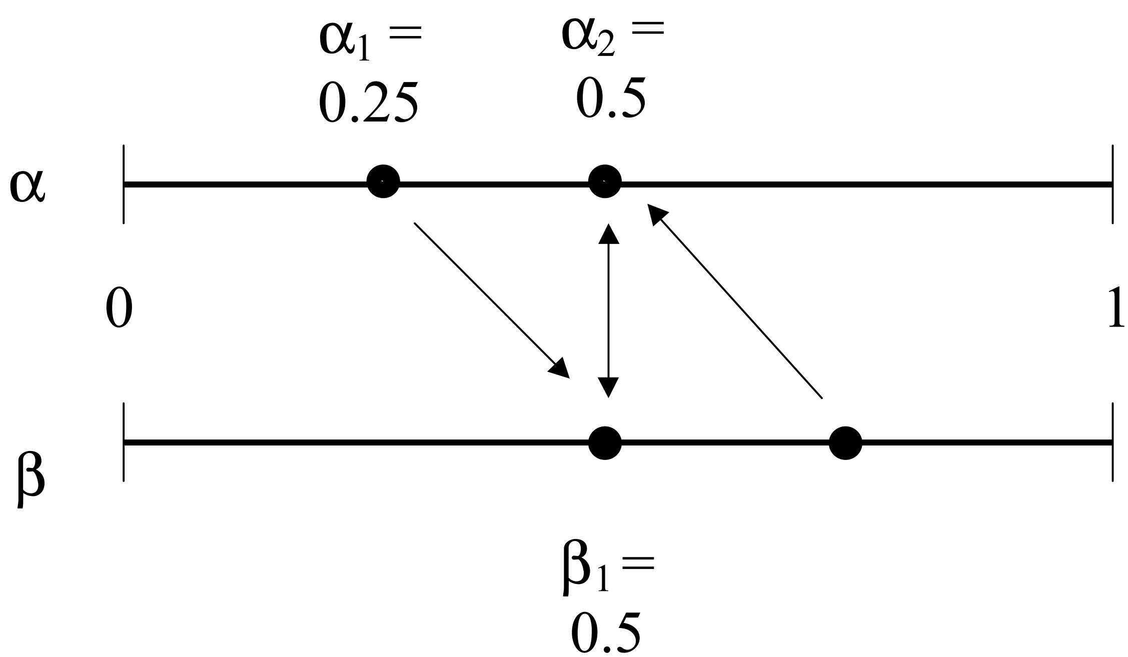

Figure 4: Matching with negative assortment. Arrows indicate preferences, and hence the double arrow between α_2 and β_1 denotes that m_2 and w_1 form a fixed pair.

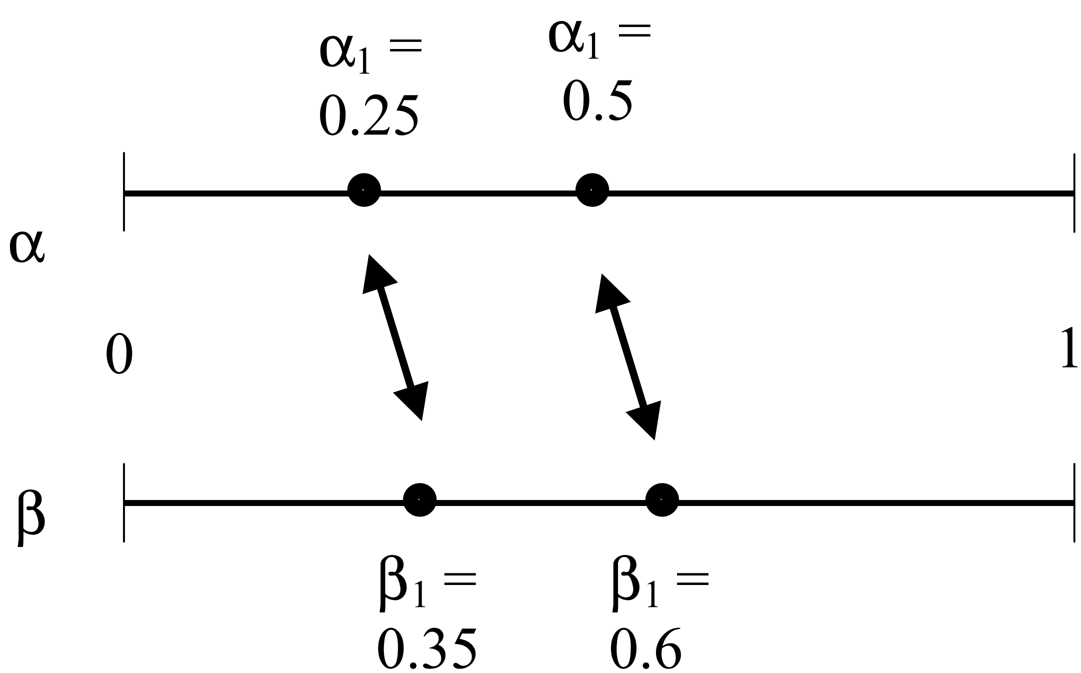

Figure 5: Matching with positive assortment. The double arrows between α_1 and $\beta_1$and α_2 and β_2 denote that m_1 and w_1, and m_2 and w_2 form fixed pairs.

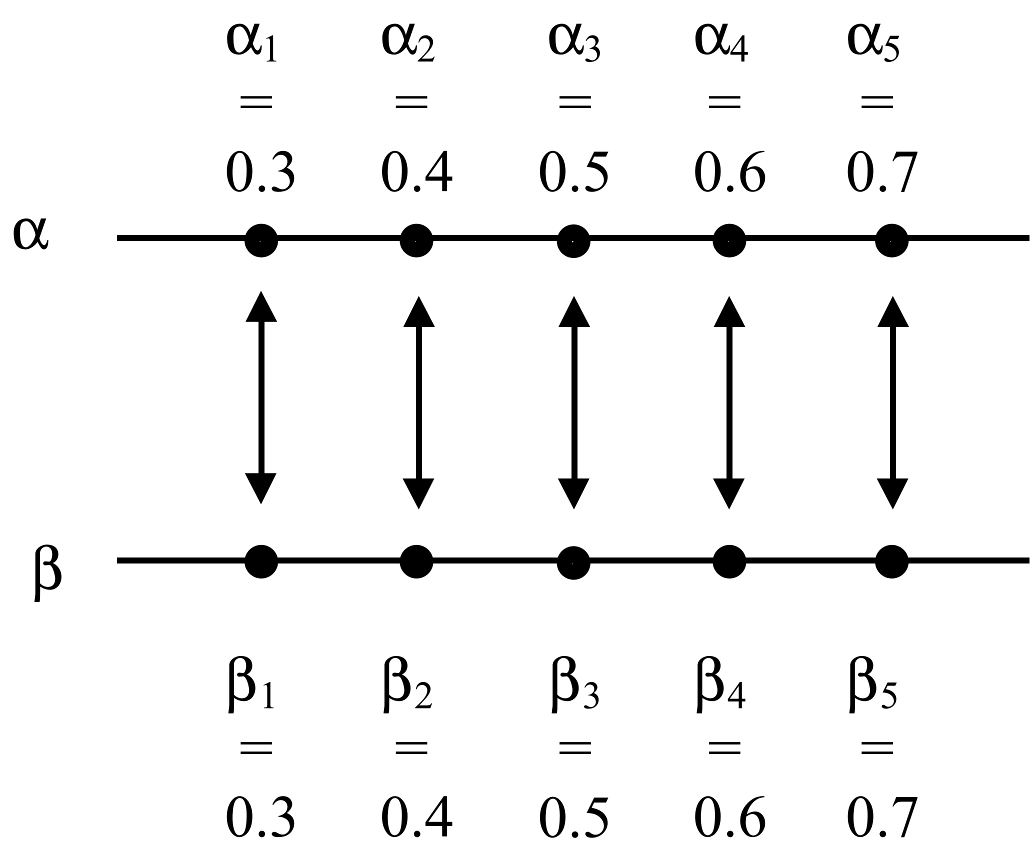

Figure 6: Matching with positive assortment. The double arrows indicate fixed pairs

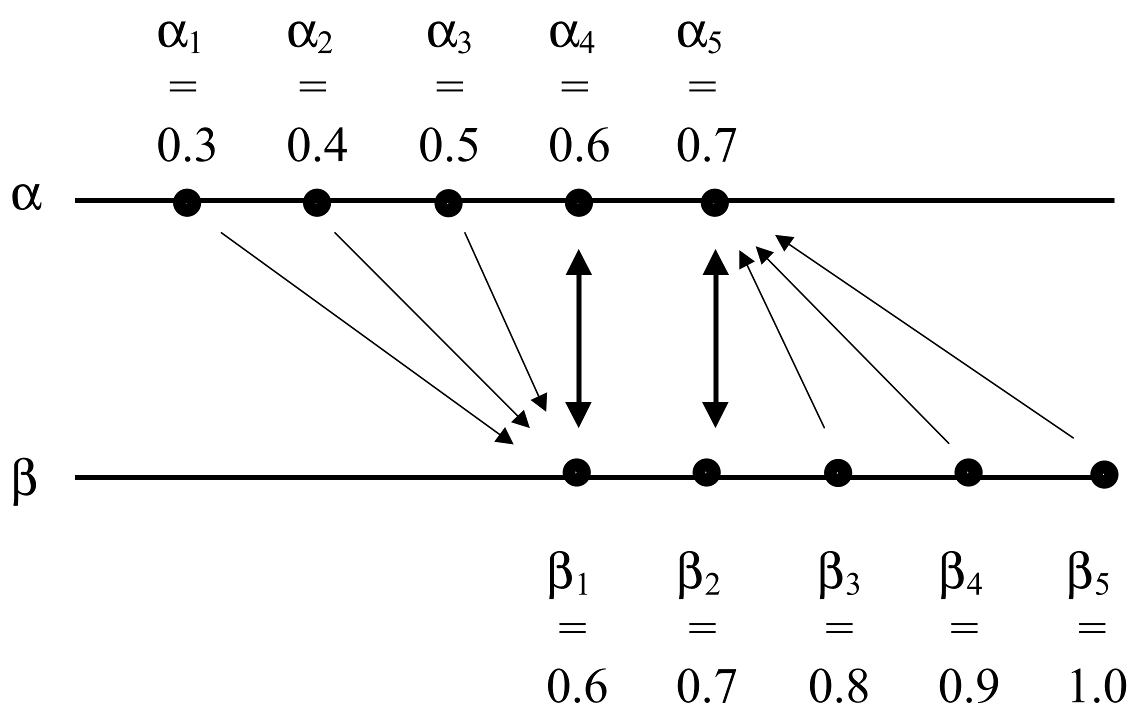

Figure 7: Matching with positive and negative assortment. Arrows denote preferences and double arrows indicate fixed pairs.

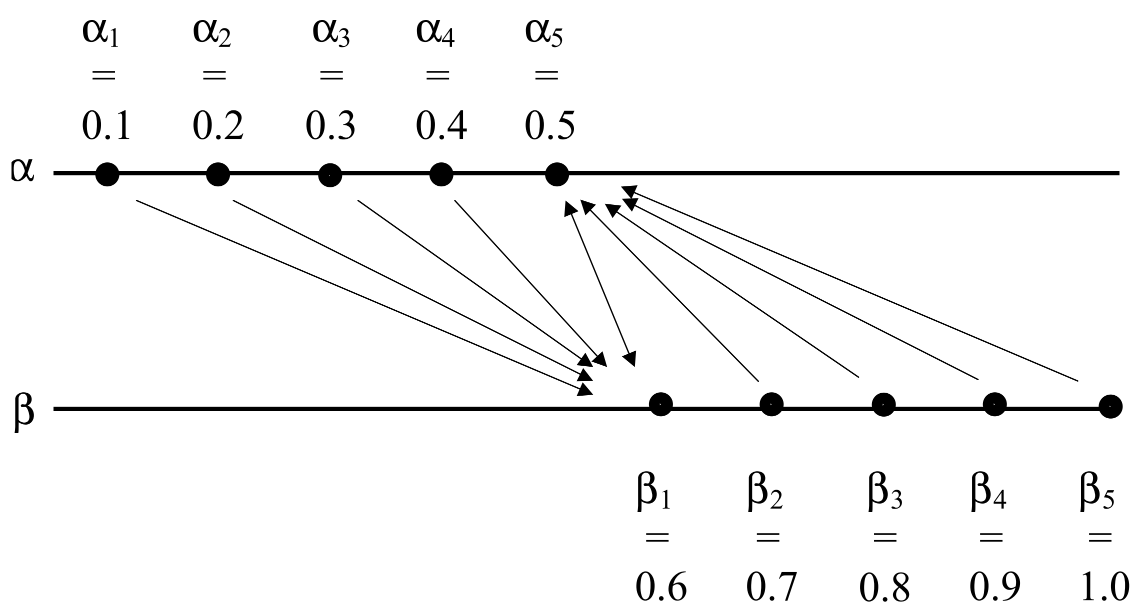

Figure 8: Matching with negative assortment. Arrows denote preferences and the double arrow indicates a fixed pair.

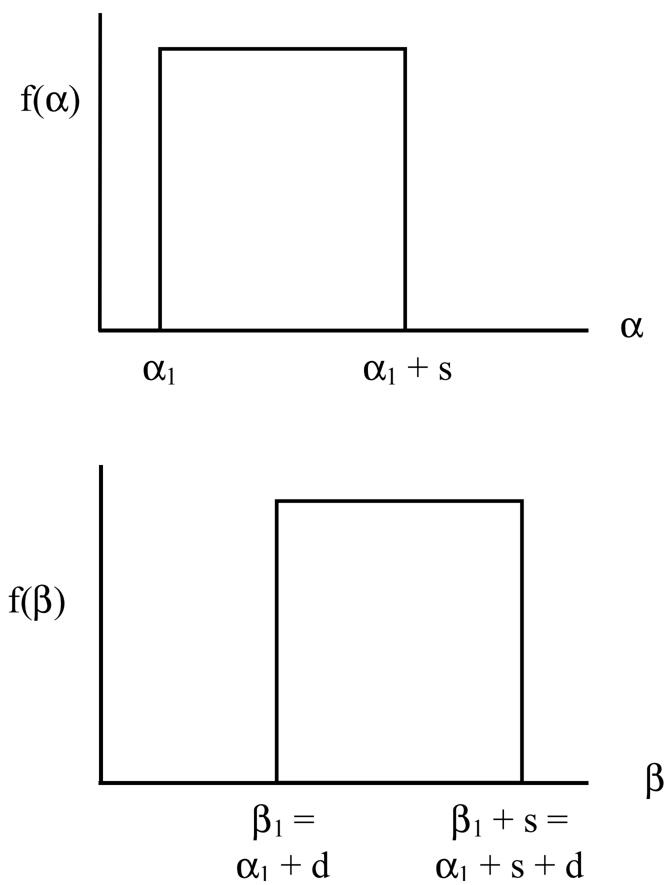

Figure 9: The distribution of women's tastes as a displacement d of the uniform distribution of men's tastes.

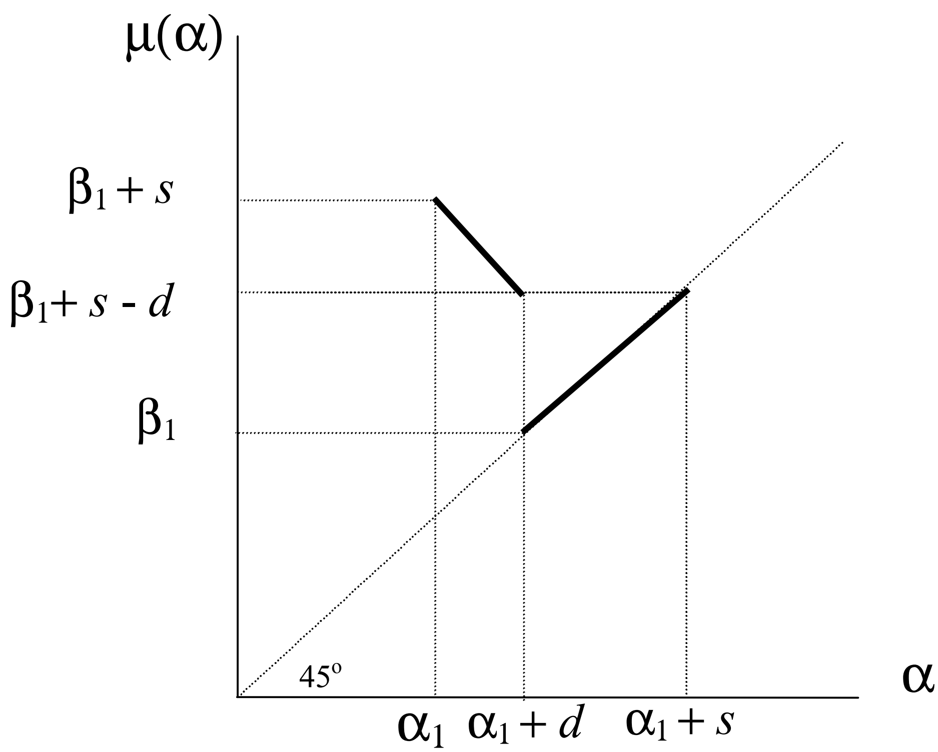

Figure 10: The matching μ when the distribution of women's tastes is a displacement d of men's tastes

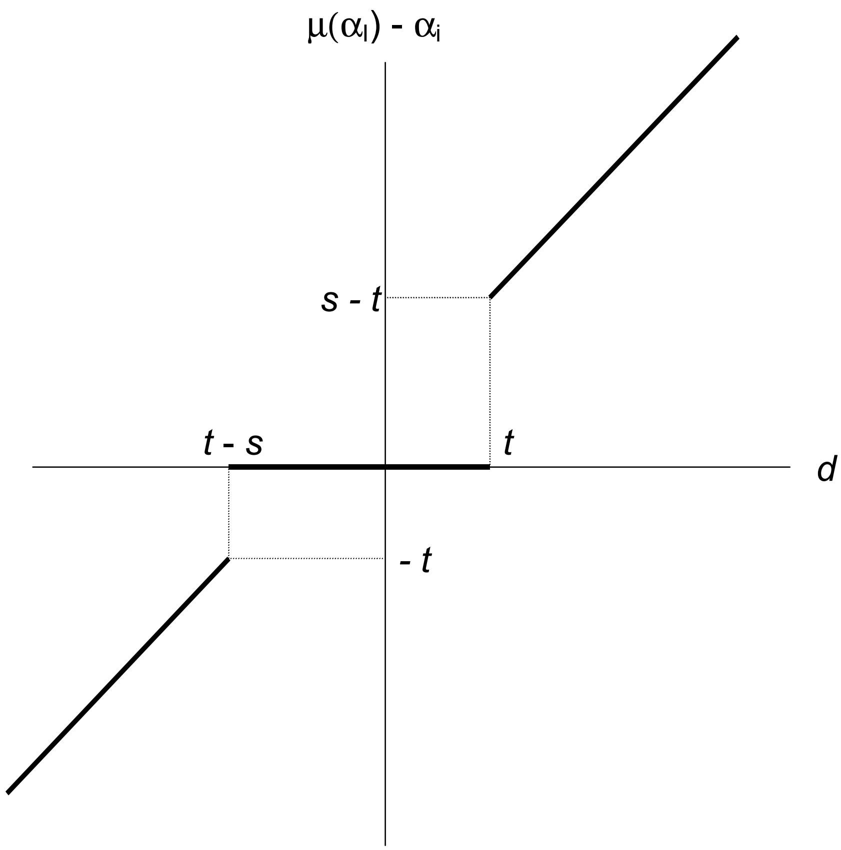

Figure 11: The effect on m_i of changes in the displacement parameter d when $\alpha_i = \alpha_1$ + t.

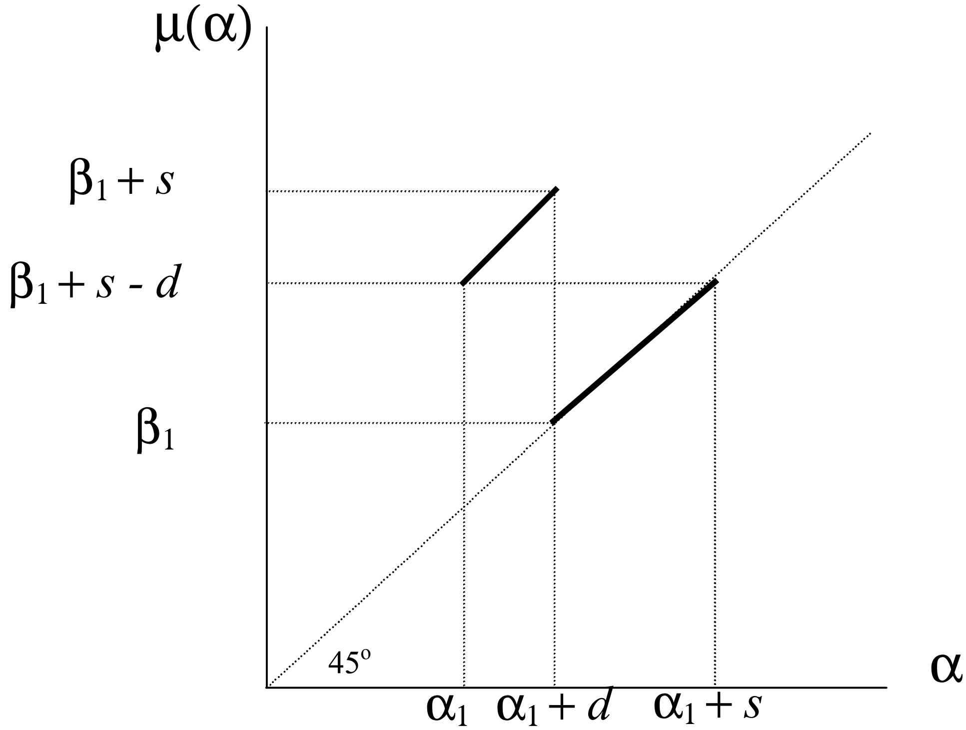

Figure 12: A possible alternative matching when the distribution of women's tastes is a displacement d of men's tastes.